OPTIMAL EXTRACTION OF WATER FROM REGIONAL AQUIFER UNDER SALINIZATION

By Ekaterina Gordon,[1] Uri Shamir,[2] Fellow, ASCE, and Jacob Bensabat[3]

ABSTRACT: A model for the optimal management of a regional aquifer under salinization is developed. The objectives of management are to maximize the total amount of water pumped for use and to minimize the total amount of salt extracted with the water. The model is based on a combination of simulation and an optimization routine, run iteratively. The simulation model uses a finite-element formulation for the flow and a streamline upwind Petrov-Galerkin formulation for the transport and computes the gradient of the state variables (heads and concentrations) with respect to the decision variables (pumping rates at wells). The gradients are then used in a Bundle-Trust nonsmooth optimization procedure to achieve an improved solution. The process ends when termination criteria are met, resulting in a good solution, which cannot be claimed to be the global optimum. The procedure is demonstrated on a 600-km^2 nonhomogeneous regional aquifer with 12 zones of differing properties and 32 pumping wells.

INTRODUCTION

The concept of ‘‘an aquifer under salinization condition’’ is frequently associated with a coastal aquifer where seawater intrudes inland when the head in the aquifer is lowered as a result of overpumping. Several models have been developed to analyze, predict, or control the seawater intrusion and to manage coastal aquifers, based on different approaches to simulation of the saltwater-freshwater interface. A sharp interface approach was used by Willis and Finney (1988) and Emch and Yeh (1998). In this approach, the two zones are distinct, separated by a sharp interface, and there is no need to deal with the spatial distribution of concentration. The second approach is to consider the transition zone between seawater and freshwater using a density-dependent model of ground-water flow and salt transport (Das and Datta 1999). In this case the concentration varies continuously over space.

Aquifer salinization is not only associated with seawater intrusion. Often, salinization results from the penetration of water bodies from other sources with salinity higher than that of the resident waters. Such sources may be irrigation water percolating over some part of the aquifer, influx of saline waters from faults in the aquifer bottom, and inflow from laterally adjacent saline water bodies. Management of an aquifer under such conditions poses special difficulties, as elaborated below.

The present study was motivated by a regional problem in Israel: the Na’aman aquifer in western Galilee, which has an area of some 600 km^2. It is located several kilometers away from and is not hydraulically connected to the sea. Its salinization is caused by intrusion of somewhat more saline water from the lower ground-water system through a system of faults. At the beginning of the management period there already existed a certain distribution of salinity in the aquifer. The saline water is displaced and dispersed because of the flow field, which is affected by pumping from a series of 32 wells and the Na’aman spring, which is the main outlet of the aquifer. Because the salinity of the intruding waters is much lower than that of seawater, the effect of density on the flow field can be neglected.

The objective of management is to make optimal use of the water pumped from the aquifer, both fresh and saline. The management model can be considered as multiobjective, where the goals are to pump as much freshwater as possible and to minimize the amount of the salt mass extracted with the water.

Das and Datta (1999) considered the multiobjective management of a regional coastal aquifer. They proposed two- and three-objective aquifer management models, where the objectives were to maximize pumping from the freshwater zone and minimize pumping from the saline zone. The salt concentration of the pumped water was considered as a constraint or a third objective.

Ground-water management models that consider water quality have been developed mostly for aquifer remediation problems (Gorelick et al. 1984; Ahlfeld et al. 1988a,b; Chang et al. 1992; Culver and Shoemaker 1992; Xiang et al. 1995). The objectives and constraints of aquifer remediation are rather different from those of ground-water resource management problems such as the one studied here. The major objectives in remediation are to minimize the cost required for reducing contaminant concentration to specified levels during a fixed time period or to minimize the total residual mass of contaminant in the aquifer at the end of the period. Water quality is considered in remediation problems at the end of time horizon, whereas ground-water resources management problems require control on the quality of the water pumped at all times. In remediation problems, both pumping and recharge in wells are considered, because one objective is to control the extent and level of pollution in the aquifer itself. In ground-water resources management problems, as considered here, recharge may be somewhat less relevant, although it should not be discarded.

Both remediation models and water resource management models use simulation and optimization. The mathematical nature of the optimization problem is determined by the response of the concentration to pumping. Gorelick et al. (1984) and Ahlfeld and Sprong (1998) studied the concentration behavior as a function of pumping/recharge rates and found that it is nonconvex and nonsmooth. Gordon et al. (1999) found the same behavior; therefore, a nonsmooth optimization technique, the Bundle-Trust algorithm (Schramm and Zowe 1992), has been used. The Bundle-Trust algorithm belongs to the subgradient method family that deals with nonlinear, nonsmooth, and certain nonconvex problems. The estimates of the physical behavior of the system, which are required for the optimization, are obtained from the simulation model. It is based on finite elements (FEs) to describe the flow in a regional, confined 2D vertically averaged, heterogeneous aquifer and on the streamline upwind Petrov-Galerkin (SUPG) method (Brooks and

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[1] Minerva Optimization Ctr., Technion, Haifa 32000, Israel.

[2] Prof., Facu. of Civ. Engrg., Technion, Haifa 32000, Israel.

[3] Dir., Envir. and Water Resour. Engrg., P.O. Box 6770, Haifa, 31067 Israel.

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Hughes 1982) for salinity transport. The SUPG method is a modified FE method used to prevent the oscillations in solution that may occur when FEs are used for transport problems with advection-dominated flow.

The simulation-optimization model was applied to an aquifer with 32 pumping wells, complex geometry, and multiple saline water sources—all similar to the Na'aman aquifer in western Galilee, Israel. (One cannot claim that it is the real aquifer, which is 3D). The model was considered as a two-objective problem: to maximize the total amount of water pumped and minimize the salt mass taken out with the water.

FORMULATION OF OPTIMIZATION MODEL

Consider a regional aquifer with a surface area of several hundred square kilometers and a number of pumping wells. There are zones of saline water in the aquifer and sources of saline water at its boundaries that will penetrate the aquifer if there is inflow from their direction. The objective of the water resource management is to pump as much water as possible from the wells, over some specified time period (years), subject to quality and quantity considerations.

The decision variables are the pumping rates. The constraints are on well pumping rates and total amount of water extracted. The last constraint does not allow pumping to exceed the influx water (water replenished by precipitation), prevent aquifer depletion, or drop below a prescribed percent of the water replenished to meet the water requirements.

The management period can be divided into time periods during which the pumping rates remain constant, each several years long. Thus the mathematical formulation of the management model is as follows:

$$\max \left\{ F_1 = \sum_{tp=1}^{N_{tp}} \sum_{ip=1}^{N_{ip}} |Q_{tp,ip}| \right\} \tag{1}$$

$$\min \left\{ F_2 = \sum_{tp=1}^{N_{tp}} \sum_{ip=1}^{N_{ip}} \int |Q_{tp,ip}| c_{tp,ip}(t)\, dt \right\} \tag{2}$$

$$Q_{\max,ip} \le Q_{tp,ip} \le 0 \tag{3}$$

$$\text{pump min} \le \sum_{tp=1}^{N_{tp}} \sum_{ip=1}^{N_{ip}} |Q_{tp,ip}| \le \text{pump max} \tag{4}$$

where $Q_{tp,ip}$ = pumping rate of well ip in time period tp, constant during the period (pumping is taken as negative to be consistent with the common formulation of ground-water models); $c_{tp,ip}(t)$ = salinity concentration at well ip in the time period tp; $Q_{\max,ip}$ = maximum pumping capacity of well ip; pump min and pump max = lower and upper bounds, respectively, on the total amount of water extracted from the aquifer; N_{ip} = number of wells; and N_{tp} = number of time periods.

The second objective, on total salt mass extracted, is motivated by practical considerations: the water pumped from the aquifer will be used somewhere (e.g., irrigation or urban supply), and its salinity load is to be minimized. This second objective substitutes for the traditional quality constraints/objectives on the concentration level that are used for ground-water resource and remediation problems.

The problem defined by (1)–(4) is nonlinear, nonconvex, and nonsmooth because of the presence of concentration in the objective function [(2)]. Classical optimization methods are based on smoothness and convexity and can fail in the search for the optimal solution while the problem is nonconvex or nonsmooth. To overcome these difficulties, the Bundle-Trust method (Schramm and Zowe 1992) was chosen. Bundle methods are a modification of classical gradient methods applicable to a nonsmooth objective function (continuous with discontinuous finite derivatives), which can handle successfully a nondifferentiable and sometimes nonconvex functional.

The central idea that distinguishes Bundle-Trust from the original gradient methods is the use of information about the function values and subgradients from previous iterations, stored in a ''bundle,'' in selecting the descent direction. This information helps to overcome local nonsmoothness and nonconvexity. The Bundle-Trust method can solve nonsmooth nonlinear problems subject to linear and box constraints by using the Powell algorithm (1985).

The box constraints [(3)] are incorporated directly into the model, whereas the constraints [(4)] on the total amount of water pumped are incorporated into the objective as a penalty term. The modified management model is a weighted sum of the two objectives and the penalty term, as follows:

$$\begin{aligned} \min \Bigg\{ L = & -\sum_{tp=1}^{N_{tp}} \sum_{ip=1}^{N_{ip}} |Q_{tp,ip}| + P_1 \sum_{tp=1}^{N_{tp}} \sum_{ip=1}^{N_{ip}} \int |Q_{tp,ip}| c_{tp,ip}(t)\, dt \\ & + P_2 \max \left[0, \text{pump min} - \sum_{tp=1}^{N_{tp}} \sum_{ip=1}^{N_{ip}} |Q_{tp,ip}| \right] \\ & + P_3 \max \left[0, \sum_{tp=1}^{N_{tp}} \sum_{ip=1}^{N_{ip}} |Q_{tp,ip}| - \text{pump max} \right] \Bigg\} \end{aligned} \tag{5}$$

subject to $Q_{\max,ip} \le Q_{tp,ip} \le 0$.

The weights P_2 and P_3 are adjusted to ensure that the bounds on total pumping are not violated, yet these terms do not create convergence difficulties (i.e., they are not too large). The value of P_1 is changed progressively to generate the trade-off curve between the two objectives.

The modified objective function is always nonsmooth owing to the incorporation of the constraints into the objective as $\max(0, f)$. The optimal solution is feasible if all constraints are satisfied within a stated tolerance.

At each iteration the Bundle-Trust method requires the calculation of the value and gradient of the objective function with respect to the decision variables: $(\partial c(x_j, t)/\partial Q_{tp,ip})$. Sensitivity theory is used to obtain these derivatives (Oblow 1978; Ahlfeld et al. 1988a; Chang et al. 1992; Xiang et al. 1995).

The simulation-optimization technique allows transferring from the simulator into the optimizer only information relevant to the optimization search: concentration and hydraulic head and their derivatives with respect to pumping rates at the wells.

SIMULATION MODEL

The flow field is essentially 2D horizontal. Density effects in the salinity range being considered (<700 ppm) are neglected. For a 2D, vertically averaged flow and transport model for a confined, isotropic, and heterogeneous aquifer the equations are as follows:

Darcy's law:

$$q = -K\nabla h \tag{6}$$

Mass balance for water:

$$\nabla \cdot (Kb\nabla h) = w - \sum_{ip=1}^{N_{ip}} Q_{ip}\delta(x - x_{ip}) \tag{7}$$

Mass balance for solute averaged over the depth of the ground-water layer:

$$\frac{\partial(ncb)}{\partial t} + q \cdot \nabla(bc) = \nabla \cdot (Dnb\nabla c) + w(c_w - c) \tag{8}$$

where h = hydraulic head (L); q = specific discharge (L/T); K = hydraulic conductivity (L/T); b = thickness of the aquifer

(L); Q_{ip} = pumping rate of jth well (L^3/T); N_{ip} = total number of pumping wells; w = leakage flux (L/T); δ = Dirac delta function at location x_{ip} (L^{-2}); n = porosity; c = concentration of solute averaged over the depth (M/L^3); c_w = solute concentration in leakage water (M/L^3); and D = hydrodynamic dispersion tensor (L^2/T), which for an isotropic aquifer is expressed as follows:

$$D_{ij} = D_d + a_T|V|\delta_{ij} + (a_L - a_T)V_iV_j/|V| \tag{9}$$

in which a_L = longitudinal dispersivity (L); a_r = transverse dispersivity (L); and V_i/V_j = velocity in the i- and j-directions, respectively. The boundary conditions for flow and transport equations can be of the first (Dirichlet) or second (Neuman) type.

The flow equations [(6) and (7)] are solved by an FE method on an irregular triangular grid. However, oscillations can appear in the solution of the transport equation by traditional FEs when the advection term is dominant. To prevent oscillations, the SUPG method (Brooks and Hughes 1982) is chosen for solving (8). The basic idea of SUPG is to modify the original FE method by adding artificial dispersion that acts only in the flow direction if the flow is advection-dominated (Péclet number $\mathsf{P} = VL/D > 2$). A detailed description of the implementation of SUPG for a ground-water simulation model can be found in Gordon et al. (2000).

CALCULATION OF DERIVATIVES

The Bundle-Trust method requires the objective function and its gradient with respect to decision variables, as do all gradient methods. According to the differentiation chain rule, the derivatives of the concentrations and hydraulic heads with respect to pumping rates at wells at each time step are needed (these are called ‘‘state sensitivity’’).

The concentration at a point at a given time depends on the pumping rates at all wells in all preceding time periods. Thus there is a derivative of the concentration at well jp and time t_{TP} with respect to the pumping rate of well ip at time period tp

$$\frac{\partial c(x_{jp}, t_{tp})}{\partial Q_{tp,ip}}, \quad \forall ip;\ tp \le TP \tag{10}$$

To calculate these derivatives, sensitivity theory is used by direct differentiation of the original system of the simulation equations. Direct differentiation of (6)–(8) results in the following system of the equations:

$$\tilde{q} = -K\nabla\tilde{h} \tag{11}$$

$$\nabla\cdot(Kb\nabla\tilde{h}) = \delta(x - x_j) \tag{12}$$

$$\frac{\partial(n\tilde{c}b)}{\partial t} + q\cdot\nabla(b\tilde{c}) - \nabla\cdot(Dnb\nabla\tilde{c}) - w\tilde{c} = -\tilde{q}\cdot\nabla(bc) + \nabla\cdot(\tilde{D}nb\nabla c) \tag{13}$$

where $\tilde{h}$, $\tilde{q}$, $\tilde{c}$, and $\tilde{D}$ = derivatives of h, q, c, and D (functions of space and time), respectively, with respect to the decision variables $Q_{tp,ip}$

$$\tilde{h} = \frac{\partial h}{\partial Q_{i,tp}}; \quad \tilde{q} = \frac{\partial q}{\partial Q_{i,tp}}; \quad \tilde{c} = \frac{\partial c}{\partial Q_{i,tp}}; \quad \tilde{D} = \frac{\partial D}{\partial Q_{i,tp}}$$

These differential equations require definition of their initial and boundary conditions. Because the initial conditions for the original problem do not depend on pumping rates, they are always zero for (12) and (13). The boundary conditions for (12) and (13) can be obtained by differentiation of the boundary conditions of the original equation. Thus, they are of the same type (Dirichlet or Neuman) at each point as for the original (7) and (8) but the value is always zero. The derivative equations [(11)–(13)] have the same structure as the original ones [(6)–(8)]. They also have the same right-hand sides and thus can be solved by the same methods, FE for derivatives of the flow equation and SUPG for derivatives of the transport equation.

The optimization and simulation models were coded in Visual C++ and run on a 350-MHz Pentium II. The optimization model used is an internal subroutine; the quadratic programming code of K. Schittkowski (personal communication, 1998). The simulation and optimization algorithms were tested separately on standard problems published in the literature and were found to perform well. A detailed description can be found in Gordon et al. (1999).

APPLICATION

The aquifer under consideration is similar in geometry, domain properties, and well locations to the Na’aman aquifer located in western Galilee, Israel (but cannot be claimed to have full and accurate data and is, therefore, defined as ‘‘similar’’). The Na’aman aquifer is shown in Fig. 1. The geology and hydrology of the aquifer were studied by Natural Resources Development, Ltd. (NRD) (1997), which also used for this purpose a 3D finite-difference model of the aquifer based on MODFLOW and MT3D (Zheng 1990). Data about aquifer properties and initial distribution of saline water were taken from this study.

Ground water is replenished by 55–60 10^6 m^3/year from precipitation in the eastern part of the aquifer, which is unconfined. The current extraction is about 30 10^6 m^3/year, and the residual of 25–30 10^6 m^3/year flows through the Na’aman spring in the western part and through the western boundary toward the sea. The Na’aman spring is a large natural outflow of saline water (600–900 ppm of chlorides) with annual discharge that reaches 25–27 10^6 m^3/year. An increase of pumping will decrease the spring flow. There are 32 pumping wells in the aquifer; most of them are located in the western part (Figs. 1 and 2) owing to the locations of the water users and to the geological structure of the domain.

The wells located in the eastern part always pump freshwater, whereas in the western part, which is confined, some of the wells pump freshwater and others pump saline water of different salinities. Saline water is used by agriculture for irrigating certain crops and for fishponds. The demand for water in the region is rising, and the increased water extraction is causing salinization of the western wells; some of them have shut down in recent years due to salinization. The management horizon is 20 years, taken as one time period; i.e., pumping rates are kept constant over this time.

Numerical Model of Aquifer

The model considers confined 2D vertically averaged ground-water flow and saline water transport. The domain of the aquifer used in the model is shown in Fig. 2. It is divided into 12 zones according to their properties, which vary by orders of magnitude. Many faults are present in the aquifer and some of them are assumed to be sources of saline water and are incorporated into the model. The main data for the simulation model are presented in Table 1.

The aquifer is replenished by freshwater in Zones 1 and 3. Regions 5–7, 9, and 11 present the faults, and saline water intrudes through them.

The boundary is impervious except in two parts: between Points 11 and 12 (Fig. 2) is a boundary with constant head (h = 0), and between Points 13 and 14 is a boundary with outflow that represents the spring located near this boundary.

The grid of the numerical model is irregular, it is denser in

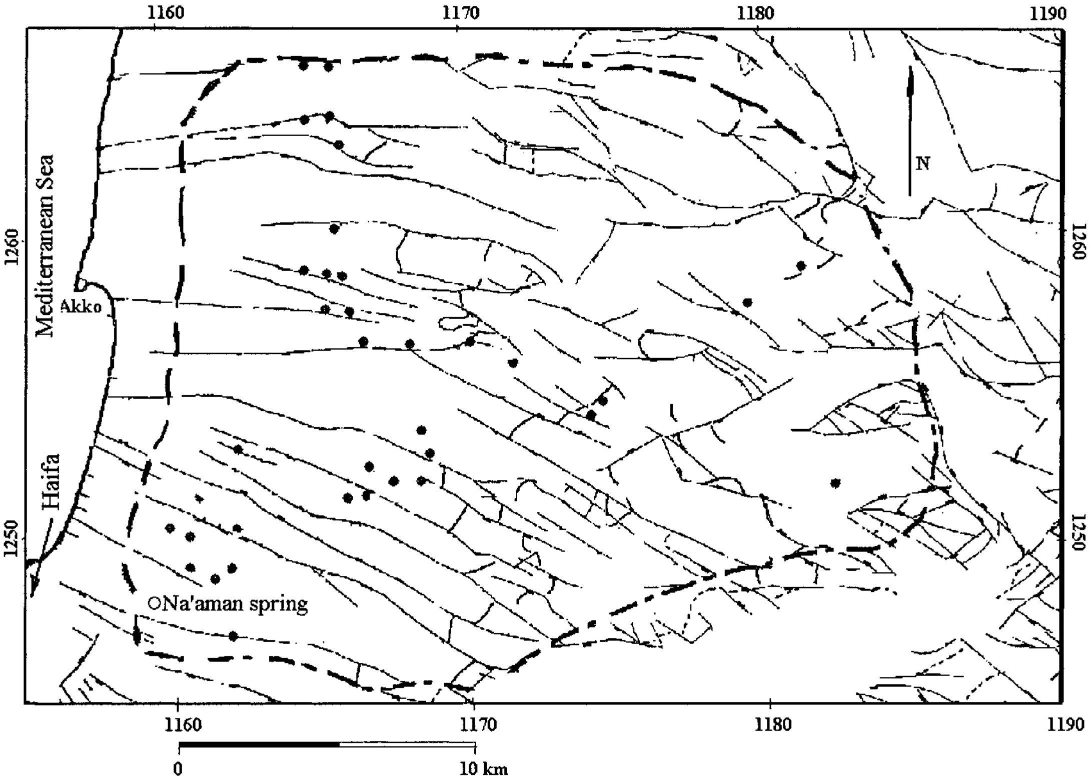

FIG. 1. Map of Na'aman Aquifer, Western Galilee, Israel (Dashed Line Indicates Aquifer Boundary; • Indicates Well Location; Lines Indicate Faults)

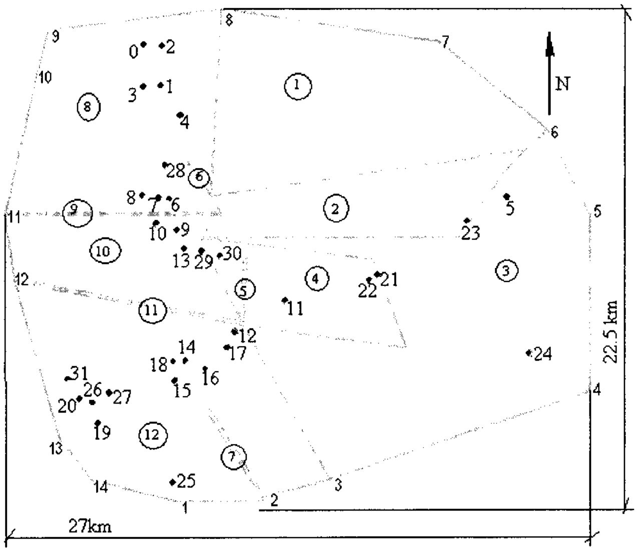

FIG. 2. Model of Aquifer (Dotted Lines Divide Domain into 12 Regions Indicated by Circled Numbers; • Indicates Well Locations)

TABLE 1. Data for Regional Aquifer Management Problem

Parameter (1)	Eastern part[a] (2)	Western part[b] (3)	Faults[c] (4)
Hydraulic conductivity K (m/day)	10	20	200–300
Porosity n	0.03	0.04	0.06
Aquifer thickness b (m)	120	120	120
Longitudinal dispersivity a_L (m)	10	20	200–300
Transverse dispersivity a_T (m)	1	2	20
Water leakage flux w (m/day)	7–9 10^{-4}	0	8–20 10^{-4}
Salinity of water leakage (mg/L)	0	0	2,000
Spatial zone discretization (m)	700–800	400–600	300–400

[a] Zones 1 and 3.
[b] Zones 2, 4, 8, 10, 12.
[c] Zones 5–7, 9, 11.

the regions around the wells and contains 1,734 nodes and 3,251 triangular elements (Fig. 3). The location of all wells and the history of their pumping rates are taken from NRD (1997). These data were used to calculate the initial salinity distribution at the beginning of the management period by simulation of the aquifer behavior averaged over the past 10 years.

Analysis of Optimal Management Strategies

The trade-off between the two objectives was investigated by combining them into a single objective [(5)], then changing the relative weights parametrically to generate the trade-off curve shown in Fig. 4. The upper limit on the total amount of water pumped from the aquifer was set equal to the average annual replenishment by precipitation (57 10^6 m^3/year). The lower limit was set equal to 31.8 10^6 m^3/year, which is slightly higher than the pumping rate today. The lower and upper limits for each well are zero and 8,000 m^3/day, respectively.

All optimization runs were started from the same initial pumping rates (Table 3), which were taken as average pumping over the 10-year period from NRD (1997). The optimization runs took between 10 and 160 iterations; each requires a simulation run. For the problem presented here the simulation takes about 10 min; thus, a full optimization run takes from several hours to a few days.

The points in Fig. 4 show the values of the two objectives. The results of optimal strategies and maximum concentration over the 20-year management period at the pumping wells for Points 1, 2, and 4 from the curve are presented in Table 3. Point 1 is the extreme left (lowest pumping), Point 4 is the extreme right (highest pumping), and Point 2 is the third from the left.

As seen in Fig. 4, the increase of pumped water amounts

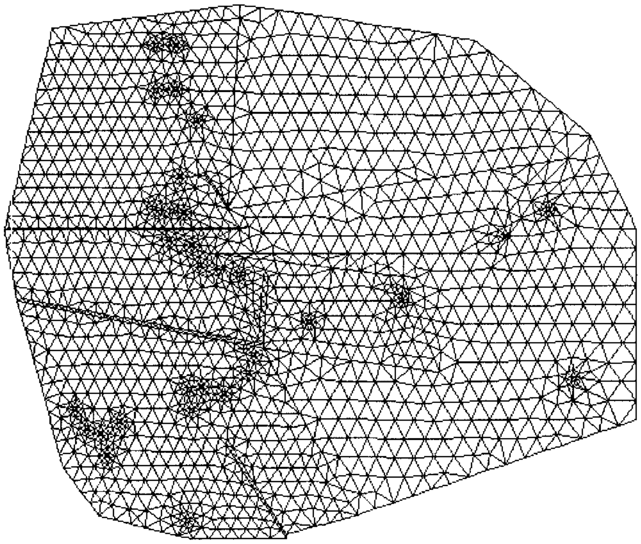

FIG. 3. Grid Discretization of Aquifer

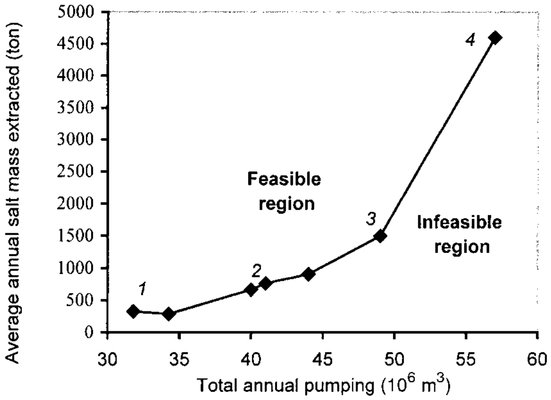

FIG. 4. Two-Objective Trade-Off Curve

TABLE 2. Results for Three Optimal Solutions

Parameter (1)	Initial value (2)	Point 1 (3)	Point 2 (4)	Point 3 (5)	Point 4 (6)
Total pumping (m^3/day)	85,430	87,170	120,234	134,246	156,226
Total mass extracted (ton/day)	5.061	0.913	2.482	4.11	12.603
Number of iterations	—	17	160	142	22

also increases the salt mass pumped from the aquifer because of the need to pump water from wells with higher salinity. One way for interpreting the results in Fig. 4 is as follows. Between Points 3 and 4 the pumping increases by 21,980 m^3/day while the amount of salt increases by 8.483 ton/day. This corresponds to an average additional salinity of the added water of 386 mg/L, which corresponds to the slope of the trade-off curve between Points 3 and 4.

As seen in Table 2, there is a significant improvement of at least one of the objectives at the optimal points from the initial pumping rates provided to the optimization search. The initial point corresponds to the real pumping, averaged over the last 10 years, and results in pumping of 31 10^6 m^3/year and salt mass extraction of 1,847 ton/year, spread over 20 years of the management period. Tables 2 and 3 and Fig. 4 show that the optimal amount pumped by each well in comparison to the nonoptimal (initial) strategy leads to

TABLE 3. Results of Optimal Strategies and Maximum Concentration over 20-Year Management Period

Well numbers (1)	Initial pumping rate (m^3/day) (2)	Optimal Pumping Rates (m^3/day)			Range of Concentration over 20 years (mg/L)		
		Point 1 (3)	Point 2 (4)	Point 4 (5)	Point 1 (6)	Point 2 (7)	Point 4 (8)
0	0	5,855	8,000	3,897	0–5	4–13	0–5
1	3,800	8,000	8,000	7,713	1–8	0–10	1–2
2	1,200	7,395	8,000	5,121	0	0	0–2
3	2,400	8,000	7,200	6,243	3–22	4–70	4–23
4	0	246	8,000	3,537	14–35	16–40	15–35
5	2,400	7,417	8,000	6,324	0	0	0
6	240	0	0	2,253	120–360	150–360	80–360
7	3,500	0	0	5,638	150–360	180–330	100–330
8	5,000	0	0	7,223	170–340	180–290	200–300
9	7,500	0	0	7,881	120–370	200–360	110–390
10	2,000	0	0	2,667	160–500	260–500	270–560
11	8,000	8,000	8,000	8,000	17–45	14–45	15–45
12	1,000	0	5,618	4,257	50–115	30–140	25–120
13	5,000	4,276	0	7,761	50–70	70–200	70–290
14	750	0	0	3,120	60–80	80–200	70–240
15	5,000	7,725	0	8,000	7–10	6–120	6–70
16	1,200	1,358	8,000	5,065	0–7	6–40	3–6
17	1,200	127	8,000	4,879	14–30	17–35	10–30
18	5,000	0	0	6,639	85–120	110–220	100–250
19	240	0	0	1,315	130–180	150–250	160–310
20	0	0	0	0	270–360	260–380	230–240
21	0	3,767	8,000	3,870	2–7	0–7	2–6
22	7,500	8,000	8,000	8,000	2–6	3–10	2–4
23	7,500	8,000	8,000	8,000	2–3	0–3	2–3
24	5,000	8,000	8,000	8,000	0	0	0
25	2,000	0	0	3,600	120–270	140–270	120–270
26	0	0	0	0	160–290	160–360	160–340
27	1,000	0	0	828	140–240	140–360	150–350
28	1,000	0	0	3,963	100–180	100–180	50–160
29	2,000	955	5,330	5,330	50–70	50–85	40–70
30	2,000	47	6,085	5,246	70–90	40–80	35–80
31	2,000	0	0	1,850	300–400	340–380	220–380

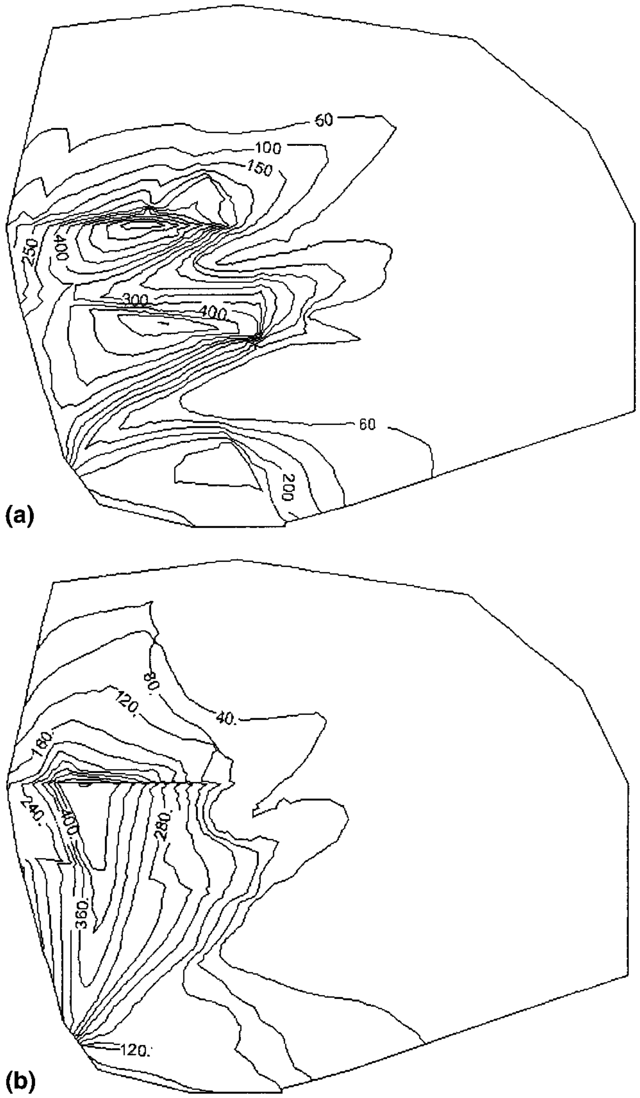

FIG. 5. At End of Management Period (20 Years) for Point 2: (a) Initial Concentration (Milligrams per Liter) Distribution; (b) Concentration Distribution (Milligrams per Liter)

- Decrease of the salt mass extracted by a factor of 5.5 without significantly changing the total pumping (corresponding to Point 1)
- Increase of total pumping by >60% without changing the total salt mass extracted (corresponding to Point 3 of the curve)

The solutions obtained are not claimed to be global optima, because there are wells where the salinity is low and the pumping rates did not reach the allowed upper limit. An expected result is low pumping rates in wells where the salinity is high and high rates where the salinity is low; this is satisfied most of the time.

At Point 4 the weight of the salt mass objective, which depends on concentration, is small, so the total pumping is at its maximum value and pumping rates are high even at wells with high salinity. Thus the increase of the pumping from 49 10^6 m^3/year (Point 3) to 57 10^6 m^3/year (Point 4) (a rise of 16%) results in an increase of salt mass extracted by a factor of 3. For the other points the changes are less drastic.

The most interesting points are probably around the middle of the curve, where the trade-off between the two objectives is significant. One of these points (Point 2) is presented in Tables 2 and 3. The optimal strategy for this case is to pump the maximum possible (8,000 m^3/day) at wells of low salinity, not pump at wells of high salinity, and gradually decrease pumping rates with an increase of salinity of the water pumped.

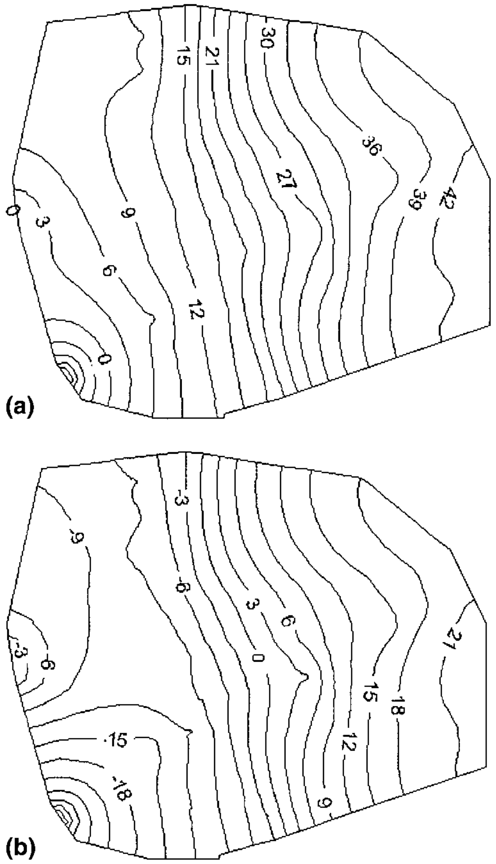

FIG. 6. Hydraulic Head Distribution (Meters) at End of Management Period for Two Extreme Points of Trade-Off Curve: (a) Minimum Pumping (Point 1); (b) Maximum Pumping (Point 4)

Fig. 5 presents the initial concentration distribution and the concentration distribution at the end of the management period (20 years) at Point 2 of the curve. For both cases there is outflow from the aquifer through the west boundary.

Fig. 6 presents the hydraulic head distributions for the extreme points of the trade-off curve (Fig. 4) of maximum and minimum total pumping. As shown in Fig. 6, the distributions are quite different. The increased pumping not only draws the head down but also changes the flow direction through the west boundary. In the case of minimum required pumping, there is outflow from the aquifer through the boundary between Points 11 and 12 (Fig. 2). In the case of maximum pumping, there is inflow into the domain through the same boundary and a head lower than zero within the aquifer exists due to intensive pumping. To prevent the change of the flow direction through the boundary, additional constraints on the hydraulic head at critical points of the domain should be imposed.

CONCLUSIONS

This study presents the development and implementation of a management model for a regional aquifer with saline water sources and zones. The objectives are to pump the maximum possible amount of water from the aquifer and extract the minimum salt mass. The second objective does not seem to have been addressed before. It is quite different from the conventional constraints on concentration level or equivalent objectives that are widely used in published models for aquifer remediation and ground-water quality management.

Our model uses a simulation-optimization approach for the solution of the optimization problem owing to the large number of dependent variables (salinity of water pumped at each time step) included in the second objective.

The Bundle-Trust optimization algorithm that was developed for nonsmooth and nonconvex problems was used in this study instead of conventional optimization methods that are appropriate for smooth problems.

The SUPG method for solution of the transport equation is used in addition to the original FE method for the flow equation. It avoids the oscillation in the solution for advection-dominated salt flow that occurs near the wells.

The model was applied to a regional aquifer similar to Na'aman in western Galilee, Israel, to determine an optimal use of 32 pumping wells located in it.

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