Water Distribution Reliability: Analytical Methods

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ABSTRACT: Probabilistic reliability measures for the performance of water distribution networks are developed and analytical methods for their computation explained. The paper begins with a review of reliability considerations and measures for water supply systems, making use of similar notions in other fields. It classifies reliability analyses according to the level of detail with which the water system is modeled, and then concentrates on methods relevant to networks. Two probabilistic measures, reachability (connection of a specific demand node to at least one source) and connectivity, are explored for use in water distribution systems. Two algorithms for their computation are presented, one for series-parallel networks and one for general networks. These measures are computed for two systems, each with ten nodes. Additionally, the probability that a given point receives sufficient supply is proposed for use as a reliability measure. For the calculation of this measure, an algorithm is provided that combines a capacitated network algorithm with a method to efficiently search through network configurations involving multiple link failures. This measure is calculated for the two sample systems.

Introduction

Traditionally, water distribution systems have been designed to be completely reliable. However, increasingly the scarcity of public money for construction and maintenance and the advanced age of many systems are causing the reliability of water distribution systems to become an important issue to water system designers and operators. Beyond a general agreement that systems should be "reliable," analysts do not concur on how reliability is defined, measured, or assessed for existing systems.

Conventionally, systems must conform to a limited set of reliability guidelines. For example, most systems are designed so that each demand point is supplied from two directions. Contingency analysis may also be performed for a few cases, e.g., to ensure that the system will still perform adequately when one pump has failed. However, reliability depends on the probabilistic occurrence of pipe and pump failures, whereas these fixed guidelines treat reliability deterministically.

Since the reliability of a system involves stochastic events, reliability shouldbeassessed by probabilistic measures. In contrast to deterministic

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Note. Discussion open until October 1, 1988. Separate discussions should be submitted for the individual papers in this symposium. To extend the closing date one month, a written request must be filed with the ASCE Manager of Journals. The manuscript for this paper was submitted for review and possible publication on March 26, 1987. This paper is part of the ***Journal of Water Resources Planning and Management,*** Vol. 114, No. 3, May, 1988. ©ASCE, ISSN 0733-9496/88/0003-0253/$l.OO + $.15 per page. Paper No. 22423.

methods, probabilistic methods implicitly or explicitly account for the likelihood and effects of each system contingency. These methods may be used for comparison and ranking of alternative system designs on the basis of reliability.

Reliability of supply must be defined in terms of selected measures (indices) that reflect the magnitude, duration, and/or frequency of service interruptions and supply shortfalls. This paper presents a survey of existing reliability measures and methods applicable to water distribution systems and develops new ones drawing on methods developed in other fields. The focus is on reliability measures that can be calculated analytically, as opposed to those calculated using simulation. However, interesting measures can be defined that cannot be calculated without a great deal of time and expense, if at all. One is therefore faced with a sometimes difficult choice of reliability measures to be used, requiring the balancing of the true relevance of a measure against the ability to actually compute it. Analytical methods are presented and developed for three useful probabilistic reliability measures, specifically: (1) The probability that a given demand point in a system is connected to a source; (2) the probability that *all* demand points in a system are connected to a source; and (3) the probability that a system can meet a specified level of flow at each demand point. The first two measures, called in this paper connectivity and reachability, respectively, can be calculated with recently developed network reliability methods. Two algorithms for these measures are presented, the first for series-parallel networks (Satyanarayana and Wood 1982), the second for general networks (Rosenthal 1977). The third probabilistic flow measure has received less attention in the literature. A method for the calculation of this measure is developed, based on an algorithm for reliability of electrical circuits by Lee (1980). Sample networks will be analyzed with each of these methods.

The causes of unreliability addressed in this paper are those within the system itself, e.g., pump malfunctions and pipe breaks. The methods presented can be used to assess the reliability of systems of moderate size, e.g., 10-25 demand points and 10-50 links.

This paper focuses on analytical methods. However, many available analytical methods involve fairly stringent assumptions that limit the applicability of these results for understanding real-life systems. Features such as limited-capacity storage tanks or operational responses to failure events are difficult to incorporate into reliability calculations for any system of even moderate size. Stochastic (Monte Carlo) simulation can provide a useful augmentation to an analytical analysis. Once an initial assessment of the reliability of a water distribution system has been performed analytically, and alternative improvement options proposed, a simulation of these options should be done to gain a better understanding of how the proposed alternative systems will be likely to behave under real-life conditions. Work using stochastic simulation for the assessment of water distribution system reliability is presented in a companion paper (Wagner et al. 1988).

This paper summarizes the research report by Wagner et al. (1986), wherein more detail can be found.

MEASURES OF RELIABILITY AND THEIR COMPUTATION

Reliability is one of the system objectives. Reliability measures should reflect the important aspects of shortfalls as they relate to different users and to the spatial and temporal distribution of their demand. One might consider many different measures of reliability. Most measures, however, may be too difficult to use for analyzing a real system. A balance must therefore be found between the measures one would like to use and those that are practical to compute.

It is interesting to examine reliability measures that have been adopted in other fields involving networks, e.g., power, telephones, and computer engineering. Some of these measures may prove useful for analyzing water distribution systems, although one must bear in mind the different physical laws that govern flow in the various networks and the different effect that shortfalls have in these services.

We present in the following, a brief review of the literature on reliability of networks in general and water supply systems in particular. More detail and a critical evaluation can be found in the research report (Wagner et al. 1986), which also contains an extensive bibliography. Mays and Cullinane (1986) also provide a good review of previous work on various aspects of water supply reliability.

Reliability analysis can be categorized according to the level of detail or aggregation that is used to represent the system. Three categories may be identified: (I) Lumped supply, lumped demand; (2) delineated supply and transmission, lumped demand; and (3) both supply and demand delineated, namely networks.

For a general study of reliability, the system may be viewed as a single supply area connected to a single demand area (Fig. 1). Reliability measures for such models have been developed by Endrenyi (1978), Billinton and Allan (1984), Shamir and Howard (1981, 1985), and Hobbs (1985b). Measures applicable to lumped systems include: (1) Loss of load expectation (LOLE)—the expected percentage of time during which load will exceed capacity; and (2) frequency of loss—the expected number of shortfall events per unit time. Bibliographies on these methods and their application appear in Billinton (1972) and the IEEE Subcommittee on the Application of Probability Methods (1978).

For a somewhat more detailed analysis, the system's supply may be modeled with more components, as in Fig. 2 for a system analyzed by Tangena and Koster (1983). Each component of the supply system is characterized by the probability function of time to failure and time to repair. When these distributions are assumed to be exponential, the mean time between failures (MTBF) and the mean time to repair (MTTR) are the only parameters needed to fully characterize the system.

Methods for analyzing power systems at this level of detail have been used by Barlow and Proschan (1975), Lie et al. (1977), and Billinton and Allan (1983, 1984). For water systems, these methods have been applied by Tangena and Koster (1983), Shamir and Howard (1985), and Hobbs (1985a).

Fault-tree analysis is also often used to analyze these partially delineated systems. Fault-tree analysis considers the different ways in which component failures lead to supply shortfall and computes the associated

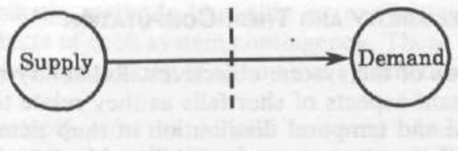


FIG. 1. System with Lumped Supply, Lumped Demand

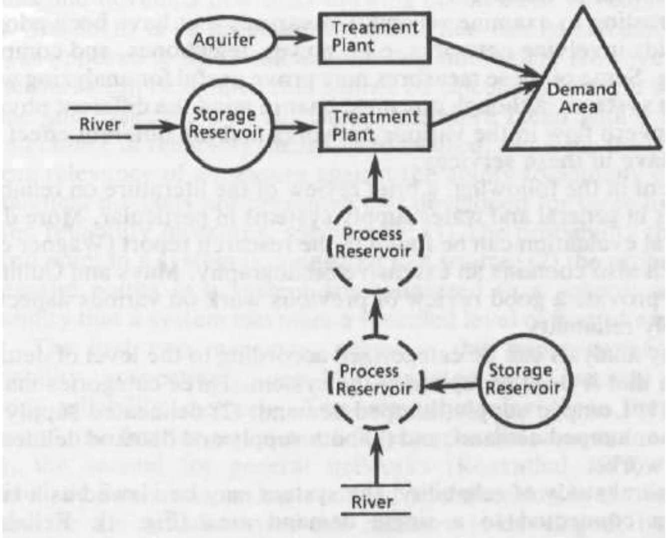


FIG. 2. Bulk Transmission System (Tangena and Koster 1983)

probabilities. Henley and Kumamoto (1981) and Ang and Tang (1984) give useful introductions to this field, and Willie (1978) and De Jong et al. (1983) provide application of these methods to power and water systems, respectively.

For a more detailed analysis that considers the distribution system itself, we need a network model, such as shown in Fig. 3. Network reliability has been studied in the fields of communications theory and operations research. Much of the work concerned the probability that a continuous path exists between two specified nodes, that all nodes in the network are connected, or that groups of nodes are connected (Ball 1980). Several researchers [e.g., Provan and Ball (1983)] have shown that calculating these reliability measures exactly is NP-hard for general networks. Methods for computing some of the simpler measures and approximations of more difficult ones have been developed in recent years for power and communications networks (Rosenthal 1977; Buzacott 1980; Satyanarayana and Wood 1982; Ball and Provan 1983; Agrawal and Barlow 1984; Agrawal and Satyanarayana 1984; Johnson 1984; Provan and Ball 1984).

Some work has also considered that links in the network have a limited

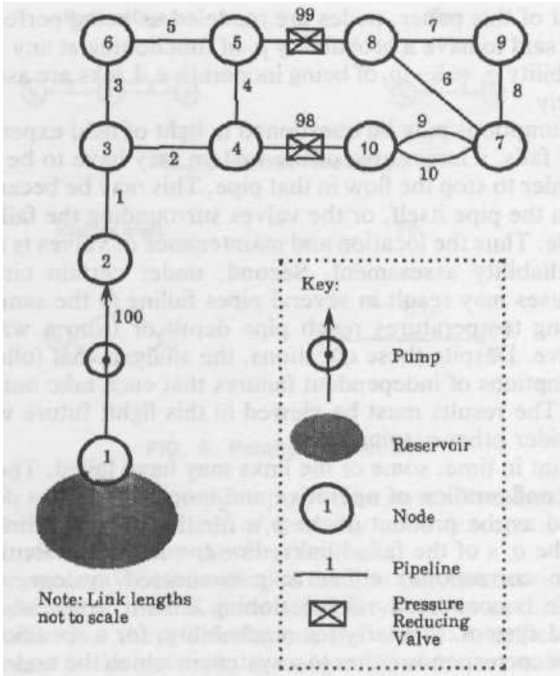


FIG. 3. Network A

carrying capacity (Willie 1979; Aneja and Nair 1980; Lee 1980; Rosenthal 1981; Shogan 1982; and Carey and Hendrickson 1984).

REACHABILITY AND CONNECTIVITY

Two analytical methods for calculation of the probabilistic reliability measures of reachability and connectivity are presented in the following. The first method is applicable only to series-parallel networks, the second to general networks. "Reachability" of a specified demand node denotes the situation in which this node is connected to at least one source. "Connectivity" denotes the situation in which *every* demand node in the network is connected to at least one source. Since any one node will be connected whenever the entire system is connected, it is obvious that the reachability for any node will always be greater than or equal to the connectivity for the network as a whole.

For water distribution systems, connection to a source is only a necessary, not a sufficient, condition to ensure that a given node is functional. For example, although a fully operational path may exist between a water source and a given demand node, if insufficient pressure exists in the system, this demand node may not receive any water. Measures of connectivity and reachability are useful for performing an initial screening of the system, identifying systems with serious problems due to insufficient redundancy, and indicating nodes with serious supply shortages. A more elaborate analysis is needed to determine whether a node that is connected can also meet its demand.

In the rest of this paper, nodes are modeled as being perfectly reliable. Each link is said to have a probability p_i of functioning at any point in time and a probability $q_i = 1 - p_i$ of being inoperative. Links are assumed to fail independently.

These assumptions may be questioned in light of field experience. First, when a pipe fails, a larger area of the system may have to be taken out of service in order to stop the flow in that pipe. This may be because there are no valves on the pipe itself, or the valves surrounding the failed pipe may be inoperable. Thus the location and maintenance of valves is an important factor in reliability assessment. Second, under certain circumstances, common causes may result in several pipes failing at the same time, e.g., when freezing temperatures reach pipe depth or from a water hammer pressure wave. Despite these questions, the analysis that follows is based on the assumptions of independent failures that each take out of service a single pipe. The results must be viewed in this light; future work may be able to consider other assumptions.

At any point in time, some of the links may have failed. The probability of any one configuration of operative and inoperative links occurring can be calculated as the product of the p_i's for the operative links times the product of the q_i's of the failed links. For connectivity calculations, each configuration corresponds either to a connected system, where *every* demand node is connected via functioning links to some source, or to a disconnected system. Similarly for reachability, for a specified node each configuration corresponds either to a system in which the node is reachable from any source via functioning links or to a system in which the node is unreachable.

Conceptually, calculating the overall probability of a given system being connected or the probability of a given node being reachable is a straightforward combinatorial problem. For any system, these probabilities can be calculated by testing each configuration individually and adding up the probabilities of each configuration that is connected or for which a given node is reachable. However, there are an exponential number of configurations, Even for a 20-link system 2^{20}, or over 1,000,000 configurations, would have to be tested. Thus methods for calculating these probability measures must efficiently search and count connected configurations. The algorithms presented in the following provide two such efficient search methods.

Series-Parallel Networks

In order to discuss series-parallel networks, it is necessary to present a few definitions. This section follows the notation and definitions from Satyanarayana and Wood (1982). A series-parallel network is an undirected network reducible to a tree (network with no loops) by performing only series and parallel reductions. A series reduction (Fig. 4) can be performed by replacing two links *(u-v* and *v-w),* incident to the same node (node *v)* by one link *(u-w).* If the probabilities of operation of each original link are p, and p_2, respectively, the probability of operation of the new link will be p_1p_2. A parallel reduction (Fig. 5) can be performed by replacing two links connecting the same two nodes by one link. If the probabilities of operation of each original link are p_1, and p_2, respectively, and the probabilities of failure are q_1 and q_2, the probability of operation of the new

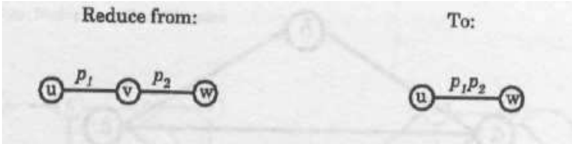


FIG. 4. Series Reduction

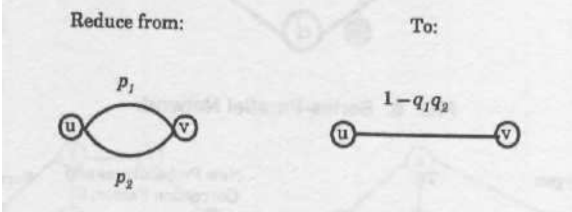


FIG. 5. Parallel Reduction

link will be $1 - q_1q_2 = P_1 + P_2 + P_1P_2$. The overall probability that the network will remain connected, or that any node (except u) will remain reachable, is not affected by these reductions.

Satyanarayana and Wood developed the first polynomial-time algorithm for the calculation of the "K-node reliability" of an undirected series-parallel network. For these calculations the analyst specifies a subset of nodes of interest in the network; these special nodes are called K-nodes. The K-node reliability is the probability that all nodes in set *K* can communicate with one another (possibly through nodes not in set K). For set *K* specified as all nodes, the K-node reliability corresponds to connectivity. For set *K* specified as two nodes, one a source and one a demand node, the K-node reliability corresponds to reachability for that node in a single-source system.

In this method, some restrictions on series and parallel reductions for components involving K-nodes exist. To maintain the correct reachability or connectivity for a network with K-nodes, two links in series can be reduced to a single link only when either: (1) Node *v* is *not* a K-node; or (2) *all three* nodes *are K*-nodes. If node *v* is not a K-node, a series reduction as shown in Fig. 4 can be performed. When all three nodes are K-nodes, the series can be reduced to a single link, as follows: (1) The probability of operation of the new link *(u-w)* becomes $p_1p_2/\{1- q_1q_2)$; and (2) the system reliability correction factor (denoted by 0) is multiplied by $(1 - q_1q_2)$. After the network has been reduced to a single link, the reliability of the network is found by multiplying the probability of operation of the final edge by the reliability correction factor O. The correction factor accounts for the necessity that the middle K-node be connected to the others, even though when the reduction is made, the middle K-node seems to "vanish."

Note that for a series-parallel network with a single source *s* and a single terminal *t,* it may not be possible to reduce the network via only series and parallel reductions to a single link *s-t.* Satyanarayana and Wood call series-parallel networks that cannot be reduced to a tree of K-nodes by series and parallel reductions "complex series-parallel" networks. A network G_K that can be reduced to a tree of K-nodes by only series and parallel reductions is called "series-parallel reducible." For example in

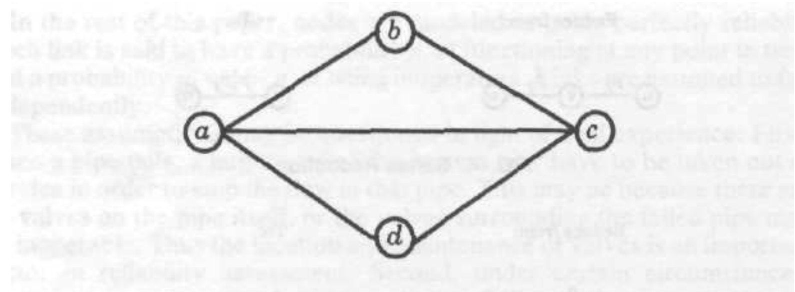


FIG. 6. Series-Parallel Network

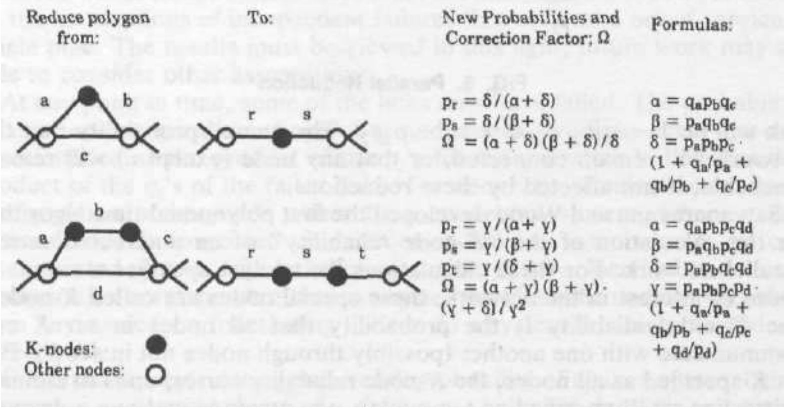
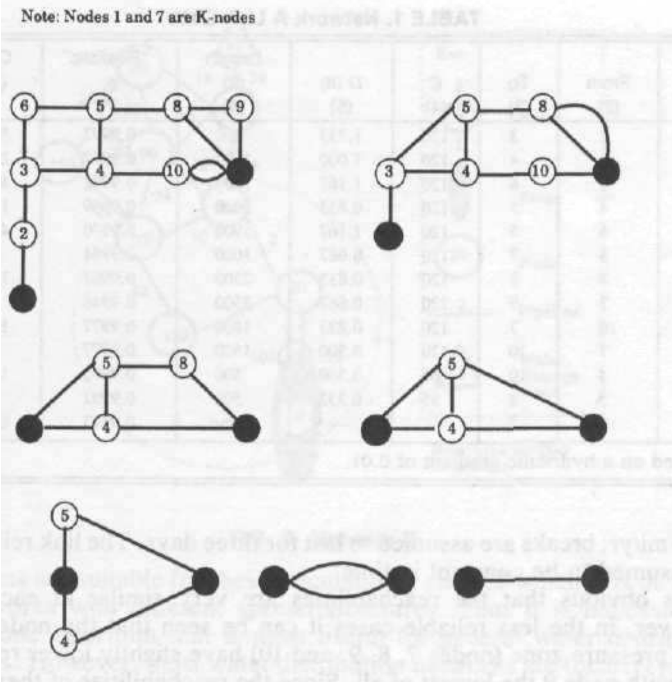


FIG. 7. Example Polygon-to-Chain Reduction (Satyanarayana and Wood 1982)

Fig. 6, if node *a* is chosen as the source node and node *c* as the terminal node [$K = (a,$ c)], the network G_K is series-parallel reducible. In contrast, if *b* is chosen as the source node and *d* as the terminal node [$K' = (b, d)$], the network G_K is series-parallel complex.

The algorithm proceeds by first making as many series and parallel reductions as possible. For a series-parallel network, if simple series and parallel reductions do not reduce the system to a simple tree of K-nodes, Satyanarayana and Wood prove that the reduced network will always contain one of seven polygons. These polygons can be reduced to a series connection of two or three links. Fig. 7 shows two such polygon-to-chain reductions. Similarly to the previously described series reduction, when all three nodes are K-nodes: (I) probabilities of operation for the "new" links (P_r ,P_s ,P_t) are calculated; and (2) factors are multiplied into the system reliability correction factor 0. In all cases, a polygon of three or more edges is reduced to two or three links connected serially.

Once the polygons have been reduced, more series and parallel connections can be identified and reduced. By repeating the reduction process, the network will be reduced to a tree of only K-nodes, (usually a single edge). For a system reduced to a single edge, the system reliability equals the P_i for this edge multiplied by the correction factor. Satyanarayana and Wood provide a formal algorithm for this process and prove that the



Reachability of node 7 $= [1 - (1 - p_a p_{99} p_f)(1 - p_r p_g)][(\alpha + \delta)(\beta + \delta)/\delta][p_1 p_{100}]$,

$p_f = 1 - q_6 q_c$

$p_g = p_{98}(1 - q_9 q_{10})$

$p_r = \delta/(\alpha + \delta)$

$p_s = \delta/(\beta + \delta)$

$\alpha = q_2 p_b q_4$

$\beta = p_2 q_b q_4$

$\delta = p_2 p_b p_4(1 + q_2/p_2 + q_b/p_b + q_4/p_4)$

FIG. 8. Reachability Reductions for Network A, Node 7

computational time involved is proportional to the number of links in the networks; i.e., this is a linear-time algorithm.

The network A in Fig. 3 is series-parallel, thus the reachability and connectivity can be calculated by this method. Network A with 13 links has $2^{13} = 8{,}192$ different combinations of operational and failed links, yet the connectivity can be calculated by hand with only a few pages of calculations, using series and parallel reductions. Fig. 8 shows the network reductions necessary to calculate the reachability for node 7. Reachability calculations for this node involve several series and parallel reductions and one polygon-to-chain reduction.

The link data for network A is given in Table 1. Numerical values for the connectivity and reachability of network A are given in Table 2 for a number of different link reliabilities. The last set of link "realistic" p_i's is based on reliability data from pipe and pumps under actual use (Damelin et al. 1972; O'Day 1982). The pump is assumed to break on average eight times a year for 50 hr each time. The pipes are assumed to have 1

TABLE 1. Network A Link Data

Link (1)	From (2)	To (3)	C (4)	D (ft) (5)	Length (ft) (6)	"Realistic" p_i (7)	Capacity[a] (gal/day) (8)
1	2	3	120	1.333	200	0.9997	5,946,000
2	3	4	120	1.000	1500	0.9977	2,790,000
3	3	6	120	1.167	1800	0.9972	4,185,000
4	4	5	120	0.833	2000	0.9969	1,727,000
5	6	5	120	1.167	1900	0.9970	4,185,000
6	8	7	120	0.667	1000	0.9984	960,000
7	8	9	120	0.833	2500	0.9961	1,727,000
8	7	9	120	0.667	3500	0.9946	960,000
9	10	7	120	0.833	1500	0.9977	1,727,000
10	7	10	120	0.500	1500	0.9977	451,000
98	4	10	65	0.500	500	0.9992	1,716,000
99	5	8	65	0.333	500	0.9992	649,000
100	1	2	—	—	pump	0.9543	6,675,000

[a]Based on a hydraulic gradient of 0.01.

break/mi/yr; breaks are assumed to last for three days. The link reliabilities are assumed to be constant in time.

It is obvious that the reachabilities are very similar at each node. However, in the less reliable cases it can be seen that the nodes in the lower pressure zone (nodes 7, 8, 9, and 10) have slightly lower reachabilities, with node 9 the lowest of all. Since the reachabilities of these nodes are so alike, the connectivity value is a good summary measure for this network. From the last case, with the varied link probabilities, it is obvious that the connectivity of this system is very close to the probability of operation of the first link—the pump.

General Networks

For non-series-parallel water distribution networks, more complex algorithms must be used to calculate reachability and connectivity. Several

TABLE 2. Connectivity and Reachability for Network A

Node (1)	Case 1 $p_i = 0.95$ for all i (2)	Case 2 $p_i = 0.97$ for all i (3)	Case 3 $p_i = 0.99$ for all i (4)	Case 4 $p_i = 0.999$ for all i (5)	Case 5 "realistic" p_i's (see Table 1) (6)
			(a) Connectivity		
—	0.8902	0.9364	0.9796	0.9980	0.9540
			(b) Reachability		
2	0.9500	0.9700	0.9900	0.9990	0.9543
3	0.9025	0.9409	0.9801	0.9980	0.9540
4	0.8979	0.9392	0.9799	0.9980	0.9540
5	0.8977	0.9391	0.9799	0.9980	0.9540
6	0.8979	0.9392	0.9799	0.9980	0.9540
7	0.8952	0.9382	0.9798	0.9980	0.9540
8	0.8952	0.9382	0.9798	0.9980	0.9540
9	0.8930	0.9374	0.9797	0.9980	0.9540
10	0.8952	0.9382	0.9798	0.9980	0.9540

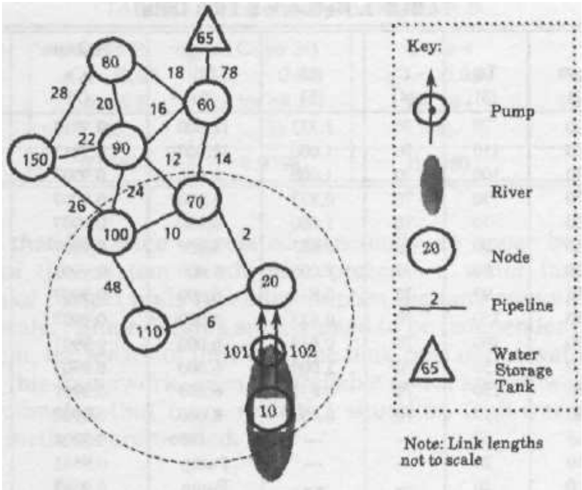


FIG. 9. Network B

algorithms are suitable for these calculations, each of which was designed for networks with different special structures. Thus it is impossible to choose one algorithm that is most useful for general water distribution networks. However, most water distribution networks are (nearly) planar, i.e., they can be drawn on a plane with (little or) no crossing of lines. Planarity limits the number of possible interconnections, thus water distribution networks are ensured to be relatively sparse.

Rosenthal (1977) presents an algorithm that can be used to calculate a number of reliability measures on general networks, including reachability, connectivity, and the *K-node* reliability. Although in the worst case the total computational effort required by this algorithm grows more than polynomially with the problem size, for "tree-like" and other sparse networks the total computational effort grows only linearly. Thus Rosenthal's algorithm should be useful for reliability calculations for many water distribution systems.

Rosenthal's algorithm decomposes the network into a number of subnetworks. Subnetworks are classified by the number of boundary nodes they contain. A boundary node is any node incident to an arc connecting the subnetwork to the larger network. In network B (Fig. 9), a subnetwork is circled. Nodes 70 and 100 are boundary nodes for this subnetwork.

For a subnetwork with two boundary nodes *(u* and *v),* called a 2-subnet, the possible configurations of this subnetwork can be divided into three separate classes: (1) "failed" (denoted *sf)* = states in which nodes are isolated from the boundary nodes; (2) *[u, v]* = states not in *sf* for which *u* communicates with *v* via the 2-subnet; and (3) *[u][v]* = states not in *sf* for which *u* does not communicate with *v* via the 2-subnet. Classes for subnetworks with more than two boundary nodes can also be identified by determining which subsets of the boundary nodes can communicate via the subnetwork and which cannot.

The algorithm is initialized by dividing the network so that each link is a separate subnetwork. The algorithm proceeds by combining in each step

TABLE 3. Network B Link Data

Link (1)	From (2)	To (3)	C (4)	D (ft) (5)	Length (ft) (6)	"Realistic" p_i's (7)	Capacity[a] (gal/day) (8)
2	20	70	70	1.333	12,000	0.9813	3,468,000
6	20	110	70	1.000	12,000	0.9813	1,627,000
10	70	100	70	1.000	6,000	0.9907	1,627,000
12	70	90	70	0.833	6,000	0.9907	1,008,000
14	60	70	70	1.000	6,000	0.9907	1,627,000
16	60	90	70	0.833	6,000	0.9907	1,008,000
18	60	80	70	1.000	6,000	0.9907	1,627,000
20	80	90	70	0.833	6,000	0.9907	1,008,000
22	90	150	70	0.833	6,000	0.9907	1,008,000
24	90	100	70	0.833	6,000	0.9907	1,008,000
26	100	150	70	1.000	6,000	0.9907	1,627,000
28	80	150	70	0.833	6,000	0.9907	1,008,000
48	100	110	70	0.667	6,000	0.9907	560,000
78	65	60	—	—	100	0.9998	2,160,000
101	10	20	—	—	Pump	0.9543	4,320,000
102	10	20	—	—	Pump	0.9543	4,320,000

[a]Based on a hydraulic gradient of 0.01 data.

two subnetworks with at least one boundary node in common. For the larger subnetwork the probability of it being in each of the possible classes is then calculated. The algorithm ends when all the subnetworks have been combined into one network. Then the probability of being in class *[u, v, w, . . . , z]* is the connectivity for the entire network.

Rosenthal does not specify the order in which the subnetworks should be combined. In fact, he indicates he "tried unsuccessfully to find a good algorithm to determine an optimal sequence of subnetwork merges to minimize . . . [the] total computational work." In one set of computational experiments on variations of a computer communications network linking some U.S. universities (ARPANET, as configured in 1974), Rosenthal used the heuristic rule: first the links in series or parallel configurations were combined, followed by successively merging 2-subnets into one common subnetwork. He was able to solve a 46-node/63-link problem with this algorithm and needed to employ only subnetworks of six boundary nodes or less.

The connectivity for the network in network B can be calculated by hand using Rosenthal's algorithm and the above heuristic rule. Network B is a portion of a test problem "Anytown" proposed by Walski et al. (1987) for optimization analysis at the June 1985 ASCE Water Resources Conference in Buffalo, New York. This subset of the Anytown network represents the central part of the city. This network is not series-parallel, e.g., the pentagon of nodes and links surrounding node 90 cannot be reduced at all with series or parallel reductions.

Table 3 presents the link data for network B. Numerical values of the connectivity of network B are given in Table 4 for a variety of link reliabilities. The connectivity calculated from the last "realistic" probability set is again very close to the probability of operation of the pump, despite the possible modifying influence of the water storage tank.

Because the nodes are modeled as perfectly reliable, the connectivity

TABLE 4. Connectivity for Network B

Case 1 $p_i = 0.95$ for all i (1)	Case 2 $p_i = 0.97$ for all i (2)	Case 3 $p_i = 0.99$ for all i (3)	Case 4 $p_i = 0.999$ for all i (4)	Case 5 "realistic" p_i's (see Table 3) (5)
0.8956	0.9383	0.9798	0.9980	0.9534

expression that has been calculated is actually an upper bound for the reliability of this system. Node 65 represents a water tank; thus the assumption of perfect node reliability implies the tank contains an infinite volume of water. Since failures are assumed to be independent of the state of the system, the length of time until the tank runs out of water cannot be modeled in this framework, even if unreliable nodes are allowed. To model this more complex (but more realistic) situation, fault-tree methods or simulation methods are needed.

Analysis

The major use of connectivity and reachability values is to provide a fast, easy-to-perform check for unreliability due to inadequate network interconnections or extremely unreliable links. For some systems, namely those that can be represented by series-parallel networks, the algorithm of Satyanarayana and Wood may be used to calculate reachability and connectivity simply and quickly. For more complicated networks, algorithms such as Rosenthal's exist for the calculations of these measures.

One possible problem with these methods is in determining the correct values for the probability of operation for each link. These values may be extrapolated from historical records of the system or estimated to be related to the length of a link with longer links being less reliable. These methods reflect the assumption that the probabilities are "instantaneous," i.e., they represent the probability of the link being operational at any point in time. Of course, once a pipe has failed it will stay failed for at least a few hours, so the probability that a pipe has failed at some moment in time is not completely independent of its previous operational state. For use as a fast first step in a reliability assessment, however, this point-in-time assumption is unlikely to be of much concern.

PROBABILISTIC SUPPLY MEASURES

A reachable node may still not receive sufficient supply at adequate pressure. Water supply is a function not just of the arrangements of links in the network but also of the amount of flow that can be carried along these links. Over time, as links randomly operate and fail, the amount of supply or the pressure of that supply will vary at any given node. Thus it would be useful to know for each node the probability distribution of these quantities. Unfortunately, methods for calculating these distributions, or even the mean and variances of these distributions, can only handle very simple systems with no more than a few nodes.

Demands for water vary over the day, the week, and the seasons, and develop over the years. As is the practice for planning and design studies, one must select a set of "design loads," i.e., one or more sets of demands at all nodes of the system, for which the analysis is carried out. For reliability analysis one can use an accepted standard, e.g., average daily rate on the maximum day of a normal year plus tire flow at one of several critical points in the system. The probability of being able to meet these demands, considering possible component failure, is a measure of system reliability. In this paper we do not address the question of how the set of design loads is selected. We assume that these design loads have been found and compute the corresponding probabilistic supply measures.

Conceptually, calculating probabilistic supply measures is again a combinatorial problem, almost exactly like the problem of calculating network connectivity and reachabilities. If each configuration of failed and operating links can be evaluated for its capacity to meet the demands, then together with the probabilities of failures, these capacities enable computation of reliability measures.

Testing for sufficient supply in a network is, however, much more difficult than testing for the connectivity of that network. Hydraulic networks are modeled by a simultaneous set of nonlinear algebraic equations. Computer programs for solving for the pressures and flows in hydraulic networks do exist (Shamir and Howard 1968; Walski 1984), but they employ relatively costly iterative techniques. It is also impossible to use the solution of the full system to deduce the solution for a reduced system, since flows are rerouted in complex ways. Even with an efficient algorithm for searching among system configurations, a large number of configurations must be tested requiring considerable amounts of computer time. Thus, in order to calculate these probabilistic supply measures, some easily solved approximation of the hydraulic performance of the network must be found.

One possible means for approximate computation of the capacity of each reduced network, with one or more links failed, is to use a capacitated network model. Each link is assigned an upper bound on its capacity. Once link capacities are fixed, determining if a given configuration can meet the specified demand without exceeding the capacities of each link is a classical transshipment network flow problem with well-known solution methods [see, e.g., Bradley et al. (1977)]. Still, finding the *probability* that a network with unreliable links will operate involves testing many individ-ual network configurations. It can be proven (Valiant 1979) that even for series-parallel networks, the problem of finding the probability that each node will receive sufficient supply is NP-hard.

One of the conceptually easiest methods for calculating probabilistic capacity measures was developed by Lee (1980). Lee's algorithm, based on lexicographic ordering, provides a search strategy among the possible configurations. At each iteration this algorithm determines a set of operational configurations and accounts for the probabilities of this set.

In the worst case and with a poor choice of initial configuration, this algorithm may search every one of the 2^L configurations (where L = the number of links in the network). However, for sparsely connected networks and networks that are not highly reliable, this algorithm is fairly efficient and therefore was chosen for this work.

The steps involved in Lee's algorithm are:

1. Find an initial feasible solution to the full network, namely a set of flows that provides the specified demand at each node without exceeding the capacity of any link.
2. Successively remove operational links and search for a set of flows in the reduced network that will meet demand without exceeding any link capacities.
3. Accumulate the probabilities of feasible configurations.
4. Repeat steps 2 and 3 for all links that participate in any feasible solution until all feasible configurations have been taken into account.

The classical spanning-tree-based transshipment algorithm [as described in Bradley et al. (1977)] was used to determine whether a set of flows could be found to meet demand without exceeding any link capacities for the individual network configurations. Feasible flows for the reduced network can be easily found as the minimum cost solution to the original network when the cost per unit flow on the arcs are as follows: *(I)* The cost for each failed arc is one unit; and *(2)* the cost for each operational arc is zero units.

The feasible flows carried on each link of the full network, when every link is operating, are supplied as an initial feasible solution to the transshipment problem for the reduced networks. If the minimum cost solution to this problem has zero cost, a feasible solution using only operational arcs is possible. If the minimum cost solution is greater than zero, no solution is possible without using failed arcs, indicating that the reduced system cannot meet the specified demand.

The computer program used for the analysis of the following sample systems employed Lee's algorithm (to search among configurations) linked to the classical transshipment algorithm (to test the individual configurations). In this program the initial feasible solution to the full network is required as a starting point. It was not necessary to implement a search for the initial feasible solution, because for networks of the size that can be handled by Lee's algorithm, it is a simple matter to find the initial feasible solution by hand.

It is not clear how to determine the capacity of each link. One upper bound on capacity is the maximum pressure drop the pipe can withstand. However, it is unlikely that there will ever be sufficient pressure in the system to obtain this limit. Another possibility is to use the maximum and minimum admissible pressures at the ends of the pipe to compute the resultant flow. More realistically, a maximum gradient or a maximum velocity in the link could be specified.

As an attempt to avoid very complicated procedures, but to still reflect practical limits on the flow in each pipe, the maximum flow in each pipe was set to the flow that would occur if the pipe was installed at a gradient of 0.01. Thus the maximum flow in each link was given by the Hazen-Williams equation (Walski 1984)

$$V = 0.55 \times C \times D^{0.63} \times S^{0.54} \quad \ldots\ldots (1)$$

or in terms of flow

$$Q = 0.646 \times V \times \pi \times \left(\frac{D}{2}\right)^2 = 0.2795 \times C \times D^{2.63} \times S^{0.54} \quad \ldots\ldots (2)$$

TABLE 5. Demands for Network A

Node (1)	Demand (gal/day) (2)
1	1,600,000
2	
3	1,200,000
4	600,000
5	400,000
6	875,000
7	600,000
8	800,000
9	400,000
10	200,000

where V = velocity (ft/sec); C = Hazen-Williams coefficient; D = pipe diameter (ft); S = slope (hydraulic gradient); Q = flow [million gallons/day (mgd)]; 0.646 = conversion factor from ft^3/sec to mgd.

If, in the actual system under normal conditions, the pipe was flowing with a gradient greater than 0.01, the actual gradient was used for S in Eq. 2. In practice this estimate worked very well, except for links between tanks and the system. For these links, it was finally easiest to determine the limits of flow by solving for the flows obtained under a few reduced configurations. For these analyses, SDP8 (Charles Howard and Associates, Ltd. 1984), a computer program for hydraulic analysis based on the method of Shamir and Howard (1968) was employed.

Network A was easy to analyze using this procedure. Table 1 presents link capacities calculated for this analysis. Table 5 contains the demand at each node. Table 6 presents the probability of sufficient supply for a number of cases. Connectivity values for each case are included for comparison.

The probability of the system being able to deliver the required supply is less than the probability of it simply being connected, particularly for the sets with less reliable links. The wide range of these probabilities is quite striking. For a system with $p_i = 0.95$, the probability of sufficient supply = 0.6586, compared with a probability of sufficient supply = 0.9920 for a system with $p_i = 0.999$.

For the last, more realistic, probability set under normal capacity conditions, the probability of sufficient supply is 0.9426, which is fairly close to the connectivity of 0.9540. For systems with higher demands, more demand points, and/or pipes operating closer to capacity under normal operating conditions, the difference in these values would be expected to be larger.

It is instructive to examine the minimum feasible configurations, namely, configurations for which the failure of any link will cause the system to fail. For capacities in the range used for these examples, links 1, 2, 3, 98, and 100 must operate for the system to operate. Link 99 (5-8) is necessary for all but two configurations under 1.2 x normal capacity (case VII), indicating this link is also quite crucial. For the normal capacity case *only,* the link 5 (4-5) is never in the minimum link set, indicating that the

TABLE 6. Supply and Connectivity Probabilities for Network A

Link probabilities (1)	Capacities (2)	Probability of sufficient supply (3)	Connectivity (from Table 2) (4)
All = 0.95	Table 3	0.6586	0.8902
All = 0.97	Table 3	0.7817	0.9364
All = 0.99	Table 3	0.9225	0.9796
All = 0.999	Table 3	0.9920	0.9980
"Realistic" set	Table 3	0.9426	0.9540
"Realistic" set	1.1 x Table 3	0.9476	0.9540
"Realistic" set	1.2 x Table 3	0.9483	0.9540
"Realistic" set	0.9 x Table 3	0.9426	0.9540

loss of this arc never causes the system to fail. However, for some of the minimum feasible configurations under 1.1 x normal capacity (case VI), link 5 is included. Thus it is expected that link 5 will add a small amount of reliability to the system. Link 10 (10-7), which is parallel to another link, is never in the minimum link set for *any* capacity condition. Thus link 10, perhaps as well as link 5, might be removed without affecting the reliability of the system to a large extent.

Calculation of the probability that every node receives sufficient supply for this series-parallel network required very little time for analysis, less than 3.0 CPU seconds on a VAX 11/750 per input set (set of probabilities, demands, and link capacities).

Calculation of the probability of sufficient supply for network B was somewhat more difficult than for network A. Network B, with 16 links and sections such as the pentagon around node 90, has much more redundancy than network A, with many more alternative feasible configurations. Thus this method, which finds feasible configurations, took considerably longer to analyze the larger network.

Table 7 contains the demand at each node of network B. SDP8 results show that the tank must supply water, and one pump must be operating for the system to meet demand at each node. When one pump is out, the tank supplies about 2.16mgd, so the capacity of the tank connection was set to 2.16 mgd. By similar reasoning, each pump capacity was set to 4.32 mgd, to allow the 6.05 demand to be met with one pump missing. Although these limits on the pump's and tank's link capacities seem to allow the system to operate without the tank supplying water, in fact the capacity limits on the pipes near the tank do not permit the network to operate without the tank in operation.

Calculation of the probability of sufficient supply for this non-series-parallel network required approximately two CPU minutes on a VAX 11/750 per case. Table 8 presents the probability of sufficient supply for a number of cases involving network B. The corresponding connectivity values are included for comparison.

TABLE 7. Demands for Network B

Node (1)	Demand (gal/day) (2)
10 (river)	—
20	720,000
60	720,000
70	720,000
80	720,000
90	1,440,000
100	720,000
110	720,000
150	290,000
65 (tank)	—

Again, a comparison of the probabilities of sufficient supply with the corresponding connectivity values indicates, as expected, the probability of the system being able to deliver the required supply is less than the probability of it simply being connected. The range of the probabilities is less than that of network A, with a probability of meeting the specified supply of 0.8463 when all links have p_i = 0.95, as compared with 0.9698 when all links have p_i= 0.99. Network B has many more minimum link feasible configurations than network A, three for network A versus 289 for network B. For the last, more realistic, probability set under normal capacity conditions, the probability of sufficient supply is 0.9514, which is quite close to the connectivity of 0.9534.

In all of the feasible configurations identified, the tank link (78) and at least one pump were operating as required. Also, links 2 and 6 (the links leading out of node 20) were operational in all feasible capacitated network supply solutions identified. Most of the configurations involved different combinations of failed links from the pentagon around node 90. 35 feasible configurations were identified with five failed links in the pentagon. Five links failed in the pentagon is the maximum possible number of failed links; six or more link failures in the pentagon cause some node to become disconnected. For all of these 35 configurations, links 2, 6, 10, 78, and at least one pump operate. Link 14 never operates in any of these 35 configurations, although it does operate in some of the other 254 identified

TABLE 8. Supply and Connectivity Probabilities for Network B

Link probabilities (1)	Capacities (2)	Probability of sufficient supply (3)	Connectivity (from Table 4) (4)
All = 0.95	Table 6	0.8463	0.8956
All = 0.97	Table 6	0.9085	0.9383
All = 0.99	Table 6	0.9698	0.9798
"Realistic" set	Table 6	0.9514	0.9534
"Realistic" set	0.9 x Table 6	0.9511	0.9534
"Realistic" set	1.1 x Table 6	0.9514	0.9534

feasible configurations. Increasing or decreasing the link capacities by 10% did not appreciably change this reliability measure for this network.

Analysis

As demonstrated by some of the previous examples, the probability that a network can provide sufficient supply at each node may be considerably less than the probability that it is simply connected. Thus except for systems in which every pipe carries supply well under its capacity, the supply-based reliability measures should be used.

The network model used for these calculations suffers from the same problems as the reachability and connectivity measures in the use of P_i-values to represent the reliability of each link. Again the uncertainty introduced by the p_i's will be very small compared to the uncertainty in the data used to estimate these values. Thus the use of these instantaneous probabilities, instead of ones accounting explicitly for the operate-fail-operate cycles, should not be of concern.

Of more concern is the use of explicit limits on pipe flow capacities. As previously discussed, pipes in networks do not truly have maximum capacities, since an increase in pump capacity can always force more water through the pipe (as long as the pipe does not burst). Additional tests were run with networks A and B in an attempt to determine how well the water distribution systems were represented by capacitated network models. This further analysis consisted of a check with SDP8 of the configurations identified as providing sufficient supply in the capacitated networks. With SDP8, fairly large networks can be solved for either supplies or pressures at each demand node. Of course, using SDP8 requires more computational effort than finding the minimum cost flows to meet supply on a capacitated network.

For network A, these and other comparisons of capacitated network models with the full network representation showed that this method does come very close to calculating the correct probability of sufficient supply. Also, by varying the pipe capacities by small amounts, useful information about the contributions made by individual components to the overall reliability can be gained.

Network B was not so easily checked. The network is too large to test with SDP8 linked into Lee's algorithm. However, the eight feasible configurations with six links out were analyzed with SDP8. (Seven or more failures will cause the network to become disconnected.) Unfortunately, all eight had pressures at one node (150) below the required 40 psi. However, node 150 was the only node below 40 psi for any of these configurations, with pressures for these configurations ranging from 16.4-36.4 psi.

A few other configurations with six failed links were examined by SDP8. These configurations also had minimum pressures at node 150 in the range of 28.7-34.9 psi. Thus the capacitated network model is perhaps somewhat imprecise at identifying feasible configurations for which one additional failure will cause some node to be undersupplied.

In all of the eight configurations with six failures, links 10, 18, and 12 were operating. A few configurations with some of these links failed were also analyzed with SDP8. Indeed, all of these configurations had nodes with pressures well below 40 psi (most had nodes with calculated negative pressures). Thus the capacitated network solutions appear, from this limited sample, to distinguish clearly infeasible configurations.

Note that for network A, the configurations not found by the capacitated network solution, or found incorrectly, all involve two or more failed links. This same pattern appears to be true for network B as well. For highly reliable components, the probability of having a number of links out at the same time is very small. Thus this method is likely to be more accurate for networks with fairly high p_i's.

In conclusion, the use of this extension of Lee's algorithm appears to provide a feasible analytical method for the calculation of the probability of sufficient supply in a network.

CONCLUSIONS

Analytical methods have been presented for the calculation of useful probabilistic reliability measures for water distribution systems. Measures of connectivity and reachability are fairly easy to calculate only for moderately sized, complex systems. Connectivity and reachability measures can be used to identify basic sources of unreliability in a system, such as lack of network interconnections or extremely unreliable links. In addition, nodes with reachabilities below those of others in the system may be initially identified as problem areas.

Following these relatively easy calculations, it is also important to calculate the more complex measures of probabilistic supply. Some difficulties were encountered with the calculation of these measures even on moderately complex networks. However, either by using a simplified (but still meaningful) representation of the network in question, or by obtaining faster and more sophisticated algorithms and software for these calculations, these probabilistic supply calculations should be possible for many water distribution systems. These measures will identify nodes in the network which, although connected, do not reliably receive the amount of water demanded.

Armed with the above information and calculations, analysts can pinpoint specific areas of concern and suggest possible improvements. To some extent these improvement options can be investigated by recalculating the connectivity, reachabilities, and probabilistic supply measures for the augmented system. However, due to the assumptions and limitations of analytical methods, more complicated analyses on more realistic representations of the system must be performed. System features not easily represented by analytical methods, e.g., tanks and possibilities of short-term adjustments in pump and supply rates, must be accounted for with other types of analyses, such as simulation.

There are a number of projects and areas for further research suggested by this work. As was discussed, the method developed for calculating probabilistic supply measures required large amounts of computer time for even moderately sized systems. However, other more sophisticated algorithms for these measures exist (Willie 1979) or could be extended from algorithms for other measures (Rosenthal 1977).

Simulation is useful for investigating in detail the most promising options for improvement of system reliability. The use of simulation for water distribution system reliability analysis is the subject of the companion paper (Wagner et al. 1988).

Appendix I. Units

To convert	To	Multiply by
inches (in)	centimeters (cm)	2.54
feet (ft)	meters (m)	0.3048
miles (mi)	kilometers (km)	1.609
gallons/day	meters³/second (m³/s)	4.39×10^{-8}

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APPENDIX III. NOTATION

The following symbols are used in this paper:

C Hazen-Williams coefficient;
D pipe diameter;
p_i probability that pipe *i* is operational-
q_i probability that pipe *i* has failed;
S slope (hydraulic gradient);
V velocity; and
Ω system reliability correction factor.