Management of Water Systems under Hydrological Uncertainty

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Abstract
Optimal management of water resources systems has been addressed mostly by deterministic models that assume perfectly known hydrological data. These models yield decisions which may perform poorly when implemented in the real world, as the problem data are revealed and differ from those assumed in the model. More sophisticated optimisation models that are stochastic in nature have been studied, but most of them require assumptions about the Probability Density Function (PDF) of the uncertain hydrological variables and their dependencies based on an historical sample. This paper opens with a brief review of a number of models that use this approach.

New optimisation methodologies have been developed in recent years that incorporate the intrinsic uncertainty and provide robust solutions, i.e., solutions that maintain feasibility over a prescribed range of parameter variability. A new non-probabilistic Robust Counterpart (RC) approach has been used to optimise the management of a regional water supply system under hydrological uncertainty. The uncertainty is captured by a user-defined ellipsoidal uncertainty set. The RC makes no assumptions about the PDF of the uncertain variables, does not require building a representative sample of scenarios, and requires the same computational burden as an equivalent deterministic model.

The methodology is demonstrated on a small hypothetical system and on a major part of the Israeli national water system. The results show considerable promise of the RC approach in terms of the tractability and the size of the model, as well as being able to show the trade-off between reliability and cost.

1. INTRODUCTION AND PERSPECTIVE

The theory and practice of water management have long recognised the need to

[1] Dedicated to the memory of Jim Dooge – mentor, colleague and friend

consider the stochastic nature of hydrological variables, while different classes of management problems depend on different variables of the forecasted future hydrology.

Some management domains require the magnitude and corresponding probability-of-occurrence or exceedence of forecasted extreme events, notably for protection against flooding and mitigation against droughts. In planning of storage for water supply, hydropower systems, and river flow control the relevant hydrological data are sequences of representative or critical seasonal to annual stream flow volumes or aquifer recharge values, while for real-time operation of these systems the relevant temporal resolution is finer and the time horizon shorter. The diversity of management issues and the corresponding hydrological data that are required has given rise to a huge range of methods and models for forecasting future hydrological variable.

The stochastic nature of hydrological data has been investigated extensively since the 1930s, with the objective of developing probability and time-series models for management of the planning, design and operation of water systems. The list of references attached to this paper (albeit brief) indicates the breadth of methodologies and applications. This paper presents a fraction of this broad range, and concentrates on our development and application work for planning of regional water supply systems under hydrological uncertainty.

1.1. Planning for Design Events

For design of drainage systems statistical analysis of historical precipitation records is used to select or construct design storms, of prescribed probability of occurrence, converted by rainfall-runoff models into design discharges at specified locations in the watershed. When peak discharge is the design variables, Unit Hydrograph methods (Dooge 1959, Nash 1960, USDA-TR55 1986) are used. When a full design discharge record is required then simulation models must be employed (EPA-SWMM; USGS-HSPF). In either case, a "design event with a prescribed probability of exceedence" is used, whose probability is selected according to the value assigned to the area or asset being protected. Howard (1976) used a statistical-analytical approach for determining the probability of untreated overflow from a reservoir that captures urban stormwater runoff and treats part of it in a plant with limited capacity before it is discharged into the receiving water body. Adams and Papa (2000) used the analytic-probabilistic approach to developed advanced and more comprehensive models for management of urban stormwater.

1.2. Design of Storage and Operation of Water Supply Systems

For planning the storage that is required in a water system the sequence of available water volumes in the sources (river, lake, and aquifer) is a crucial feature. Storage in water supply systems is designed to balance between time-varying availability at the source and the forecasted temporal variation of demands (Shamir and Howard 1981, Watkins and Vogel 1997, Ajami et al. 2008, Chung et al. 2009, Watkins and

McKinney 1997). The sequential order of available quantities, in particular runs of low values, will determine the size of storage that will be required to meet the demands with prescribed reliability. Therefore, the use of historical records must respect the serial dependence of values. The historical record itself can be used if it is long enough to contain an acceptable statistical sample of the relevant design events (e.g., size, depth duration of shortfalls), while synthetic hydrology methods purport to generate longer series based on statistical properties calculated from the historical data (Matalas 1967, Valencia and Schaake 1973, Hirsch 1979, Vogel an Stedinger 1988, Watkins and Vogel 1997, Mirus et al. 2011) so that the synthetic record displays a richer variability than the historical record (a matter of some debate). Brekke et al. (2009) extended the analysis to conditions of climate change.

1.3. Capacity Expansion of Desalination in Israel

For planning of the capacity expansion of desalination in Israel to 2050, the 75-year historical sequence (1935-2009) of annual recharge into the aggregate national reservoirs (aquifers and the Sea of Galilee) was "cycled around" so that it starts at one of the historical years, goes to the end of the series and then continues from first year. Each randomly generated set of values of the other stochastic variables (population, per capita demand, initial volume in storage, operational rules for management of the sources) is "met" by 75 sequences (same sequence, each started on a different historical year), thereby generating 75 values of the required desalination capacity that is required to "close the gap" with a prescribed probability (Gamzin et al. 2011). This stochastic simulation approach generated the forecasted (with probabilities and reliabilities) annual sequences of desalination development, storage in the sources (aquifers and Lake Kinneret), shortages and spills.

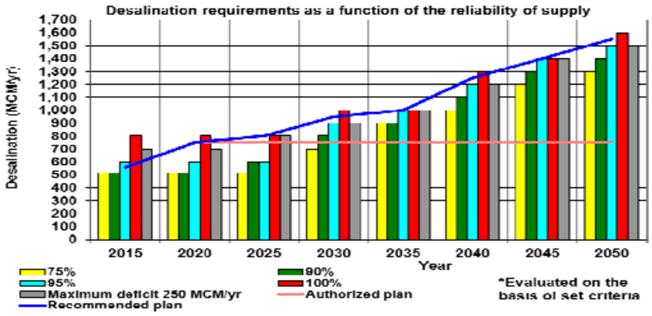


Fig. 1 Israel Water System: Desalination Capacity Expansion 2015-2050, for different prescribed reliabilities (Israel Water Authority 2012)

In this paper, we concentrate on a somewhat different planning paradigm, one that is not based on a probabilistic description of the hydrology but rather on a "domain of uncertainty". We address optimal operation of a water supply system that contains sources, consumers and an interconnecting conveyance system, over periods of time that range from months and seasons to years and decades.

Operation of regional water supply systems under uncertainty by other methods has been addressed by Mulvey et al. (1995), Escudero (2000), and Jenkins et al. (2004).

3. ROBUST COUNTERPART (RC) APPROACH

3.1. Motivation and Basic Paradigm

Consider the (demo) system depicted in Figure 2, fed from two aquifers whose recharge is uncertain, supplemented from a more expensive desalination plant that has a capacity to produce 120 mcm/year. The system is to be operated over a 10-year period, with two unequal seasons per year (3-month summer with high demands, the rest of the year with lower demands), to feed two consumer zones whose demands are known. The most significant decision is the desalination production over the years, which is used to meet supply reliability constraints when the aquifers fall short and/or when demands cannot be met by water from the aquifers. The other decision variables are seasonal pumping from the aquifers and deliveries through the network, all subject to capacity limitations, and state equations for aquifer volumes that are constrained between given limits. The recharge values are uncertain with partial statistical information. More specifically, the mean and the covariance of the recharge are assumed known, but the complete PDF is not known. The mean and the covariance matrix are estimated on the basis of historical hydrological data.

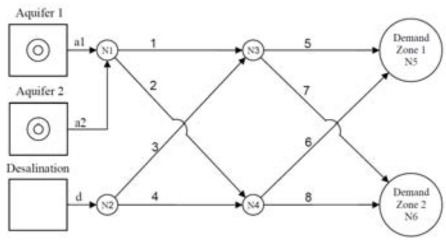


Fig. 2 Demo water supply system

The objective is to minimise the total cost of seasonal water production and delivery over the 10-year period: pumping from the aquifers, production in the desalination plant (quantities and their salinities), and transfer through the network to the consumers - to meet their growing demands over the years with salinities that do not exceed prescribed maximum limits.

The management problem recognises hydrological uncertainty by requiring that the optimal solution be feasible within a defined space of values of the stochastic recharges, without assigning a probability function over this space (Ben-Tal et al. 2009).

3.2. Capturing Hydrological Uncertainty

Figure 3 depicts the joint space of recharges into the two aquifers, showing the min-max ranges of each and a set of superimposed ellipses. The interior of the ellipse defines the range of recharges for which the optimal solution must be feasible. The decision maker can choose to narrow the ellipse, for example all the way to a single point at the average values of the recharges, have it bounded exactly within the min-max box, or expanded beyond. The size of the ellipse is given by a single parameter Θ that is specified by the decision maker.

The use of an ellipse relates to the solvability of the mathematical formulation of the robust counterpart optimisation algorithm and provides flexibility in controlling the size of the uncertain domain within which the solution is to remain feasible - by a single parameter; for details see Housh et al. 2012. Definition of the feasible as an ellipse also reflects the assumption that there is only a low probability for all stochastic variables (two in this demo problem) to take on simultaneously extreme values, so the corner points can be excluded. If the recharges to different aquifers are correlated then their covariance matrix is used to change the orientation of the ellipse, as shown in Figure 4, for positive (ρ >0) or negative (ρ <0) correlation.

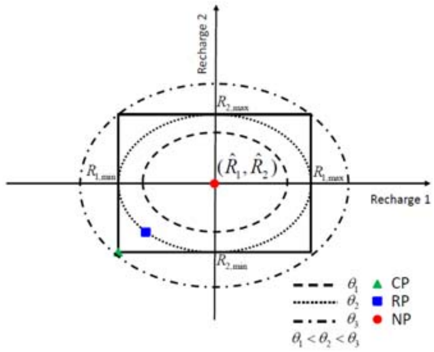


Fig. 3 Schematic joint space of recharges to the two aquifers, and delineation of the feasibility space (ellipses with different values of the parameter Θ)

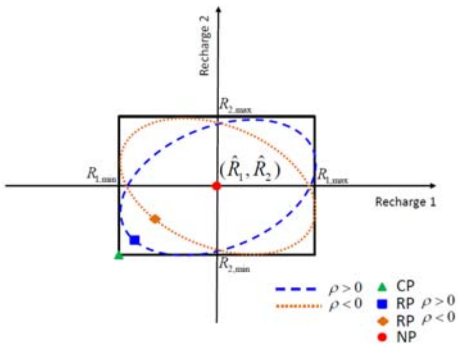


Fig. 4 Joint space of correlated recharges (positive ρ >0 and negative ρ<0)

Expansion/contraction (the size of the ellipse) reflects the decision maker's risk attitude: a risk-averse person can expand the range to safeguard against low recharge values, while an optimistic person might choose to narrow the range, thereby demonstrating more confidence in the mean recharge values, possibly even going as far as counting on a constant recharge equal to the historical average - a decision that is expected to lead to shortfalls.

The RC solution for different values of the reliability parameter Θ is seen in Figure 5. The conservative policy (CP) does not depend on the aquifers at all, and uses the full desalination capacity of 120 mcm/year throughout the time horizon. At the other extreme, the nominal policy (NP) assumes that the average recharge will be available year after year, and can therefore delay the use of desalination until demands can no longer be met reliably from the aquifers. NP reaches full use of 120 mcm/year only in year 6 (as the demands have grown). Following the expansion line for Θ =1 we see that in year 3 desalination has risen to 100 mcm/year. The vertical distance of this value from the 75 mcm/year for the NP is a measure of the extra caution this RP takes relative to the NP. The vertical distance of 30 mcm/year from the 120 mcm/year for the CP is a measure of this RP's policy confidence of getting some water from the aquifers.

Each solution was tested by 1,000 simulations with uniformly distributed random values for the two (statistically uncorrelated) recharges. Figure 6 summarises the results for the NP and RP (Θ =3) policies. It shows the final water levels in the two aquifers at the end of the 10-year period and the total cost without and with a shortage penalty added.

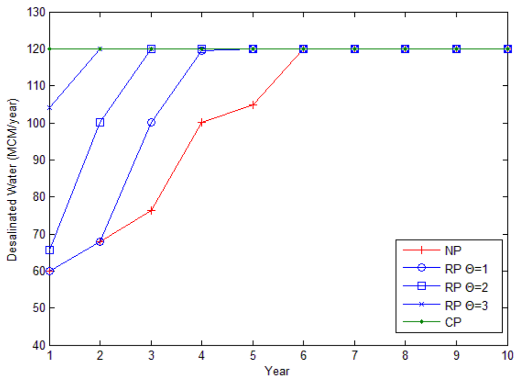
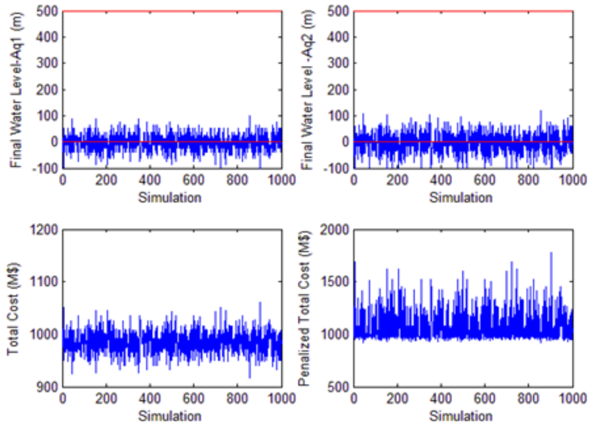
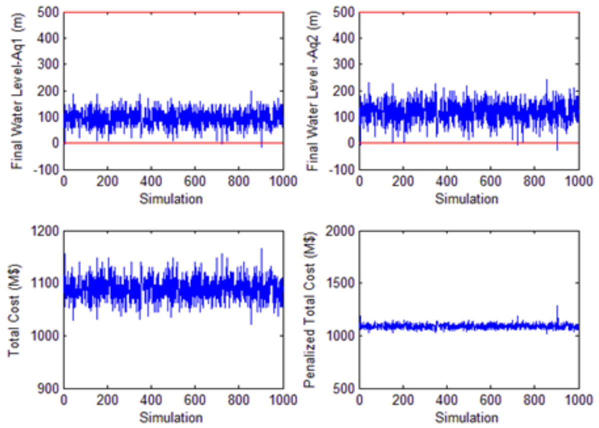


Fig. 5 Desalination capacity expansion paths for four reliability policies: NP=nominal policy, using average recharge values, RP= robust policy with different values of the reliability coefficient Θ, CP=conservative policy that depends only on desalination



(a) Nominal Policy (NP)



(b) Robust Policy (RP, Θ =3)

Fig. 6 Results of 1,000 simulations for the Nominal Policy (NP) and a Robust Policy (RP, Θ =3). For (a) and (b) the top panels are final water levels in the two aquifers and the bottom panels are the total cost without (left panel) and with shortage costs added (right panel)

From these results one observes: water levels for the NP oscillate around the reference value (denoted by a zero level) and are positive only 47% of the time, while the RP policy maintains levels above zero almost all the time (99.7%). The cost of the NP policy is higher than for the NP policy, but when penalties for shortage are added the NP policy competes better with the NP policy, and its total cost is much more stable.

These results are summarised in Figure 7 as trade-off between cost and reliability.

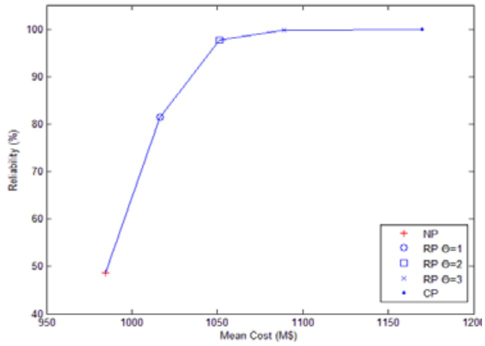


Fig. 7 Tradeoff between cost and reliability

This is the tool that is placed in the hands of the decision maker with the following observation: the point for Θ =2 improves the reliability from 82% to 98% for a cost increment of M$16, while going beyond it to the CP point adds a mere 0.3% to the reliability for a cost increment of M$125. Moving to the NP policy (Θ =0) drops the reliability to an unacceptable level of 47%. Therefore the RP (Θ =3) policy appears to be a good compromise solution in balancing between reliability and cost.

A deterministic optimisation model for flows and salinities was applied to the demo system (Figure 2) and to the central part of the Israeli national water supply system, depicted in Figure 8: three aquifers, four desalination plants (the one on top is a connection to northern part of the national system), 9 demand zones, 14 network nodes (Housh 2011, Housh et al. 2012a, 2013). An efficient optimisation algorithm was developed, using a number of manipulations that reduce running time substantially (Housh et al. 2012b). This model was used as a kernel of the RC model discussed above.

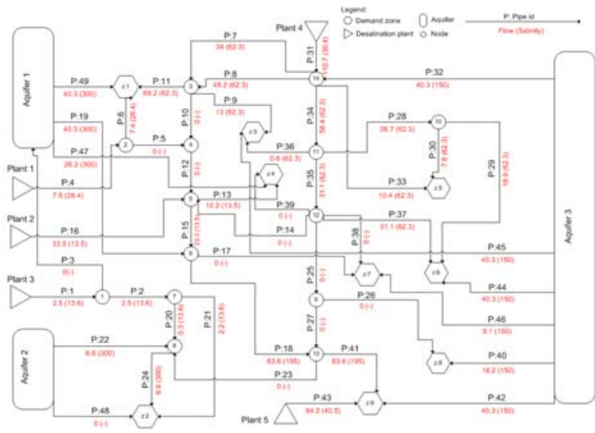


Fig. 8 Example of one season's flows and salinities in the central part of the Israeli National Water System

Additional approaches, methods and algorithms for management of regional systems under uncertainty have been developed by Housh (2011) and are presented in Housh et al. (2012b, 2013).

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References

Adams B.J. and Papa F. (2000) *Urban Stormwater Management Planning with Analytical Probabilistic Models*, Wiley.

Ajami N. K., Hornberger G. M., and Sunding D. L. (2008) Sustainable water resources management under hydrological uncertainty. *Water Resources Research*, 44(11) DOI: 10.1029/2007WR006736.

Ben-Tal A., El Ghaoui L., and Nemirovski A. (2009). Robust Optimization. Princeton University Press.

Brekke L.F., Maurer E.P., Anderson J.D., Townsley E.S., Harrison A. and Pruitt T. (2009) Assessing reservoir operations risk under climate change. *Water Resources Research* 45(4) DOI: 10.1029/2008WR006941.

Dooge J.C.I. (1959) A general theory of the unit hydrograph. *Journal of Geophysical Research*, 64(2): 241-256. DOI: 10.1029/JZ064i002p00241

EPA-SWMM (on the web). Storm Water Management Model. http://www.epa.gov/athens/wwqtsc/html/swmm.html (Retrieved January 2014)

Escudero L. F. (2000) WARSYP: A Robust Modeling Approach for Water Resources System Planning Under Uncertainty. *Annals of Operations Research*, 95:313-339.

Gamzin D., Zaide M. and Shamir U. (2011) *Statistical analysis of the gap between demand and natural supplies at the national level, to 2050* (in Hebrew) The Israeli Water Authority, for the Master Plan 2012 (in English) http://www.water.gov.il/Hebrew/Planning-and-Development/Planning/MasterPlan/DocLib4/MasterPlan-en-v.4.pdf

Hirsch R.M. (1979) Synthetic Hydrology and water supply reliability. *Water Resources Research* 15(6): 1603-1615.

Housh M., (2011) Optimal Management Regional Water Resources Systems under Uncertainty, Ph.D. thesis in Hydrodynamics and Water Resources Engineering, Technion - Israel Institute of Technology.

Housh M., Ostfeld A., and Shamir U. (2012) Optimal multiyear management of a water supply system under uncertainty: Robust counterpart approach, Water Resources Research, 47, W10515, doi:10.1029/2011WR010596.

Housh M., Ostfeld A., and Shamir U. (2012a) Seasonal multi-year optimal management of quantities and salinities in regional water supply systems. *Environmental Modeling and Software*, 37: 55-67

Housh M., Ostfeld A., and Shamir U. (2012b) Limited multi-stage stochastic programming for managing water supply systems. *Environmental Modeling and Software*, 41: 53-64.

Housh M., Ostfeld A., and Shamir U. (2013) An implicit mean-variance approach for optimal management of a water supply system under uncertainty. *Journal of Water Resources Planning and Management Division*, 139(6), 634–643.

Howard CDD (1976) Theory of Storage and Treatment-Plant Overflows. *Journal of the Environmental Engineering Division, ASCE,* 102(4) 709-722

Israel Water Authority (2012) Long-Term Plan for the National Water Sector, Part A: Policy Document, Version 4, August 2012. Available at: http://www.water.gov.il/Hebrew/Planning-and-Development/Planning/MasterPlan/DocLib4/MasterPlan-en-v.4.pdf

Jenkins M. W., Lund J. R., Howitt R. E., Draper A. J., Msangi S. M., Tanaka S. K., Ritzema R.S., and Marques G.F. (2004) Optimization of California's water supply system: Results and insights. *Journal of Water Resources Planning and Management*, 130(4): 271-280.

Matalas N. (1967) Mathematical assessment of synthetic hydrology. Water Resources Research. 3(4): 937-945.

Mirus B,. Loague K., Cristea N.C., Burges S.J., and Kampf S.K. (2011) A synthetic hydrologic-response dataset. Hydrological Processes. 25(23):3688–3692.

Mulvey J. M., Vanderbei R. J., and Zenios S. A. (1995) Robust Optimization of Large-Scale Systems. *Operations Research*, 43(2): 264-281.

Shamir U. and Howard C.D.D. (1981) Water Supply Reliability Theory. *Journal of the American Water Works Association*, July 1981 pp. 379-384.

USDA-TR55 (1986) Urban Hydrology for small watersheds. Technical Release 55.

USGS-HSPF (on the web). *HSPF–Hydrologic Simulation Program Fortran.* http://water.usgs.gov/software/HSPF/ (Retrieved January 2014).

Valencia D.R. and Schaake J.C. (1973). Disaggregation processes in stochastic hydrology. *Water Resources Research,* 9(3): 580-585.

Vogel R.M. and J.R. Stedinger (1988) The Value of Stochastic Streamflow Models in Overyear Reservoir Design Applications, *Water Resources Research*, 24(9) 1483-1490.

Watkins D. W., and McKinney D. C. (1997) Finding Robust Solutions to Water Resources Problems. *Journal of Water Resources Planning and Management*, 123(1): 49-58.