# Implicit Mean-Variance Approach for Optimal Management of a Water Supply System under Uncertainty

Mashor Housh, Ph.D.[1]; Avi Ostfeld, F.ASCE[2]; and Uri Shamir, F.ASCE[3]



**Abstract:** This study addresses the management of a water supply system under uncertainty. Water is taken from sources that include aquifers and desalination plants and conveyed through a distribution system to consumers under constraints of quantity and quality. The replenishment into the aquifers is stochastic, whereas the desalination plants can produce a large and reliable amount, but at a higher cost. The cost is stochastic because it depends on the realization of the replenishment into the aquifer. A new implicit mean-variance approach is developed and applied. It utilizes the advantages of implicit stochastic programming to formulate a small size and easy to solve convex external optimization problem (quadratic objective and linear constraints) that generates the mean-variance tradeoff without the need to solve a large-scale problem. The results are presented as a tradeoff between the expected value versus the standard deviation. At one end of the tradeoff curve, dependence on the aquifer results in low expected cost and higher cost variability. At the other end, when all of the water is taken from desalination, the cost is high with no variability (deterministic). **DOI: 10.1061/(ASCE)WR.1943-5452.0000307.**


**CE Database subject headings:** Water supply; Water distribution systems; Uncertainty principles; Optimization.

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## Introduction

The planning and management of real-life water resources projects are always conducted under uncertainty, such as uncertain demands, flows, yields, costs, and benefits. A common approach is to neglect the uncertainty and replace it with a deterministic estimator. Philbrick and Kitanidis (1999) have shown the limitation of this approach for water resource systems. They present three models for reservoir management and demonstrate that a deterministic formulation does not perform as well as a stochastic formulation. Recently, the consideration of uncertainties has become a standard step in modeling of water resources. Numerous models have been developed for the management optimization of reservoirs, in which inflows, net evaporation, hydrologic, and economic parameters and system demands are considered to be random variables (Labadie 2004).

In stochastic programming (SP) (Birge and Louveaux 1997), the uncertain parameters are modeled as random variables with prescribed probability density functions (PDFs). Generally, there are two types of SP: implicit and explicit stochastic programming (ISP and ESP).

To understand the concepts of the ISP and the ESP, one must first consider the following unconstrained optimization problem: $f(x,R) \to \min$, where $x$ is the vector of decision variables and $R$ is the stochastic process. A common tool to present the stochastic process is that of scenarios (Dupačová et al. 2000) that are particular representations of how the process may be realized. The stochastic process, $R$, is approximated by a finite number, $k_f$, of scenarios represented by the scenario set $\Omega$. Each scenario $R^k \in \Omega \ \forall\, k = 1, \ldots, k_f$ has probability of $p^k$, where

$$\sum_{k=1}^{k_f} p^k = 1$$

The ISP approach implies that decisions can be made only after the value of $R$ becomes known. The stochastic aspects of the problem are implicitly included and each scenario is used as input for a deterministic optimization problem. Thus, the optimization problem depends on $R^k$ as a fixed parameter: $f(x,R^k) \to \min \ \forall\, k$; after solving this optimization problem for each scenario $k$, the decisions corresponding to each scenario $x^k$ are obtained. These decisions construct the objective value set $\min_{x^k} f(x^k,R^k) \in \Omega_f \ \ \forall\, k$ and the optimal solution set $\arg\min_{x^k} f(x^k,R^k) \in \Omega_x \ \forall\, k$. The set of optimal values, $\Omega_f$, and the optimal set of solutions, $\Omega_x$, are treated as stochastic elements and their probabilistic behavior is used to derive decision rules for implementation.

ISP solves multiple deterministic problems; hence, an efficient deterministic module is required. Hiew et al. (1989) and Crawley and Dandy (1993) applied the ISP approach for the optimal management of a linear multireservoir model with uncertain inflows. In many applications of the management of multireservoirs, a linear model is not applicable because of the existence of nonlinearities in the objective functions and constraints. In these cases, various nonlinear algorithms are utilized. Barros et al. (2003) compared the performance of successive quadratic programming (SQP) and

[1]Faculty of Civil and Environmental Engineering, Technion—Israel Institute of Technology, Haifa 32000, Israel; and Postdoctoral Associate, Univ. of Illinois at Urbana-Champaign, 2524 Hydrosystems Laboratory, 301 N. Mathews Ave., Urbana, IL 61801. E-mail: mashor@illinois.edu; mashor@tx.technion.ac.il

[2]Associate Professor, Faculty of Civil and Environmental Engineering, Technion—Israel Institute of Technology, Haifa 32000, Israel (corresponding author). E-mail: ostfeld@tx.technion.ac.il

[3]Emeritus Professor, Faculty of Civil and Environmental Engineering, Technion—Israel Institute of Technology, Haifa 32000, Israel. E-mail: shamir@tx.technion.ac.il

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successive linear programming (SLP) on a large scale Brazilian hydropower system. Peng and Buras (2000) applied the general reduced gradient (GRG) to a nonlinear model. Cai et al. (2001) presented a framework for solving a large-scale nonlinear water management model by a genetic algorithm (GA) that can be embedded within the ISP approach.

ESP incorporates the PDF of the stochastic process into the optimization problem. Hence, in an ESP formulation, each possible decision results in a stochastic objective function $f(x,R)$ with a given PDF. Because it is not possible to minimize a PDF of $f(x,R)$, one must apply a statistical operator to the PDF (e.g., expectation, variance, or quantile) before the optimization is performed.

## Implicit Mean-Variance Motivation

The primary advantage of ISP over ESP is the ability to solve each scenario separately, as opposed to ESP, in which all scenarios are simultaneously optimized.

One of the more common approaches is to optimize the expected value of the objective (Vasiliadis and Karamouz 1994; Seifi and Hipel 2001; Kracman et al. 2006). However, optimizing the expected value does not hedge against risk. Decision makers are and should be interested in optimizing risk measures jointly with, or sometimes instead of, the expected value.

Following the scenario-based robust optimization (RO) (Mulvey et al. 1995), Watkins and McKinney (1997) incorporated the cost variance as a measure of risk within an ESP framework. In this framework, the objective function was defined as a weighted sum of the expected value and standard deviation of the cost (mean variance). In financial applications, this approach is considered traditional, in which the variance of the outcomes serves as a measure of risk (Markowitz 1959). Watkins and McKinney (1997) demonstrated the advantages of using the mean-variance framework as a measure of risk in the optimization model.

When other objectives than the expected value are optimized (e.g., mean variance), one cannot solve the ISP formulation by separately solving each scenario (as described in the following). Hence, the primary advantage of ISP over ESP is lost. Creating the mean-variance tradeoff by using classical ISP requires solving many large-scale optimization problems, which makes this approach impractical.

In the current study, the implicit mean-variance (IMV) approach is developed and demonstrated. In IMV, the advantage of ISP (the ability to separately solve each scenario) is utilized to formulate a small convex external optimization problem to create the mean-variance tradeoff without the need to solve large-scale optimization problems.

The primary disadvantage of the implicit methods (classical ISP and IMV) is that the resulting optimal operational policies are unique to each scenario. Because the solution is different for each scenario and the true scenario is not known, one cannot decide which solution to implement. On the other hand, the ESP solution is not scenario dependent; i.e., the optimal solution is inherently implementable. However, computational difficulties in ESPs (especially when optimizing a nonlinear model) have led modelers of large systems to rely on ISP techniques (Labadie 2004). Despite this drawback, ISPs remain applicable tools for the analysis of complex systems. The solution obtained from the ISPs can be used to derive an implementable decision (not a scenario dependent decision). Developing implementable decisions from the output of ISP has been the subject of several studies. Hiew et al. (1989) used multiple regression analysis to derive an implementable decision for a multireservoir operation problem from an ISP solution. Raman and Chandramouli (1996) used artificial neural networks and Saad et al. (1992) employed principal component analysis to derive the implantable decision from the ISP solution.

On the mean-variance tradeoff created by classical ISP or by the proposed IMV method, each point is comprised of a set of optimal decision vectors (one for each scenario). These decision vectors serve as intermediate information toward deriving an implementable decision, as discussed previously.

A noteworthy property of IMV is that it can elucidate the shape of the entire tradeoff curve (which is useful in itself and is required to settle on a preferred tradeoff) by utilizing the minimum expected cost solution and a small sized convex external optimization problem. Because most of the studies on ISP considered the minimum expected cost solution (as shown in previous examples), this means that with the addition of a little computation (solving the external problems), these studies can be extended by the IMV approach to produce the entire mean-variance tradeoff for the decision maker.

The mean-variance tradeoff is comprised of all noninferior solutions; i.e., the solutions with the least value of the variance for the specified value of the expectation. Each point thus generated has its corresponding decision vectors. The decision maker is faced with selecting a point (or, even better, a range for sensitivity analysis) that has their preferred balance between the two objectives; at the same time, they must examine the corresponding decision vectors and the implantable decision derived from them to ascertain whether they are willing to accept them or require further analysis.

The rest of this paper is organized as follows: the next section contains the formulation of a deterministic model for optimizing the operation of a water supply system (WSS) with its objective function and constraints. Next, the ISP for the deterministic WSS is developed, followed by presentation of the IMV approach. In the last section, IMV is applied to the stochastic version of the WSS model.

## Deterministic Formulation of the WSS Management Model

This section presents the deterministic formulation of a seasonal multiyear model for the management of water flow and salinity in water supply systems. Fig. 1 depicts a small hypothetical WSS (two source nodes, four junction nodes, two demand nodes, and nine links) that were used in developing and testing the model, whereas the system shown in Fig. 2 is the central part of the Israeli National Water Supply System, to which the method has also been applied.

Water is taken from sources (aquifers, reservoirs, and desalination plants) and conveyed through a distribution system to consumers. The operation is subject to constraints on water levels and water qualities in the aquifers, the capacities of the pumping and distribution systems, the production capacity of the desalination plants, and a limit on salinity removal ratio of the plants. The objective is to minimize the total present value of the operation cost, which includes the cost of desalination, pumping, delivery, and an extraction levy from the aquifers. The objective function and some of the constraints are nonlinear, leading to a nonlinear optimization problem.

The model does not include detailed hydraulics (i.e., the energy equations or Kirchoff's second law); it is implicitly assumed that the short-term hydraulic operation within the season is feasible with the seasonal quantities that are prescribed by the model. Still, the cost of conveyance is related to the hydraulic characteristics of the network links.

The network representation in the model can be classified according to the physical laws that are explicitly considered in the model constraints (Ostfeld and Shamir 1993; Cohen et al. 2000).



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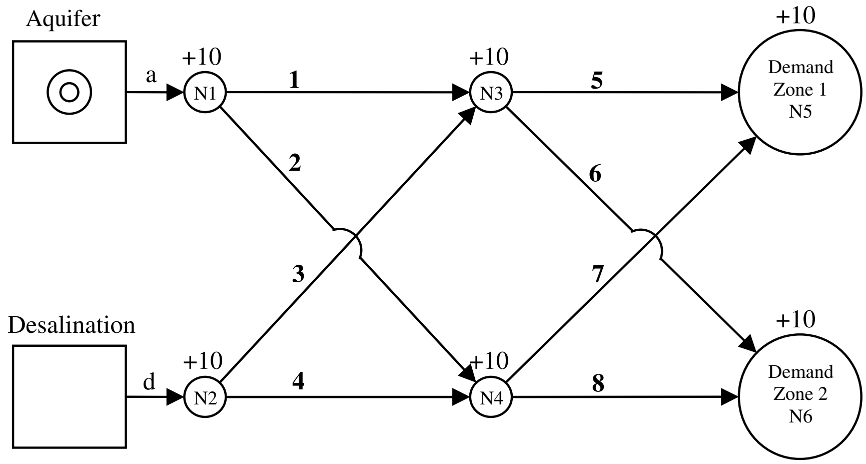


**Fig. 1.** Simple example of a WSS

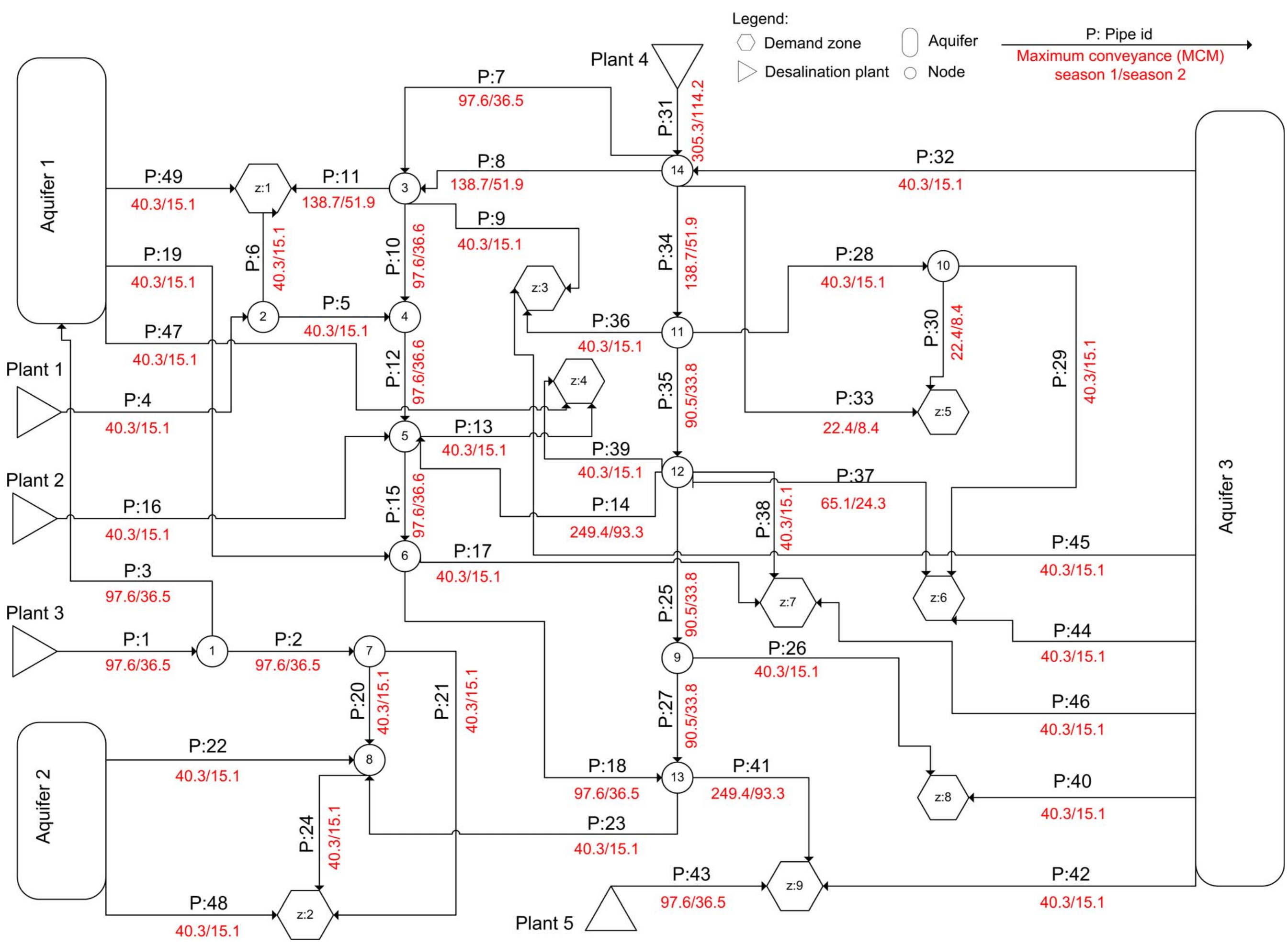


**Fig. 2.** Layout of WSS and conveyance capacity of the links in Seasons 1 and 2 (mcm, rounded to one decimal place, to simplify the presentation)

Ostfeld and Shamir (1993) defined flow-quality models as those that consider the balance of the flows and mass of quality parameters, but without explicit inclusion of the hydraulics. According to this definition, the developed model is a flow-quality model. Several flow-quality models have been developed in the last decade, (Tu et al. 2005; Yates et al. 2005; Zaide 2006). The model presented here differs by the spatial and temporal resolution and by the inclusion of more hydraulic characteristics of the network. The study by Housh et al. (2012) provides full details.

In the model detailed in the following, $p$, $d$, $a$, $z$, $S$, and $Y$ denote pipe, desalination plant, aquifer, demand zone, season, and year, respectively.



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### Objective Function

The objective of the multiyear model is to minimize the present value of the total cost:

$$\cos t = \sum_{Y} \frac{\sum_{S}(\sum_{d} CD_{d}^{S,Y} + \sum_{a} CE_{a}^{S,Y} + \sum_{p} CC_{p}^{S,Y})}{(1+i)^{Y}} \quad (1)$$

where $\cos t$ = total operation cost (\$); $CD_{d}^{S,Y}$ = desalination cost (\$/season); $CE_{a}^{S,Y}$ = extraction levy (\$/season); $CC_{p}^{S,Y}$ = conveyance cost (\$/season); $i$ = annual discount rate (dimensionless).

The desalination cost, $CD$, is comprised of a constant price per unit of desalinated water plus a variable cost that depends on the salinity removal ratio:

$$CD_{d}^{S,Y} = \left(\alpha_{d} + \frac{1}{(100 - RR_{d}^{S,Y})^{\beta_{d}}}\right) \times Q_{d}^{S,Y} \quad (2)$$

where $\alpha_{d}$ = parameter for the desalination cost function (\$/m$^3$); $Q_{d}^{S,Y}$ = desalination amount (m$^3$/season); $RR_{d}^{S,Y}$ = removal ratio (%); and $\beta_{d}$ = parameter for the desalination cost function (dimensionless).

The extraction levy from the natural sources, $CE$, depends on the water level in the source:

$$CE_{a}^{S,Y} = \overline{CE}_{a}^{S,Y} \times Q_{a}^{S,Y} \quad (3)$$

$$\overline{CE}_{a}^{S,Y} = \left(1 - \frac{h - h_{\min}}{h_{\max} - h_{\min}}\right)_{a}^{S,Y} \times (\overline{CE}^{\max})_{a}^{S,Y} \quad (4)$$

where $\overline{CE}_{a}^{S,Y}$ = specific levy (\$/m$^3$); $Q_{a}^{S,Y}$ = pumping amount from the aquifer (m$^3$/season); $h_{a}^{S,Y}$ = water level (m); $(h_{\min})_{a}^{S,Y}$ = minimum allowed water level (m); $(h_{\max})_{a}^{S,Y}$ = maximum allowed water level (m); $(\overline{CE}^{\max})_{a}^{S,Y}$ = maximum levy (\$/m$^3$).

The conveyance cost, $CC$, depends on the head loss in a link given by the Hazen-Williams equation, with the average flow of the season and the elevation difference between its two ends:

$$CC_{p}^{S,Y} = \frac{X_{p}^{S,Y} \times \left(\frac{Q_{p}^{S,Y}}{w^{S,Y}}\right)}{200} \times 0.736 \times w^{S,Y} \times \mathrm{KWHC}^{S,Y} \quad (5)$$

$$X_{p}^{S,Y} = \Delta Z_{p} + \Delta Hf_{p}^{S,Y} \quad (6)$$

$$\Delta Hf_{p}^{S,Y} = 1.526 \times 10^{7} \times \left(\frac{Q_{p}^{S,Y}}{w^{S,Y} c_{p}^{Y}}\right)^{1.852} \times D_{p}^{-4.87} \times L_{p} \quad (7)$$

where $X_{p}^{S,Y}$ = head loss (m); $Q_{p}^{S,Y}$ = total discharge for the season (m$^3$/season); $w^{S,Y}$ = number of pumping hours per season (h/season); $\mathrm{KWHC}^{S,Y}$ = pumping cost (\$kW · h); $\Delta Z_{p}$ = elevation difference (m) between the intake and discharge ends of the pipe; $\Delta Hf_{p}^{S,Y}$ = energy head loss (m); $c_{p}^{Y}$ = Hazen-Williams head loss coefficient (dimensionless); $D_{p}$ = link diameter (cm); $L_{p}$ = link length (km).

### Constraints

For the natural sources, aquifers and reservoirs, the water and salinity mass balance constraints are:

$$R_{a}^{S,Y} - Q_{a}^{S,Y} = SA_{a} \times (h_{a}^{S,Y} - h_{a}^{(S,Y)-1}) \quad (8)$$

$$(C_{R})_{a}^{S,Y} \times R_{a}^{S,Y} - C_{a}^{(S,Y)-1} \times Q_{a}^{S,Y} = SA_{a} \times [C_{a}^{S,Y} \times h_{a}^{S,Y} - C_{a}^{(S,Y)-1} \times h_{a}^{(S,Y)-1}] \quad (9)$$

where $R_{a}^{S,Y}$ = recharge (m$^3$); $SA_{a}$ = storativity multiplied by area (m$^2$); $h_{a}^{S,Y}$ = water level (m); $C_{a}^{S,Y}$ = salinity (mg/L Cl$^-$); $h_{a}^{(S,Y)-1}$ = water level in the previous season (m); $C_{a}^{(S,Y)-1}$ = salinity in the previous season (mg/L Cl$^-$); $(C_{R})_{a}^{S,Y}$ = salinity of the recharge water (mg/L Cl$^-$).

The salinity of the desalinated water is

$$C_{d}^{S,Y} = C_{\text{sea}} \times \left(\frac{100 - RR_{d}^{S,Y}}{100}\right) \quad (10)$$

where $C_{d}^{S,Y}$ = salinity of desalinated water (mg/L Cl$^-$); $C_{\text{sea}}$ = salinity of sea water (27,000 mg/L Cl$^-$); $RR_{d}^{S,Y}$ = removal ratio (%).

For each season $S$ in year $Y$, the following linear equation system ensures water conservation at the network nodes:

$$A \times Q = b \quad (11)$$

where $A$ = junction node connectivity matrix of the directed graph that represents the topology of the network; $Q = [Q_{\text{source}}, Q_{\text{pipes}}]^{T}$; $b = [0, Q_{\text{demand}}]^{T}$; $Q_{\text{Source}}$ = vector of discharges leaving source nodes; $Q_{\text{pipes}}$ = vector of discharges leaving intermediate nodes; $Q_{\text{demand}}$ = vector of outgoing discharges at demand nodes. For example, the water supply network shown in Fig. 1 has two source nodes, four intermediate nodes, and two demand nodes. The junction node connectivity matrix for this network is $A \in \mathbf{R}^{6\times10}$. The vectors $Q$ and $b$ are $Q = [Q_{a}, Q_{d}, Q_{1}, \ldots, Q_{8}]^{T}$ and $b = [0, \ldots, 0, Q_{z=1}, Q_{z=2}]^{T}$.

Salt mass conservation at network nodes is satisfied by the following equation set:

$$\begin{aligned} & A^{0} \times D_{Q} \times C^{0} = 0, \qquad A^{0} \in \mathbf{R}^{N_{2}\times(M+n_{3})} \\ & A^{0} = \left(A \middle| \begin{matrix} 0 \\ -I \end{matrix}\right) \\ & C^{0} = [C_{\text{source}}, C_{\text{pipes}}, C_{\text{demand}}]^{T} \\ & D_{Q} \in \mathbf{R}^{(M+n_{3})\times(M+n_{3})} \text{ diagonal matrix} \\ & D_{Q} = \mathbf{diag}([Q_{\text{source}}, Q_{\text{pipes}}, Q_{\text{demand}}]) \end{aligned} \quad (12)$$

where $C_{\text{source}}$ = salinity leaving source nodes; $C_{\text{pipes}}$ = salinity in the links leaving intermediate nodes; $C_{\text{demand}}$ = salinity supplied at demand nodes; $n_{3}$ = number of demand nodes; $I \in \mathbf{R}^{n_{3}\times n_{3}}$ = identity matrix. For the network in Fig. 1, the matrix $A^{0} \in \mathbf{R}^{6\times12}$, the matrix $D_{Q} \in \mathbf{R}^{12\times12}$, and the vector $C^{0}$ are defined as $C^{0} = [C_{a}, C_{d}, C_{1}, \ldots, C_{8}, C_{z=1}, C_{z=2}]^{T}$.

The model assumes total mixing at network nodes, so the salinities in all links leaving a node are equal. This dilution condition is given by the following linear equation system:

$$B^{0} \times C^{0} = 0 \quad (13)$$

Each row of $B^{0}$ indicates equal salinity for two outgoing edges that share the same inflow node, i.e., each row has only two nonzero elements $+1$ and $-1$; when three links leave the same node, there are two rows, each with two nonzero elements, $+1$ and $-1$. For the network in Fig. 1, $B^{0} \in \mathbf{R}^{4\times12}$.

### Operation bounds

Bounds are listed in Eq. (14) on flow, salinity, and state variables, reflecting both physical and operational limits:



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$$
\begin{aligned}
&(h_{\min})_a^{S,Y} \le h_a^{S,Y} \le (h_{\max})_a^{S,Y}; \qquad && (C_{\min})_a^{S,Y} \le C_a^{S,Y} \le (C_{\max})_a^{S,Y} \\
&0 \le Q_p^{S,Y} \le (Q_{\max})_p^{S,Y}; && 0 \le Q_a^{S,Y} \le (Q_{\max})_a^{S,Y} \\
&(Q_{\min})_d^{S,Y} \le Q_d^{S,Y} \le (Q_{\max})_d^{S,Y}; && (RR_{\min})_d^{S,Y} \le RR_d^{S,Y} \le (RR_{\max})_d^{S,Y} \\
&(C_{\min})_z^{S,Y} \le C_z^{S,Y} \le (C_{\max})_z^{S,Y}
\end{aligned} \tag{14}
$$

where water quantities are in $m^3$/season, salinities are in mg/L $Cl^-$, and elevations are in m; $()^{\min}$, $()^{\max}$ = lower and upper bounds, respectively.

The preceding deterministic model was developed in the study by Housh (2011). Sensitivity analysis was performed to check its performance and behavior under various conditions. The sensitivity analysis results showed the rational behavior of the model under changes of parameters and initial conditions. The study by Housh (2011) provides more details on the deterministic model.

## Implicit Stochastic Programming

This section presents classical ISP and its application for the model in the previous section.

### Efficient Deterministic Module

The efficient deterministic model (EDM) described in the previous sections results in a general nonlinear optimization problem of the form

$$
\begin{aligned}
&\min_x F(x,R) \\
&\text{Subject to:} \\
&g_i(x,R) = 0 \quad \forall\ i = 1,\ldots,m_i \\
&g_j(x,R) \le 0 \quad \forall\ j = 1,\ldots,m_j
\end{aligned} \tag{15}
$$

where $x$ = vector of decision variables that include the flow and the salinity variables in the network ($Q_{\text{pipes}}$, $C_{\text{pipes}}$), the aquifer variables ($Q_a$, $C_a$, $h_a$), and the desalination variables ($Q_d$, $C_d$, $RR_d$) for each season of all years; $R$ = multiyear recharge vector; $F$, $g_i\ \forall\, i$, $g_j\ \forall\, j$ include linear and nonlinear functions.

The general nonlinear optimization problem, as defined in Eq. (15), can be solved with one of the existing general nonlinear programming solvers, such as SQP (Fletcher 1985) or an interior point algorithm (Waltz et al. 2006; Byrd et al. 2000).

A set of mathematical strategies was developed to increase the computational efficiency of the optimization: (1) reduction of the model size, (2) concise representation of the variables and constraints, and (3) an efficient finite-difference scheme entitled the time-chained-method (TCM) for computing the derivatives required by the optimization algorithm. The TCM utilizes the multistage structure of the model to reduce the computational time from $O(T_f^2)$ to $O(T_f)$, where $T_f$ is the number of years (Housh et al. 2012). A sensitivity analysis for the WSS deterministic model was performed to check its performance and behavior under various conditions. The sensitivity analysis results showed rational behavior of the model under changes of parameters and initial conditions, as shown in the work by Housh (2011).

These mathematical strategies substantially improve the solvability and efficiency of the optimization problem, a critical property when the model is run multiple times as a deterministic module of the ISP approach.

The detailed description of these strategies exceeds the scope of this paper. The following refers to the solution of the optimization problem defined in Eq. (15), with a predetermined fixed vector, $R$, as an EDM. The EDM was programmed in *MATLAB* and utilizes the interior-point algorithm with conjugate gradient of the *MATLAB* FMINCON nonlinear optimization suite.

### Incorporation of Uncertainty

There is considerable uncertainty in the recharge vector, $R$, due to its dependency upon climate variability (Ajami et al. 2008), seasonal effects, and climate change (Grantz et al. 2007). In this study, the uncertainty in the recharge is considered, although the authors recognize there is often significant uncertainty in other variables, such as demands and desalination costs. Once the recharge is modeled as a stochastic process, scenario based SP can be applied for the WSS model.

The scenario-based SP assumes that the distribution of the stochastic process $R$ is a finite discrete probability space; that is, the particular representation of how the process may materialize. The stochastic process $R$ is approximated by a finite set of scenarios, $R^k \in \Omega\ \forall\, k$, with associated probability $p^k$, where $\sum_{k=1}^{k_f} p^k = 1$.

Various scenario generation methodologies can be used to provide the scenario set, such as Monte-Carlo sampling, principal component sampling, moment matching, and bootstrapping. A survey of methods is provided by Dupačová et al. (2000).

An important aspect of solving stochastic models is the sequence of alternating decisions and observations. ISP separately seeks an optimal solution for each scenario. Hence, ISP assumes that the decision maker waits until the uncertainty is revealed before making a decision; i.e., delaying all decisions until the last possible moment, after all uncertainties have been revealed. The optimal solution relies upon all of the information about the future for each scenario; therefore, this approach provides a set of scenario solutions. Because of this assumption, the approach cannot be implemented directly and is a passive approach.

The optimal solution (solution for each scenario) of ISP constructs the objective values set, $\Omega_F$, which contains the optimal objective value for each scenario, and the optimal solutions set, $\Omega_x$, which individually holds the optimal decisions for each scenario. These sets can be analyzed probabilistically and a decision rule is used to aggregate these solutions to a single outcome for implementation (Hiew et al. 1989).

In ISP, one must solve multiple deterministic problems (one deterministic problem for each scenario); hence, the previously described EDM constitutes an efficient building block for ISP for solving multiple deterministic problems. The ISP formulation is

$$
\begin{aligned}
&\min_{x^k} F(x^k,R^k) \quad \forall\ k = 1,\ldots,k_f \\
&\text{Subject to:} \\
&g_i(x^k,R^k) = 0 \quad \forall\ i = 1,\ldots,m_i; \forall\ k = 1,\ldots,k_f \\
&g_j(x^k,R^k) \le 0 \quad \forall\ j = 1,\ldots,m_j; \forall\ k = 1,\ldots,k_f
\end{aligned} \tag{16}
$$

Eq. (16) is individually solved for each scenario $k$ to construct the objective value set $\min_{x^k} F(x^k,R^k) \in \Omega_F\ \forall\, k$ and the optimal solution set $\arg\min_{x^k} F(x^k,R^k) \in \Omega_x\ \forall\, k$.

The formulation Eq. (16) can be solved for predefined criteria of the optimal objective values set, $\Omega_F$. For example, the expectation, $E[\cdot]$ of $\Omega_F$, i.e., $E[\Omega_F]$, can be minimized, and all of the decisions corresponding to each scenario are determined in the same optimization problem:



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$$\min_{x^k \forall k} E[F(x^k, R^k)]$$
$$\text{Subject to:}$$
$$g_i(x^k, R^k) = 0 \quad \forall\ k = 1, \ldots, k_f \quad \forall\ i = 1, \ldots, m_i \tag{17}$$
$$g_j(x^k, R^k) \le 0 \quad \forall\ k = 1, \ldots, k_f \quad \forall\ j = 1, \ldots, m_j$$

The EDM cannot be utilized to solve Eq. (17), because all of the decisions of all of the scenarios are in the same optimization problem. However, if both Eqs. (16) and (17) are solved with the same scenarios, the optimal solution obtained from Eq. (17) would be the same as that obtained from Eq. (16). The minimum expectation is also obtained by individually minimizing each scenario. Obviously, in this case, one should choose to solve Eq. (16) and utilize EDM.

In the general case, when a statistical operator other than the expectation is applied, separately minimizing each scenario will not yield the same solution. One may consider, for example, the variance, $\sum[\cdot]$ of $\Omega_F$; i.e., $\sum[\Omega_F]$. In this case, the optimal solution cannot be individually obtained by each scenario; rather, the following optimization problem has to be solved:

$$\min_{x^k \forall k} \sum[F(x^k, R^k)]$$
$$\text{Subject to:}$$
$$g_i(x^k, R^k) = 0 \quad \forall\ k = 1, \ldots, k_f \quad \forall\ i = 1, \ldots, m_i \tag{18}$$
$$g_j(x^k, R^k) \le 0 \quad \forall\ k = 1, \ldots, k_f \quad \forall\ j = 1, \ldots, m_j$$

The minimum variance is zero. A zero variance is obtained when all members of $\Omega_F$ have the same value, i.e., all of the scenarios have the same objective value. Because each scenario has its own decision vector, the model can increase the costs to obtain $\Omega_F$ that has the minimum variance of zero.

A zero variance can be obtained by an infinite number of solutions. For instance, if the same objective value of for all scenarios is attained, and because the model is able to increase costs, a solution that increases the objective values of the scenarios by a constant is also optimal and produces zero variance. Among all optimal solutions that produce zero variance, the most interesting is the solution that produces zero variance with the smallest expectation; this solution can be obtained by increasing the cost of all of the scenarios to match the minimum optimal cost of the most severe scenario. In this case, $E[\Omega_F]$ is equal to the minimum cost of the most severe scenario and $\sum[\Omega_F]$ is zero.

## Mean-Variance Approach

### *Mean-Variance by Classical ISP*

ISP can be used within a multiobjective approach to produce the Pareto front between the variance and the expectation; i.e., the assembly of feasible points from which one cannot move to improve the expectation without worsening (increasing) the value of the variance. This tradeoff covers the interval from the point of minimum expectation to the point of zero variance. The expectation associated with variance of zero is equal to the minimum cost of the most severe scenario, as elaborated previously.

The bounds of the tradeoff interval are obtained from the solution of Eq. (16), which requires separately solving each scenario. This is because the mean of the objective values results in the minimum expectation ($E_{\min}$, left point on the tradeoff) whereas the maximum of the objective values results in the severe scenario cost ($E_{\max}$, right point on the tradeoff). For each point $E_i$ in this tradeoff interval, the following optimization problem is solved to obtain one point on the tradeoff curve. In this formulation, the epsilon-constraint approach for multiobjective optimization is utilized (Miettinen 1999), which converts the multiobjective optimization problem into a one-dimensional problem by reformulating some of the objectives as constraints:

$$\min_{x^k \forall k} \sum[F(x^k, R^k)]$$
$$\text{Subject to:}$$
$$E[F(x^k, R^k)] \le E_i \tag{19}$$
$$g_i(x^k, R^k) = 0 \quad \forall\ k = 1, \ldots, k_f \quad \forall\ i = 1, \ldots, m_i$$
$$g_j(x^k, R^k) \le 0 \quad \forall\ k = 1, \ldots, k_f \quad \forall\ j = 1, \ldots, m_j$$

To obtain a point on the tradeoff curve, one has to solve Eq. (19), in which all of the decisions corresponding to each scenario are determined in a single optimization problem, and the EDM cannot be utilized [Eq. (19) is obtained from Eq. (18) by adding one constraint to the first objective, i.e., the minimum expectation].

### *Implicit Mean-Variance*

Deriving the optimal mean-variance tradeoff curve by classical ISP requires solving the large scale optimization problem [Eq. (19)] for each point on the tradeoff curve. In Eq. (19), all of the decisions corresponding to each scenario are determined in the same optimization problem; solving each scenario individually would not result in the optimal solution of Eq. (19), as explained previously.

The IMV approach developed in this study formulates a small convex external problem to create the mean-variance tradeoff without the need to solve a large-scale problem of the form defined in Eq. (19).

IMV is assisted by Eq. (16) (where each scenario is solved individually) to formulate a convex external problem that can be solved efficiently.

### *IMV Framework*

The IMV framework is comprised of the steps described in the following.

In Step 1, EDM is utilized to solve Eq. (16) for each scenario, and the results are stored as follows:

$$F_*^k = \min_{x^k} F(x^k, R^k) \quad \forall\ k = 1, \ldots, k_f \tag{20}$$

In Step 2, the tradeoff covers the interval from the point of minimum expectation to the point with variance of zero. The expectation with zero variance is equal to the minimum cost of the most severe scenario. Thus, $E_{\min} = \sum_{k=1}^{k_f} F_*^k / k_f$ (left point) and $E_{\max} = \max(F_*^{k \forall k})$ (right point).

In Step 3, for each $E_i$ in the tradeoff interval, the following problem is solved and one point is obtained on the tradeoff curve. The values of $E_i$ are chosen based on the required resolution in the tradeoff curve. For example, to obtain a tradeoff curve with 11 points (i.e., nine new points in addition to $E_{\min}$ and $E_{\max}$), $E_i$ can be defined as $E_i = 0.1(E_{\max} - E_{\min}) \times i + E_{\min}$ $\forall\ i = 1.9$:

$$\min_{F^k \forall k} \sum_{s=1}^{k_f} p^k \times (F^k - E)^2$$

Subject to:

$$E = \sum_{k=1}^{k_f} p^k \times F^k \quad (21)$$

$$E \le E_i$$

$$F^k \ge F_*^k \quad \forall\ k$$

In Eq. (21), the variance is minimized, whereas $F^k$ (the cost of each scenario) is the decision variable and the constraints $F^k \ge F_*^k$, $\forall\ k$ maintain the feasibility of the original problem.

The size of this optimization problem is smaller than the original because it includes one variable, $F^k$, for each scenario instead of a vector of decision variables, $x^k$. The optimization problem [Eq. (21)] is convex and easy to solve because the objective function is quadratic and the constraints are linear.

In the external problem, only the tradeoff itself is produced instead of the decision, $x^k\ \forall\ k$. To obtain the decision for a chosen point on the tradeoff, a goal seeking problem is formulated for that scenario:

$$\min_{x^k} [F(x^k, R^k) - F_{\text{opt}}^k]^2 \quad \forall\ k$$

Subject to:

$$g_i(x^k, R^k) = 0 \quad \forall\ i = 1, \ldots, m_i; \forall\ k \quad (22)$$

$$g_j(x^k, R^k) \le 0 \quad \forall\ j = 1, \ldots, m_j; \forall\ k$$

where $F_{\text{opt}}^k$ = optimal solution of Eq. (21).

## Application

### *IMV versus Classical ISP*

A water system with nine demand zones, three aquifers, five desalination plants, and 49 pipes (Fig. 2) is solved in this study; the structure of this system mimics the central part of the Israeli National Water System. The year is divided into two seasons that are labeled "winter" (265 days with low demands and stochastic recharge given by a scenario tree), and "summer" (100 days with high demands and zero recharge). The full parameter set appears in the appendix.

The aquifer recharge is considered stochastic, given by a finite number of scenarios. For water resources management models, it is often possible to generate a large number of scenarios obtained by simulation with stochastic models by using historical data and expert forecasting of more extreme scenarios. However, to demonstrate the method developed in this study, it is assumed that the annual aquifer recharge is given by a probability mass function (PMF) of an independent random variable (no serial correlation). Hence, the scenarios set for the multiyear recharge vector and its corresponding probabilities are obtained by using multiplication, i.e., multiplying the probabilities of the independent random variable.

The PMF is given in Table 1, where $r_t$ is the recharge in year $t$ and $R^t$ is the vector of recharges up to year $t$.

For a three-year horizon, the scenario tree has eight scenarios (i.e., paths in the tree), where each of the nodes, 2–15, receives the values of the low recharge if the node index is odd, and those of the high recharge otherwise.

**Table 1.** Probability Mass Function of the Recharge

| Recharge | Low recharge | High recharge |
|---|---|---|
| $r_{1,t}$, $r_{2,t}$, $r_{3,t}$ (mcm) | 60, 10, 110 | 360, 280, 550 |
| $\text{Prob}(r_{a=1..3,t} \mid R^t_{a=1..3})$ | 0.5 | 0.5 |

In classical ISP, to obtain a point on the tradeoff curve, one solves Eq. (19), in which the decisions corresponding to all scenarios are determined by one optimization problem. For the WSS in Fig. 2, for each year, there are eight different decision vectors with 62 variables each. This results in an optimization problem with $8 \times 3 \times 62 = 1{,}488$ decision variables. Decision vectors are subject to 60 nonlinear constraints and 124 linear constraints and bounds. This implies for each point on the tradeoff, an optimization problem with 2,976 linear and 1,440 nonlinear constraints.

In the application, the model was run for three years, so the number of decision variables in ISP is not large. In general, the time horizon should be extended to more years. It is tractable to solve IMV with an extended horizon, whereas in the classical ISP it is not, because the optimization problem size increases exponentially with the time periods. For the demonstration in this paper, a horizon of only three years was chosen so the ISP formulation could still be solved and compared with the IMV.

IMV formulates an equivalent external optimization problem that yields the optimal Pareto front without the need to solve the optimization problem, as described previously. The external problem [Eq. (21)] is quadratic, having eight decision variables (one corresponding to each scenario) and nine linear constraints. Thus, it is much easier to solve than ISP. The steps involved in solving the IMV are described in the following.

In Step 1, solve Eq. (16) for each scenario. Each scenario corresponds to solving an optimization problem with $3 \times 62 = 186$ decision variables, $3 \times 60 = 180$ nonlinear constraints, and $3 \times 124 = 372$ linear constraints and bounds. The optimal objective value for each scenario is shown in Fig. 3.

The results in Fig. 3 show, for example, that Scenario 4 with high-low-low recharge costs less than Scenario 5, which is a low-high-high recharge scenario, although Scenario 5 has, overall, more water in the aquifers. This is because the high recharge value in Scenario 4 in the first year is larger than the required demand and the aquifers can store some water for future years. On the other

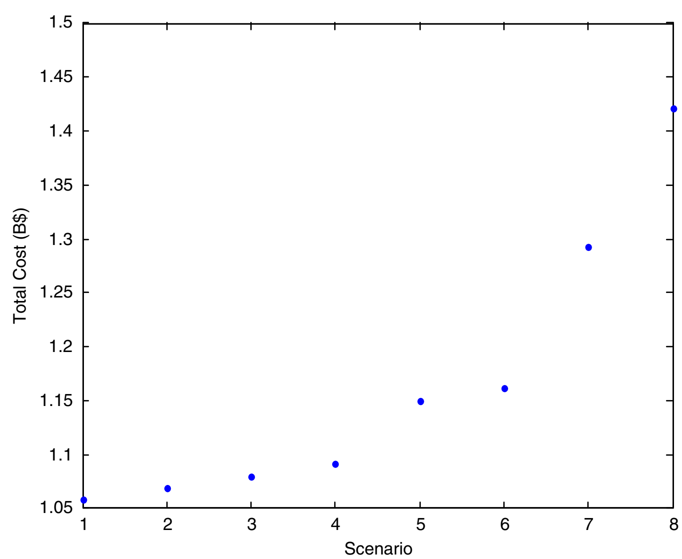


**Fig. 3.** Total cost ($billion) for each scenario





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hand, in Scenario 5, the low recharge is in the first year; hence, despite knowing that the future holds high recharge, more water from the desalination plant in the first year needs to be used, which leads to a high cost in the first year, and consequently, to a higher total cost.

In Step 2, the bounds for the mean-variance tradeoff interval are obtained from the solutions to the scenarios in Fig. 3. The mean of the objective values results in the minimum expectation ($E_{\min} = 1.17$B$, leftmost point) and the maximum of the objective values yields the most severe scenario cost ($E_{\max} = 1.42$B$, rightmost point).

In Step 3, for selected points in the interval $1.17 \le E_i \le 1.42$, the following quadratic optimization problem is solved (with eight decision variables and nine linear constraints):

$$\min_{F^{k}\forall k} \frac{1}{8}\sum_{s=1}^{k_f}\left(F^k - \frac{1}{8}\sum_{k=1}^{k_f}F^k\right)^2$$

Subject to:

$$\frac{1}{8}\sum_{k=1}^{k_f}F^k \le E_i \tag{23}$$

$$F^k \ge F_*^k \quad \forall\ k = 1.8$$

The tradeoff curves obtained by IMV [Eq. (23)] and the one obtained from the classical ISP formulation [Eq. (19)] are compared in Fig. 4.

The tradeoff curves are practically identical, as shown in Fig. 4. This clearly demonstrates the advantage of IMV over classical ISP for producing the mean-variance tradeoff.

To obtain a point on the tradeoff by classical ISP, an optimization problem with 1,488 decision variables is solved. Hence, to obtain the tradeoff in Fig. 4, 10 optimization problems, each with 1,488 decision variables, are solved. In IMV eight problems are solved with 186 decision variables and an external problem with eight decision variables is solved for each point in the tradeoff. A particularly noteworthy feature is that the eight optimization problems in IMV are solved only once and there is no need to solve these problems for each point in the tradeoff curve.

Furthermore, it is well known that solving a number of subproblems obtained by decomposition is computationally superior to solving a large-scale optimization problem (Rockafellar and Wets 1991; Mulvey and Ruszczynski 1995). Hence, solving eight problems with 186 decision variables each is "cheaper" than solving one optimization problem with 1,488 decision variables. This decomposition is critical when one considers the long time horizon and large number of scenarios. For example, when the time horizon is 11 instead of three, the number of decision variables in ISP is 1,396,736, whereas in IMV, the large-scale optimization problem is decomposed into 2,048 problems with 682 decision variables each. Next, an external quadratic problem with only 2,048 variables is utilized to produce the tradeoff curve. There is no available optimization solver that is capable to solve a general nonlinear optimization problem (similar to that described in this paper) with 1,396,736 variables.

The IMV approach is computationally cheaper than classical ISP in producing the mean-variance tradeoff. IMV can be utilized to produce the mean-variance tradeoff for problems in which the ISP formulation is intractable, as demonstrated previously.

### Tradeoff Analysis

Because water from the aquifers costs less than from desalination, the tradeoff between the expected value of the cost and its variability (SD) shows that as the cost rises (corresponding to less water taken from the stochastic sources and more from desalination), the variability decreases. Fig. 5 compares the first point ($1.17 billion) with the middle point ($1.28 billion) in the tradeoff. The first year total water withdrawal from the aquifers corresponding for the minimum expectation solution (first point on the tradeoff) ranges between 272–422 mcm, whereas for the middle point, the range is much lower at 270–335 mcm.

The lower withdrawal from the aquifers in the middle point indicates that the increments in the cost are expressed by less water withdrawn from the stochastic sources and more from the desalination, so as one moves to the right along the points on the tradeoff graph, the solutions become more robust as they decrease the reliance on the stochastic sources. The cost variability can be viewed as a measure for the robustness of the solution.

As indicated earlier, the solution obtained by IMV cannot be implemented, because the decisions have different optimal values for different scenarios. For example, in the minimum expectation

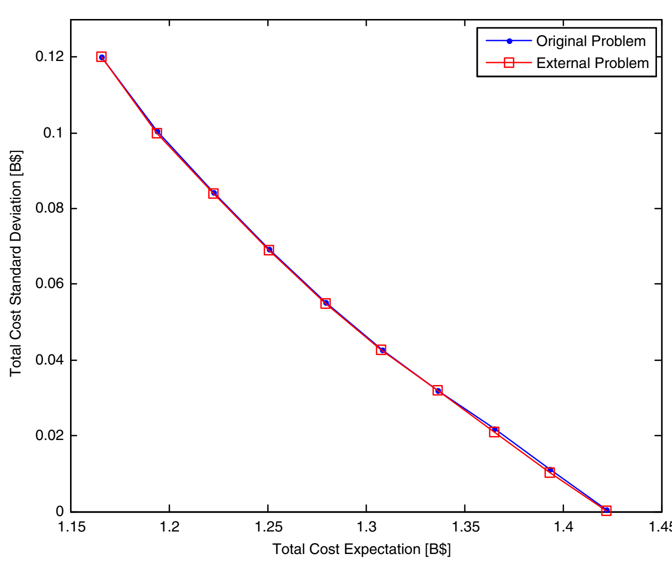


**Fig. 4.** Validation of the IMV tradeoff curve

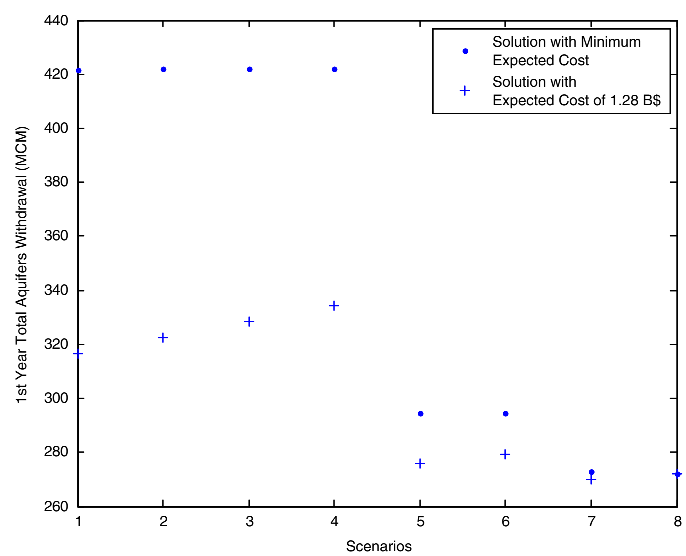


**Fig. 5.** Aquifer withdrawals (mcm): comparison of different points on the tradeoff curve



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solution ($1.17 billion), among the eight scenarios, there are three different values for the first year optimal withdrawal from the aquifers. The first value (420 mcm) corresponds to Scenarios 1–4 that begin with a high recharge, whereas the remaining scenarios that start with low recharge are further subdivided according to the recharge in the second year.

It may be possible to identify an implementable decision that is a nearly optimal solution for a wide range of possible realizations (Labadie 2004). Moving to the right along the tradeoff range makes the solution more implementable, i.e., the values obtained from different scenarios are closer. Thus, deriving a heuristic decision rule for implementation would be made easier by moving along the tradeoff curve.

## Discussion

The mean-variance approach can be attributed to the scenario-based robust optimization suggested by Mulvey et al. (1995). A robust solution is defined as one that results in low variability. Robustness (less variability) was projected in the WSS application as smaller amounts of aquifer withdrawal, resulting in less reliance on the uncertain water source and greater reliability.

Still, choosing the variance as a measure of robustness will not always be correct. For example, one may consider a case in which there is a high levy on extraction from the aquifers. In this case, seeking a solution with less variability results in more water withdrawal from the stochastic sources, resulting in less reliability and less robustness.

The variability is not always a good measure of reliability and robustness; this may depend on the problem parameters (e.g., extraction levy). In fact, a similar conclusion was proposed by Watkins and McKinney (1997), in which the variance was used as measure of risk with two-stage ESP. They state,

"(1) This reduces the variability in cost across scenarios (by moving all costs towards that of the "max cost" scenario), but of course is illogical from a decision-making point of view. (2) Nonetheless, since the first-stage decision remains nearly the same, this limitation may not be important in this case."

Although Watkins and McKinney's conclusion was derived for ESP, the first statement is still valid for ISP. However, the second statement will not be valid, because the ISP approach does not constrain the first stage decision to have the same value as two-stage ESP. Thus, understanding how the cost variability yields different optimal solutions must be investigated before considering the variance as a measure of robustness in the ISP approach.

## Summary and Conclusions

1. IMV substantially reduces the computational burden when producing the mean-variance tradeoff and provides almost the same tradeoff as that obtained from the original optimization problem.
2. The IMV approach facilitates the building of the mean-variance tradeoff within the ISP approach without the need to solve a large-scale optimization problem. Producing the tradeoff Pareto front for a large-scale real system such as the one considered in this study would be intractable without the utilization of IMV.
3. Many studies have considered the minimum expected cost solution within the ISP approach. Because IMV only requires the minimum expectation solution for formulating the external problem, these studies may be extended by the IMV approach to produce the entire mean-variance tradeoff for a decision maker (adding a small computation of solving the external convex optimization problems).
4. The inability of the mean-variance tradeoff to provide decisions for implementation is well documented in the literature; for example, this is discussed in the study by Labadie (2004) and the references therein. Still, as demonstrated in the current example, moving along the tradeoff toward solutions with less variability produces solutions from which an implementable decision is easier to derive. This is because of the proximity of scenario solutions when the variability is small.
5. Robustness (implying less variability; Mulvey et al. 1995) resulted in the WSS application as smaller amounts of aquifer water were withdrawn, reducing reliance on the uncertain water source.
6. A conclusion to be drawn from the discussion is that understanding the considerations of cost variability solutions in making decisions is essential. Selecting the variance or SD as a measure of robustness may lead to decision errors for cases in which fewer variability requirements artificially/inefficiently increase the cost of the scenarios to close vicinity, thereby providing the existence of less variability.

**Table 2.** Demand Data for the Large WSS

| Demand ($z$) | Season 1 (mcm) | Season 2 (mcm) |
|---|---|---|
| 1 | 116.9 | 50.1 |
| 2 | 9.1 | 3.9 |
| 3 | 53.9 | 23.1 |
| 4 | 36.4 | 15.6 |
| 5 | 18.2 | 7.8 |
| 6 | 90.3 | 38.7 |
| 7 | 9.1 | 3.9 |
| 8 | 18.2 | 7.8 |
| 9 | 198.1 | 84.9 |

## Appendix. Parameter Set Description

The daily pumping hours are 14 and 16 h/day, respectively, for the first and second seasons; hence, the seasonal pumping hours are $w^{S=1} = 3{,}710$ h/season and $w^{S=2} = 1{,}440$ h/season. The seasonal capacities of the pipes are shown in Fig. 2, based on pipe diameters, lengths, Hazen-Williams coefficients, topographic difference. and a hydraulic loss of 4‰. The energy cost for the first season is $\text{KWHC}^{S=1} = 0.09$ \$/kW · h and that for the second is $\text{KWHC}^{S=2} = 0.11$ \$/kW · h.

**Table 3.** Aquifer Data for the Large WSS

| Aquifer ($a = 1.3$) | $(Q_{\max})_a^S$ (mcm) | $(h_{\max})_a^S$ (m) | $(h_{\min})_a^S$ (m) | $h_a^0$ (m) | $C_a^0$ (mg/L $Cl^-$) | $SA$ (mcm/m) | $(\overline{CE}^{\max})_a^{S,Y\text{a}}$ (M$/mcm) |
|---|---|---|---|---|---|---|---|
| 1 | 130 | 100 | 1 | 2 | 300 | 65 | 0.2 |
| 2 | 111 | 100 | 1 | 3 | 300 | 37 | 0.2 |
| 3 | 290 | 100 | 17 | 19 | 150 | 25 | 0.2 |

[a]The recharge salinity is 150; recharge in the second season is 0.



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The seasonal demands for the nine demand zones are given in Table 2. The maximum allowed water salinity in all zones is set to 220 mg/L $Cl^-$. The maximum desalination amounts of Plants 1 to 5 are $(Q_{\max})_d^S = [30, 100, 100, 200, 100]$ mcm, respectively, whereas no desalination plants have requirements for a minimal supply, i.e., $(Q_{\min})_d^S = 0$ mcm. The removal ratio of the plants is between $(RR_{\max})_d^S = 99.95\%$ and $(RR_{\min})_d^S = 99.75\%$, yielding a product salinity in the range of 13.5–67.5 mg/L $Cl^-$. The desalination cost parameters are $\alpha_d = 0.7$ M\$/mcm and $\beta_d = -10^6$ (dimensionless), implying constant desalination cost per mcm in all plants. Data for the three aquifers are given in Table 3. The maximum allowed salinity in all aquifers is set to 350 mg/L $Cl^-$.

## References


Ajami, N. K., Hornberger, G. M., and Sunding, D. L. (2008). "Sustainable water resource management under hydrological uncertainty." *Water Resour. Res.*, 44(11).

Barros, M., Tsai, F., Yang, S.-L., Lopes, J., and Yeh, W. (2003). "Optimization of large-scale hydropower system operations." *J. Water Resour. Plann. Manage.*, 129(3), 178–188.

Birge, J. R., and Louveaux, F. V. (1997). *Introduction to stochastic programming*, Springer, New York.

Byrd, R. H., Gilbert, J. C., and Nocedal, J. (2000). "A trust region method based on interior point techniques for nonlinear programming." *Math. Program. B*, 89(1), 149–185.

Cai, X., McKinney, D., and Lasdon, L. (2001). "Solving nonlinear water management models using a combined genetic algorithm and linear programming approach." *Adv. Water Resour.*, 24(6), 667–676.

Cohen, D., Shamir, U., and Sinai, G. (2000). "Optimal operation of multi-quality water supply systems—I: Introduction and the Q-C model." *Eng. Optim.*, 32(5), 549–584.

Crawley, P., and Dandy, G. (1993). "Optimal operation of multiple reservoir system." *J. Water Resour. Plann. Manage.*, 119(1), 1–17.

Dupačová, J., Consigli, G., and Wallace, S. W. (2000). "Scenarios for multistage stochastic programs." *Ann. Oper. Res.*, 100(1–4), 25–53.

Fletcher, R. (1985). "Practical methods of optimization." *Constrained optimization*, Vol. 2, Wiley, New York.

Grantz, K., Rajagopalan, B., Zagona, E., and Clark, M. (2007). "Water management applications of climate-based hydrologic forecasts: Case study of the Truckee-Carson river basin." *J. Water Resour. Plann. Manage.*, 133(4), 339–350.

Hiew, K., Labadie, J., and Scott, J. (1989). "Optimal operational analysis of the Colorado-Big Thompson project." *Computerized decision support systems for water managers*, J. Labadie, et al., eds., ASCE, Reston, VA, 632–646.

Housh, M. (2011). "Optimal management of regional water resources systems under uncertainty." Ph.D. thesis, Technion-Israel Institute of Technology, ⟨http://mashorhoush.wix.com/index⟩ (Jun. 8, 2013).

Housh, M., Ostfeld, A., and Shamir, U. (2012). "Seasonal multi-year optimal management of quantities and salinities in regional water supply systems." *Environ. Modell. Soft.*, 37, 55–67.

Kracman, D. R., McKinney, D. C., Watkins, D. W., Jr., and Lasdon, L. S. (2006). "Stochastic optimization of the highland lakes system in Texas." *J. Water Resour. Plann. Manage.*, 132(2), 62–70.

Labadie, J. W. (2004). "Optimal operation of multi-reservoir systems: State-of-the-art review." *J. Water Resour. Plann. Manage.*, 130(2), 93–111.

Markowitz, H. (1959). *Portfolio selection*, Yale University Press, New Haven, CT.

*MATLAB* [Computer software]. Natick, MA, MathWorks, Inc.

Miettinen, K. (1999). *Nonlinear multi-objective optimization*, Kluwer Academic Publishers, Boston.

Mulvey, J. M., and Ruszczynski, A. (1995). "A new scenario decomposition method for large-scale stochastic optimization." *Oper. Res.*, 43(3), 477–490.

Mulvey, J. M., Vanderbei, R. J., and Zenios, S. A. (1995). "Robust optimization of large-scale systems." *Oper. Res.*, 43(2), 264–281.

Ostfeld, A., and Shamir, U. (1993). "Optimal operation of multi-quality networks. I: Steady-state conditions." *J. Water Resour. Plann. Manage.*, 119(6), 645–662.

Peng, C. S., and Buras, N. (2000). "Practical estimation of inflows into multireservoir system." *J. Water Resour. Plann. Manage.*, 126(5), 331–334.

Philbrick, C. R., and Kitanidis, P. K. (1999). "Limitations of deterministic optimization applied to reservoir operations." *J. Water Resour. Plann. Manage.*, 125(3), 35–142.

Raman, H., and Chandramouli, V. (1996). "Deriving a general operating policy for reservoirs using neural network." *J. Water Resour. Plann. Manage.*, 122(5), 342–347.

Rockafellar, R. T., and Wets, R. J. B. (1991). "Scenarios and policy aggregation in optimization under uncertainty." *Math. Oper. Res.*, 16(1), 119–147.

Saad, M., Turgeon, A., and Stedinger, J. R. (1992). "Censored-data correlation and principal component dynamic programming." *Water Resour. Res.*, 28(8), 2135–2140.

Seifi, A., and Hipel, K. (2001). "Interior-point method for reservoir operation with stochastic inflows." *J. Water Resour. Plann. Manage.*, 127(1), 48–57.

Tu, M., Tsai, F. T., and Yeh, W. W. (2005). "Optimization of water distribution and water quality by hybrid genetic algorithm." *J. Water Resour. Plann. Manage.*, 131(6), 431–440.

Vasiliadis, H., and Karamouz, M. (1994). "Demand-driven operation of reservoirs using uncertainty-based optimal operating policies." *J. Water Resour. Plann. Manage.*, 120(1), 101–114.

Waltz, R. A., Morales, J. L., Nocedal, J., and Orban, D. (2006). "An interior algorithm for nonlinear optimization that combines line search and trust region steps." *Math. Program.*, 107(3), 391–408.

Watkins, D. W., and McKinney, D. C. (1997). "Finding robust solutions to water resources problems." *J. Water Resour. Plann. Manage.*, 123(1), 49–58.

Yates, D., Sieber, J., Purkey, D., and Huber-Lee, A. (2005). "WEAP21—A demand priority and preference-driven water planning model. Part 1: Model characteristics." *Water Int.*, 30(4), 487–500.

Zaide, M. (2006). "A model for multiyear combined optimal management of quantity and quality in the Israeli national water supply system." M.Sc. thesis, Technion—I.I.T., Israel, ⟨http://urishamir.wri.technion.ac.il/⟩ (Aug. 6, 2013).