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Incorporating reliability in optimal design of water distribution networks—review and new concepts

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Much of the effort in optimal design of water distribution networks (WDNs) has focussed so far on minimizing cost alone, with little emphasis on reliability or on investigating the tradeoff between cost and reliability. This is a consequence of the difficulty in defining reliability measures which are meaningful and appropriate, while still of a form which can be incorporated directly into optimization models. This paper will deal with these issues. It contains three parts: (1) conceptual discussion of reliability definitions from different points of view (system versus consumers), (2) a literature survey of existing techniques to incorporate reliability in the optimal design of WDNs, and (3) a new concept for explicitly including reliability in the optimal design of WDNs.

INTRODUCTION

Minimum cost and reliability are important considerations in the design of water supply systems. In this paper we concentrate on water distribution networks (WDNs), review past work on reliability, and propose some new concepts and methods which should improve the way in which reliability is incorporated directly into models for optimizing the design of networks.

It is a well-known fact that the minimum cost network for a single loading condition is a tree,[1] unless a constraint is added on minimum diameters or minimum flows. Still, WDNs for urban areas are designed as looped systems. The rationale is that there should be at least two paths from the sources to each demand node, so that in the case where one fails the other remains intact. There are other reasons for looping networks, primarily avoidance of no-flow segments, but reliability is always quoted as a major factor in justifying the extra cost of looping the system.

If reliability is to be incorporated into models for optimal design of WDNs, we must first define reliability, and make certain that the definition is such that it can be accommodated in an efficient optimization algorithm. Previous work, which will be reviewed in the next section, has frequently used measures of reliability defined on the network itself—such as connectivity and reachability[2]—and did not consider the perspective of the consumers who are affected by the system's failure to meet demand quantities or pressures.

A good measure of reliability must be defined from the point of view of the consumer. It should reflect the drop in level of service, or, even better, the damage incurred to the consumer, when failure occurs. The probability of failure of one or more components may be known, or assessed, but a considerable computational effort is required to determine the residual level of service once failures occur. Furthermore, the number of possible failure modes is very large, so it is not feasible to compute the residual capacity for all failure modes in advance. The approach we have taken, which will be described later, is to incorporate into the design optimization model constraints which guarantee that, when a component fails, the residual system can meet the demands with adequate pressures.

The purpose of this paper is to review previous work on WDN reliability, classify the methods for analyzing a network's reliability and for optimizing the

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network with reliability considerations, and then to propose some new concepts that define reliability and incorporate them directly into models for optimizing the overall cost of a network. Tradeoff between cost and reliability can then be evaluated.

Nowadays, the quality of the water supplied must also be considered when measuring reliability of service. However, inclusion of water quality in considerations of reliability goes beyond the scope of the present paper, and is the subject of ongoing work.[3]

LITERATURE SURVEY

This section describes and comments on approaches for assessing the reliability of existing WDNs and for incorporating reliability measures in optimal design of new networks. It consists of five parts:

1. Techniques for assessing the reliability of an existing network.
2. Techniques that link simulation with a general optimization algorithm.
3. Methods based on decomposition.
4. Chance constrained approach.
5. Methods relying on Graph Theory procedures.

Techniques for assessing the reliability of an existing network

As stated in previous sections, it is difficult to define reliability in distribution systems, and even more difficult to include it explicitly in optimization models. A necessary first step is to be able to assess the reliability of an existing system.

Simulation has been a way to deal with some of these difficulties. Wagner *et al.*[2] used analytical methods to assess the reliability of a water network. Reachability and connectivity have been used as the reliability measures. These criteria, taken from Graph Theory, are concerned with the probability of nodes being connected. They can be used primarily to identify parts of the system with low link reliability. Analytical methods have also been used by Hobbs and Biem[4] and Biem and Hobbs,[5] and by Shamir and Howard,[6,7] for bulk water supply systems. Hobbs and Biem[4] and Biem and Hobbs[5] used frequency-duration analysis and a Markov model to assess reliability. Both methods were compared with results obtained by Monte Carlo simulation. Shamir and Howard[6,7] based their analysis on shortfalls factors.

Stochastic simulation for reliability evaluation has been used by Damelin *et al.*,[8] Wagner *et al.*[9] and Bao and Mays.[10] Damelin *et al.*[8] simulated the operation of a water supply system reliability. Wagner *et al.*[9] used simulation to supplement their analytical analysis of network reliability. Bao and Mays[10] defined reliability as the probability that the system meets consumers' demands for flow and pressure. They used a Monte Carlo simulation model built of a random-number generator and a hydraulic steady state simulator.

Entropy has been used as a surrogate for reliability by Awumah *et al.*[11,12] and by T. T. Tanyimboh and A. B. Templeman (1991, pers. comm.). It is still an open question what entropy means in terms of reliability—how it should be incorporated and measured, and what is the tradeoff between entropy and cost. Shamsi[13] and Quimpo and Shamsi[14] proposed a model based on time-varying connectivity concepts to locate areas with low link reliability. This model is then used as a tool for the priorization of maintenance strategies.

Simulation alone cannot, however, guarantee that an optimal solution is found, or even determine how good is the solution reached at the end of the simulation analysis. It is, therefore, important to develop methods that incorporate reliability measures directly into optimization models. It is expected that this can be done only at the expense of certain simplification, and simulation must, therefore, be used to examine solutions reached by optimization.

Techniques that link simulation with a general optimization algorithm

Techniques that combine simulation and optimization divide the overall algorithm into two levels. At the lower level a given system is analyzed for reliability and cost, while in the upper level the system is modified according to the information provided by successive runs of the simulation. The upper level is a general-purpose optimization package, such as MINOS[15] or GRG2.[16] The optimization algorithm uses values of the objective function generated in successive runs of the simulation, and information on constraint violations, to determine the next solution to be tested. This approach has been used by Su *et al.*,[17] Duan *et al.*[18] and Cullinane *et al.*[19]

Su *et al.*[17] developed a model which combines the hydraulic steady-state simulation network solver, KYPIPE,[20] with a reliability model based on a minimum cut set method, for the definition of system and nodal reliabilities. These are linked to an optimization routine, GRG2,[16] which changes at each iteration the values of the decision variables. Reliability is defined as the probability of satisfying consumers' demands, for various possible minimum cut sets. A minimum cut set is defined as the minimum set of system components, which, when they fail, cause system failure.

Duan *et al.*[18] combined KYPIPE, RAPS (reliability analysis of pumping systems)—developed by Duan and Mays[21], GRG2, integer programming and a heuristic rule in an overall optimal planning, design and operation model of a water distribution system. Reliability is calculated using RAPS, which analyses the performance of pumping stations. It uses a continuous-time Markov process, in a frequency and duration analysis framework, for the computation of reliability measures, such as failure probability, cycle time between failures, expected duration of a failure and expected unserved demand. These reliability measures computed by RAPS are set as threshold constraints in the optimization model.

Cullinane *et al.*[19] combined a hydraulic availability measure, defined as the relative time during which demand can be supplied at or above the required consumers' pressure, with KYPIPE and GRG2. Availability appears in the model as constraints with minimum admissible required levels.

The optimization algorithm in such packages does not contain any consideration of the specific model being optimized. This is both the strength and weakness of using it for network optimization. On the one hand, one can depend on a package which is well tested and requires no tailoring to the problem under study. On the other hand, precisely because the optimization method is completely general, it cannot take advantage of the special structure of the model or the specific forms of the functions involved. Furthermore, because no specific features of the model are incorporated into the optimization algorithm, the only criteria for optimality of the solution are those determined from the sequence of values returned by the simulation.

This should not be construed as criticism of using a combination of simulation and optimization. In the absence of a better approach, this one should definitely be used, since it can provide improved solutions. The cost of running such a combined model may be considerable, but as a percentage of the potential savings, this is probably well justified.

Still, the quest must continue for methods that construct and solve directly an optimization model that incorporates reliability in its constraints and/or objective function.

Methods based on decomposition

Under this approach, the LPG method[1] is used as the basic tool for solving the optimal design problem of a water distribution system, with reliability measures embedded as additional constraints. Authors using this scheme are Goulter and Coals,[22]Goulter and Bouchart,[23] Fujiwara and De Silva,[24] Fujiwara and Tung[25] and Bouchart and Goulter.[26]

Goulter and Coals[22] presented two quantitative methods for incorporating reliability measures in the optimal design of a water distribution system. The first considers the probability of isolating a node due to simultaneous failures of all the links incident to that node, and the second minimizes the deviations in reliability of all the pipes connected to a node, with probability of failure of a link given by the Poisson probability distribution. The first concept assumes that failure at a node occurs only when the node is isolated, and the second assumes that each link connected to a node is capable of supplying the entire demand of that node, in the case that all other links incident to that node fail. These two assumptions are usually not fulfilled in reality.

Goulter and Bouchart[23] combined the probability of pipe failure of each link, and the probability of demand exceeding the design values, for a fixed flow pattern throughout the network, into a single heuristic reliability criterion called 'the probability of no node failure.' This reliability measure is set at a minimum required level in the optimization model.

Fujiwara and De Silva[24] proposed a heuristic model, for a single source and a single demand pattern, based on three main stages: (1) solving the optimal design problem of a WDN, given the flows in the links; (2) assessing two reliability measures—the complement of the ratio of the expected minimum total shortfall in flow to total demand, and the maximum flow supplied under a single link failure, by using a linear maximum flow model; and (3) applying a heuristic rule, flow is modified and the new flow distribution is returned to stage (1). This iterative procedure ends when reliability is greater than a minimum required level.

Fujiwara and Tung[25] improved the model of Fujiwara and De Silva[24] by: (1) solving a nonlinear maximum flow model which takes into account, in addition to continuity (Kirchoff law no. 1), the conservation of energy (Kirchoff law no. 2); and (2) improving reliability directly by increasing pipe size, instead of the increase of flow in some of the links.

Bouchart and Goulter[26] used the expected volume of deficit to consumers as the reliability measure of the network, taking into account the fact that frequently shortfalls are connected not just with the failure of system components, but also with the locations of valves throughout the distribution network, which have to be shut down during repair, and thus cause the isolation of parts of the system.

Chance constrained approach

Lansey *et al.*[27] developed a chance constrained model for the inclusion of reliability in the optimal design of water networks. The model takes into account uncertainties, both in consumers' demands for quantities and pressures, and for pipe roughness coefficients, but it does not consider pipe failures. The model is solved using GRG2.

Methods relying on Graph Theory procedures

Graph Theory procedures for assessing network reliability have been used extensively in electrical and communication engineering, but only little for water supply systems. This is probably a result of the nonlinear linkage between graph properties of a water network, and its hydraulic performance. For example, measures from Graph Theory, like reachability and connectivity, which are based on the probability of paths existing between nodes in a network, impose only necessary conditions in a water supply systems for delivering needed water capacities at sufficient pressures; they do not guarantee that the residual network (after failure) can indeed meet demands. Authors using this approach are Jacobs and Goulter,[28] Kessler *et al.*[29] and Ormsbee and Kessler.[30]

Jacobs and Goulter[28] presented an integer goal programming framework, aimed at maximizing the regularity of a water network layout, where a network is said to be 'regular' if all its nodes have the same number of arcs connected to it. The model does not consider network costs or the fulfillment of hydraulic laws. It is only concerned with finding the most regular network, given the number of edges and nodes, since regularity has been shown by Jacobs and Goulter[31] to result in a less invulnerable network.

Kessler *et al.*[29] and Ormsbee and Kessler[30] presented a model for explicitly incorporating reliability in the optimal design model of a water network. This method involves two major stages. The first stage requires splitting a given 'two node connected' network, fed by a single source, into two disjoint spanning trees, using an algorithm developed by Itai and Rodeh.[32] These two trees assure a path from the source to each node in case of a single one-link failure. The second stage involves solving an optimal design problem that considers as constraints the simultaneous operation of the two disjoint spanning trees. This assures a specified level of service (e.g. adequate flows, at or above a minimum required pressure) for each consumer throughout the system in the case of a random single-link failure. The method is limited to networks that are supplied by only one source, and rely on heuristics, or external design considerations, in choosing the two disjoint spanning trees.

A NEW CONCEPT FOR INCLUSION OF RELIABILITY IN OPTIMAL DESIGN OF WDNS

Reliability analysis of a WDN should consider two types of failures: (1) failures of system components such as pipes, pumps, valves, etc.; and (2) failures in meeting consumers' demands, which are usually for flow and pressure, but may also be for quality in multiquality distribution networks. These two types of failures should not be considered separately for reliability assessment of a water distribution network, as they are strongly connected. The problem of combining these two types of failures into meaningful and yet computable measures is not trivial. The literature survey has highlighted some of the methods adopted over the last few years for dealing with this problem.

Our approach for definition and inclusion of reliability in the optimal design problem of a water distribution system, with concern to the problems mentioned above, is the following.

1. We define a surrogate to the reliability of a water distribution system as the maximum damage to consumers, until repair, in the case of a single component (pipe, pump, valve) failure. This definition takes into account three basic conceptual issues. (a) The consumers throughout a water distribution system are usually not of the same type, therefore, not meeting their requirements will cause different consequences. For example, the result of disruption in water supply to a hospital is not the same as not filling a swimming pool. We assume that it is possible to assign each consumer a damage function, which defines the damages caused, when user requirements are not met. (b) This surrogate measure to reliability is based upon a deterministic approach. We seek to assure a minimum level of system performance when one hydraulic element fails. (c) The consideration of the failure of only one hydraulic element is justified because the probability of failure of a single hydraulic component is low, and if the probability independence is assumed, then simultaneous failure of more than one element at the same time is small.

2. Retaining a minimum level of service to consumers, in case of a single random component failure, is embedded in the optimal design of a WDN, through the following stages:

(a) In the first stage we formulate an optimal design problem of a water distribution system in which the objective function is minimization of the sum of system components' cost and the damages to consumers, under different demand loading conditions. The constraints are on continuity of flow and energy (i.e. Kirchoff laws nos 1 and 2, respectively); pressure head at consumption nodes; length of each pipeline (this is a consequence of the mathematical formulation of the model, in which each pipeline is expressed as the sum of its segment lengths, where the cost of each segment is associated with a specific commercial pipe diameter;)[1] and power of pumping stations. The decision variables are the vector of flows in all pipes for each loading condition, pumping heads for each pumping station and loading condition, the pipeline's segment lengths, and the maximum power of each pumping station. This formulation resembles a goal programming framework, in which we seek to minimize the sum of total cost and deviations from user requirements.

The method of solution requires one to deal with a highly nonsmooth, nonconvex objective function as an 'outer' problem and a linear 'inner' problem[33] (more details concerning the solution method require a full mathematical formulation, which is beyond the scope of this paper). At this stage we do not consider failure of system components, and assume deterministic user demands.

(b) In the second stage we identify Backup networks for each of the loading conditions. These Backups are subnetworks of the system, which will be responsible for retaining a prescribed level of service when a failure occurs.

(c) In the third stage, the hydraulic laws and consumer demands are formulated separately for each of the Backups. By doing this, this, the level of service required from each of the Backups for each of the loading conditions is explicitly defined.

(d) In the fourth stage the Backups are added to the block of constraints of the optimization model.

The final result of these four stages can be expressed schematically for n loading conditions, as follows:

minimize {system cost + damages to consumers}
subject to:

Loading condition #1

System constraints
Backups constraints

•
•

Loading condition #n

System constraints
Backups constraints

This formulation creates an expanded model which minimizes the total cost plus damages, subject to explicit constraints on residual network performance when components fail.

The method proposed above raises the following issues:

(1) What damage functions should be used, and how do they affect the solvability of the model?
(2) How should the Backups be selected?
(3) What are the ways to examine the tradeoffs between cost and reliability?

Our findings to date are as follows:

1. Since the overall optimal design problem, without inclusion of damage functions, is nonsmooth,[33] one should use linear or at most quadratic functions at each consumer node to model the penalty associated when demands deviate.

(2) For each loading condition two Backups are defined. These can be found by searching for two spanning trees in the system whose distance is maximum; the distance between two trees is defined as the number of edges contained in one tree, but not in the other. If these trees are used, then the system is one level redundant or invulnerable in the case of a one-system component failure. Additional components, if such exist, which do not belong to the spanning trees can be added to either of these two trees, creating the Backups.

An example of a network, based on Walski *et al.*,[34] with two Backups, is shown in Fig. 1. These two Backups have been chosen so that they have a maximum distance between them. The first (called

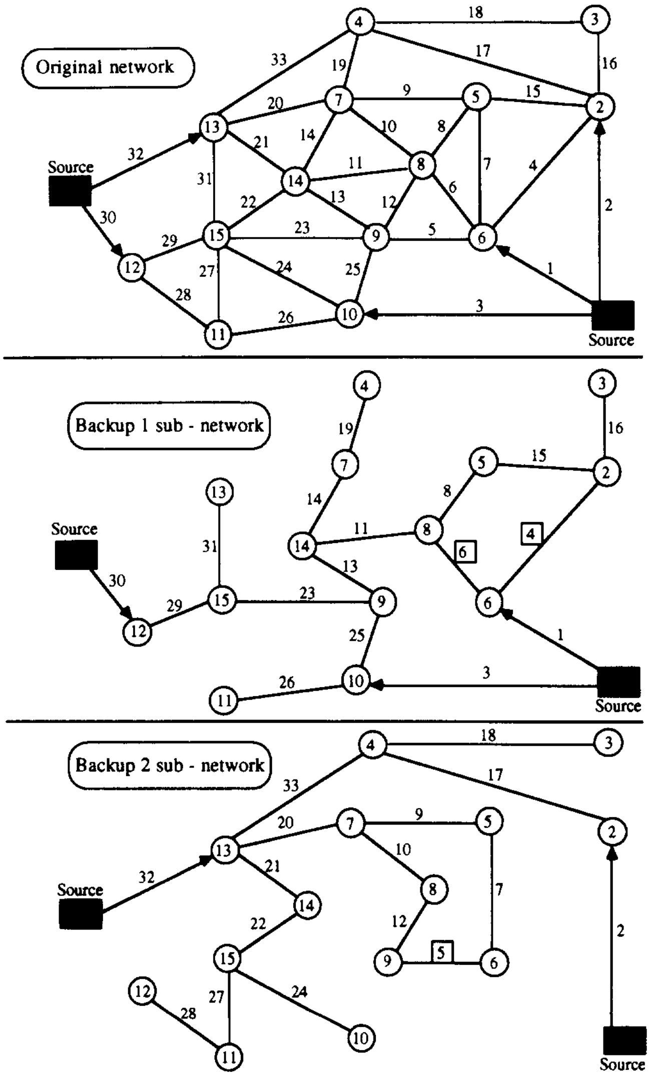

Fig. 1. Network example, showing the original network, Backup 1 and Backup 2.

Backup 1) consists of a spanning tree plus two additional pipes, 4 and 6; the second (called Backup 2) consists of another spanning tree, with the additional pipe 5. The network is one level redundant. For a given network, several pairs of Backups may be generated. The selection of a pair is not trivial. It should probably rely on external heuristics judgment of the designer, or maybe linked through the optimization model to a scanning genetic algorithm based procedure.

3. The tradeoff between cost and reliability may be evaluated by changing the selection of the Backups for given consumers' demands, or by reducing consumers' requirements for a given pair of Backups.

The concept of using Backup subnetworks to cope with the residual performance of water networks in case of failures is not limited to optimal design. It may also be incorporated in the optimal operation of water networks by creating a 'library of Backups' to be used for different failure scenarios.

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