

OPTIMAL OPERATION OF MULTIQUALITY NETWORKS. I: STEADY-STATE CONDITIONS

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ABSTRACT: A model is developed for optimal operation of a multiquality water-supply system, under steady-state conditions. The system contains: sources of different qualities, treatment facilities, pipes, and pumping stations. The objective is to minimize total cost, while delivering to all consumers the required quantities, at acceptable qualities and pressures. A special approximation of the equation for water quality in pipes is used, which enables the model to select the flow directions in pipes as part of the optimization. The steady-state operation of an example system is optimized: it supplies six consumers from three sources, two of them with treatment plants, and has three pumping stations and 10 pipes. The optimization is carried out with GAMS/MINOS, which employs a projected augmented Lagrangian algorithm. The example system has been analyzed through a base run and four additional runs, aimed at studying the effects of modifications in key data. The optimal solutions of the five cases demonstrate the response to changes in economic and operational conditions.

INTRODUCTION

In recent years there is a growing interest in models for simulation (Males et al. 1985; Shah 1985; Liou and Kroon 1987; Grayman et al. 1988; Cohen 1992), optimal design (Cohen 1987), and optimal operation (Liang and Nahaji 1983; Schwartz et al. 1985; Sinai et al. 1985; Reike et al. 1987; Ostfeld 1990; Cohen 1992) of multiquality water supply systems. The interest in quality stems from three types of circumstances:

1. Use of waters from sources with different qualities in a single system, which serves to mix and convey them.
2. Concern over quality changes, as water flows through the network, due to decay and growth of various chemical and biological constituents.
3. Accidental entry of low-quality water into drinking-water-supply systems.

Our work deals directly with the first of these cases, but some of the methods and computational tools may be useful for the other two.

More and more frequently, "water" can no longer be considered a single commodity, and water networks are becoming multicommodity systems. Waters of different qualities are taken from sources, possibly treated, then mixed in the system, and a blend is supplied to the consumers.

We consider optimal operation of an existing water supply system with sources of different qualities, treatment plants, pumps, and pipes, which serves a number of consumers. In this paper we deal with steady state operation; operation under unsteady conditions is the topic of a companion paper (Ostfeld and Shamir 1993). Under steady-state conditions we assume

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Note. Discussion open until May 1, 1994. Separate discussions should be submitted for the individual papers in this symposium. To extend the closing date one month, a written request must be filed with the ASCE Manager of Journals. The manuscript for this paper was submitted for review and possible publication on May 29, 1992. This paper is part of the *Journal of Water Resources Planning and Management*, Vol. 119, No. 6, November/December, 1993. ©ASCE, ISSN 0733-9496/93/0006-0645/\$1.00 + \$.15 per page. Paper No. 4145.

at the supply sources (wells, reservoirs) fixed heads and concentrations of the quality parameters. Therefore they do not affect the model formulation. Under unsteady conditions they will become important. Water quality is expressed by the concentrations of certain constituents. For simplicity we shall refer to them as "pollutants"¹ although some, such as chlorine, may in fact be desirable.

The paper consists of two main parts: (1) Formulation of the model for optimal operation of a multiquality system under steady-state conditions; and (2) application to a system with three sources, two treatment plants, and 10 pipes, serving six consumers, with a single pollutant considered. The model is solved with GAMS/MINOS [general algebraic modeling system (Brooke et al. 1988)/mathematical in-core nonlinear optimization system (Murtagh and Saunders 1982)] on a personal computer.

MATHEMATICAL MODEL FORMULATION

Models of optimal operation of networks can be classified according to their consideration of time and of the physical laws that are included explicitly. In time the distinction is between policy and real-time models. Policy models are run off-line, in advance, and generate the operating plans for a number of typical and/or critical operating conditions.

Real time (on-line) models are run continuously in real time, and generate an operating plan for the immediate coming period. The classification with respect to the physical laws that are considered explicitly as constraints is:

1. *Q-H* (discharge-head) models: quality is not considered, and the network is described only by its hydraulic behavior.
2. *Q-C* (discharge-quality) models: the physics of the system are included only as continuity of water and of pollutant mass at nodes. Quality is described essentially as a transportation problem, in which pollutants are carried in the pipes, and mass conservation is maintained at nodes. Such a model can account for decay of pollutants within the pipes and even chemical reactions, but does not satisfy the continuity of energy law (Kirchoff Law No. 2), and therefore there is no guarantee of hydraulic feasibility and of maintaining head constraints at nodes.
3. *Q-C-H* (discharge-quality-head) models: quality constraints, and the hydraulic laws, which govern the system behavior, are all considered.

The model described herein is a policy model, concerned with the optimal steady-state operation of a multiquality undirected distribution network, under *Q-C-H* constraints. The distribution network contains sources of different qualities, pumping stations, treatment plants, and supply nodes. The optimal operation problem is to minimize the total cost of water, treatment, and energy, in compliance with the physical laws that govern the system behavior and with constraints at supply nodes, on water quantities, qualities, and pressures.

The equations that describe the quality in the network are based on the following main assumptions:

1. Mixing at nodes is complete and instantaneous.
2. The quality parameters are conservative; they do not decay with time, nor do they interact with each other.

The aforementioned second assumption constitutes a limitation on the applicability of the model. It may be possible to consider nonconservative (but noninteracting) pollutants—at some extra computational cost—by adding functions describing their attenuation during flow in a pipe, as has been done in water-quality simulation models (Liou and Kroon 1987; Grayman et al. 1988; Cohen 1992). This has not been tried in our work.

The nodes of the network are divided into three types:

1. *TSN* (treatment source nodes, indexed *isn*): nodes where there is a facility in which water quality can be improved.
2. *NTSN* (nontreatment source nodes, indexed *ntsn*): nodes where there is no facility to improve water quality, and therefore water quality is assumed fixed.
3. *IN* (internal nodes, indexed *in*): internal nodes of the distribution system, which include the consumption nodes, and nodes at which two or more arcs meet.

A is the set of arcs (indexed a), where most of them are pipes. A subset of the set A is A_{pumps} , on which pumping stations are located. Each arc is assigned an arbitrary positive direction. However, since arcs are generally undirected, the actual flow can end up being in either direction. Exceptions are where the direction of flow is known in advance; for example, arcs which connect the sources to the distribution system are always directed away from the sources.

CONSTRAINTS

The constraints are divided into three groups:

1. *Q-H* constraints: describe the hydraulics of the network and the head requirements at internal nodes.
2. *C* constraints: describe the pollutants behavior within the distribution network, and the quality requirements of the consumers at supply nodes (in the example described later we considered only one quality parameter).
3. Fixed values of three types: heads at sources (*TSN* and *NTSN*), inlet qualities to *tsn* nodes, and fixed qualities at *ntsn* nodes.

Q-H CONSTRAINTS

The hydraulics of the network are formulated in the form given by Kessler and Shamir (1989).

1. Continuity of flows at internal nodes (Kirchoff's law No. 1 for water):

$$RIM^{RN}q = W \dots\dots\dots (1)$$

where RIM^{RN} = reduced incidence matrix with respect to a reference node RN , in an undirected network; q = vector of discharge along arcs; W = vector of consumptions (except for the reference node RN) at nodes, ($W_i = 0$ if node i is not a consumption or a supply node).

2. Continuity of energy (Kirchoff's law No. 2):

$$FloopAHpipe(q) = LHloop \dots\dots\dots (2a)$$

where $Floop$ = fundamental loop matrix for a given (arbitrarily selected) spanning tree; $AHloop$ = head difference for a set of fundamental loops and paths ($AHloop = 0$ for loops, and equals the head difference between the end nodes of an open path); $AHpipe(q)$ = vector of heads losses or gains along arcs as follows;

$$\Delta H_{pipe}(q) = -\Delta H_{f_a}(q_a) \quad \text{if } a \in Apumps \quad \forall a \in A \dots\dots\dots (2b)$$

$$\Delta H_{pipe}(q) = \Delta H_{p_a}(q_a) - \Delta H_{f_a}(q_a) \quad \text{if } a \in Apumps \quad \forall a \in A \dots (2c)$$

where

$$\Delta H_{f_a}(q_a) = \Omega_a f_a \left(R_a, \frac{\varepsilon_a}{D_a} \right) \frac{L_a}{D_a^5} |q_a| q_a \quad (2d)$$

$\Delta H_{f_a}(q_a)$ - head loss along arc a , calculated by the Darcy-Weisbach formula; f_a , $(R_a, \frac{\varepsilon_a}{D_a})$ = friction factor, which depends on the Reynolds number (R_a), and the relative roughness ($\frac{\varepsilon_a}{D_a}$), where ε_a = a roughness coefficient (a measure of the roughness of the pipe); D_a = internal diameter of the pipe; L_a — arc length; q_a = discharge in arc a ; Ω_a = a constant which depends on the units used for L_a , D_a , and q_a . Assuming $f_a + R_H$ means that we expect in advance the flow in every arc to be turbulent, and therefore f_a to be constant. This assumption is reasonable in most water distribution systems, and is made herein. $\Delta H_{p_a}(q_a)$ = head gain at pumping station on arc a , given by:

$$\Delta H_{p_a}(q_a) = (\alpha_0^a q_a^2 + \beta_0^a q_a + \gamma_0^a) + (x_a - y_a) \quad \forall a \in Apumps \dots (2e)$$

where $\alpha_0^a, \beta_0^a, \gamma_0^a$ = coefficients for the a th pumping station at its maximum efficiency

$$\{q_a^i, \Delta H_{p_a}(q_a^i)\} \in \eta_a^{i,max} \quad j = 1 \dots, \text{ and } q_a^{min} \leq q_a^i \leq q_a^{max} \dots (2f)$$

q_a^i = the i 'th feasible discharge at the a th pumping station, which gives the maximum efficiency ($\eta_a^{i,max}$) among all possible combinations of pumps at the station; q_a^i is bounded between maximum (q_a^{max}) and minimum (q_a^{min}) possible discharges; x_a, y_a = dummy variables, which incur high (artificial) penalties in the objective function. These variables make it possible to obtain a mathematical solution to the problem even when the physical system cannot meet all the head constraints. If it can, then in the solution $x_a = y_a = 0$. If it cannot, then some of these variables will differ from zero, and it is necessary to examine the possible reasons for this infeasibility and correct it. Similar to the usage of the dummy variables x_a, y_a it is possible to make the intercept of the pumping curve $Q-H$, i.e., $-y_a$, a decision variable. Physically this is interpreted as the ability to change the pump rotation speed, and allowing the intercept γ_0^a to be set by the optimal solution. 3. Head constraints.

a. At selected internal nodes (usually at consumer nodes):

$$H_{in}^{min} \leq H_{in} = P^{RN} \Delta H_{pipe}(q) \leq H_{in}^{max} \quad \text{for some } in \in IN \quad (3a)$$

where H_{in} = total head at selected internal nodes; P^{RN} = path matrix, connecting the reference node RN with the prescribed subset of internal nodes; $H_{in}^{min}, H_{in}^{max}$ = minimum and maximum total heads allowed at the selected internal nodes, respectively.

b. Heads at source nodes are known:

$$H_{sn} = Head_{sn} \quad \forall sn \in SN \quad \dots \dots \dots (3b)$$

4. Discharge constraints for arcs. The purpose of these constraints is to limit the discharges to be within reasonable levels. Reasonable discharges are, for example, those which correspond to velocity values in the range of 0.5-4 m/s, or to hydraulic gradients in the range of 2-15‰:

$$q_a \in (q_a^{\min}, q_a^{\max}) \quad \forall a \in A \quad \dots \dots \dots (4)$$

For directed arcs $q_a^{\min} = 0$ and for undirected $q_a^{\min} < 0$.

C CONSTRAINTS

1. Continuity of loads (Kirchoff's law No. 1) at nodes, for quality parameter k

$$\sum_{a \in \xi^-(in)} C_{pipe_a}^k q_a = C_{in}^k q_{in} \quad \forall k \in K \quad \dots \dots \dots (5)$$

where K = set of water quality parameters, indexed k ; $\xi^-(in)$ = set of arcs connected to node, in ; $C_{pipe_a}^k$ = concentration of quality parameter k in arc a , computed by (6b); C_{in}^k = concentration of quality parameter k at node in ; q_{in} = discharge at node in ($q_{in} > 0$ when water is withdrawn from the node).

2. Quality in arcs. In a multiquality distribution network two or more types of water are mixed at nodes, called dilution nodes. One of the assumptions of the model is that dilution at these nodes is complete and immediate. The concentration along every arc, for steady-state conditions, is that of its upstream node:

$$C_{pipe_a}^k = C_{in}^k \quad a \in \xi^-(in) \quad \forall in \in IN, k \in K \quad \dots \dots \dots (6a)$$

where $\xi^-(in)$ is the set of arcs with flow out of node, in .

Eq. (6a) can be omitted at nodes with only one outgoing arc (Sinai et al. 1985). Embedding (6a) in a general nonlinear optimization problem of a multiquality undirected network requires the addition of binary variables to insure that the concentration along arcs, in steady-state conditions, is always the concentration of the upstream node. Since we deal with an undirected network, and do not wish to introduce binary variables to a nonlinear constrained optimization problem, we need to use a smoothing approximation for the concentrations equations, which allows the solution algorithm to reverse flow directions during the course of iterations, and still fulfill continuity of loads at internal nodes [(5)]. The equation we use is based upon a smoothing approximation, developed by Cohen (1992), and refined in our study. The equation is:

$$C_{pipe_a}^k = \frac{C_{ii}^k}{1 + \exp[-q_a PEN_a(q_a)]} + \frac{C_{jj}^k}{1 + \exp[q_a PEN_a(q_a)]} \quad \dots \dots \dots (6b)$$

$$PEN_a(q_a) = \frac{LN(NMAX)}{1 + |q_a|}; \quad \forall ii, jj \in IN, k \in K \quad \dots \dots \dots (6c)$$

where ii, jj = end nodes of pipe a \ C_{ii}^k, C_{jj}^k = concentrations of quality parameter k at nodes ii and jj , respectively; LN = natural logarithm; and $NMAX$ = a very large number. We used the maximum number possible for the PS/2 mod 80, which is $= 10^{22}$. The general form of (6b) is shown in Fig. 1. Table 1 gives the value of the concentration in the pipe, by (6b) for the range of: $-2,000$ (m^3/hr) to $+2,000$ (m^3/hr), with an accuracy better than 10^{-8} . Eq. (6b) need not be applied to arcs in which the direction of flow is fixed, such as pipes out of sources. The term $PEN_a(q_a)$ produces a sufficiently large penalty product $[r_{fi}/EA_{fi}(g_o)]$ even when q_a approaches zero.

3. Minimum and maximum concentrations allowed at internal nodes.

$$C_{in}^{k,\min} \leq C_{in}^k \leq C_{in}^{k,\max} \quad \text{for some } in \in IN, k \in K \dots \dots \dots (7)$$

where $C_{in}^{k,\min}, C_{in}^{k,\max}$ = minimum and maximum concentrations, respectively, of water-quality parameter k at some internal nodes. Such constraints

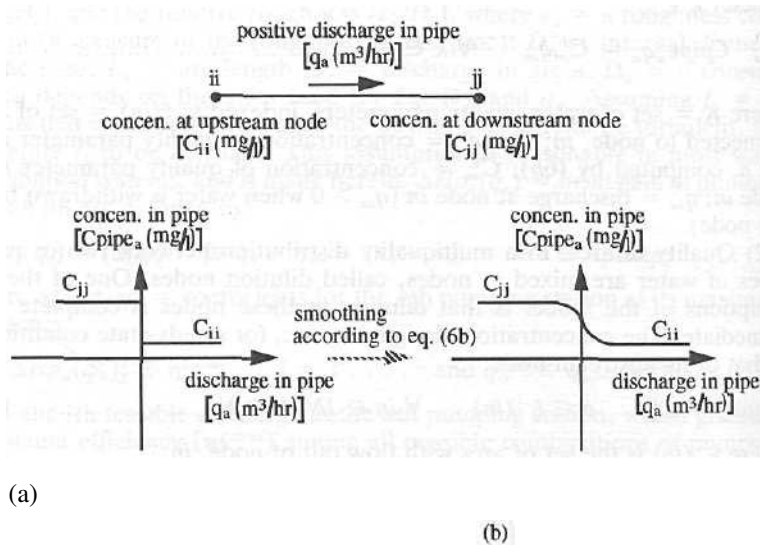


FIG. 1. Graphical Representation of Eq. (6b): (a) Discontinuity of Concentration along Arc when Flow Direction Reverses, Before Smoothing; (b) After Smoothing Discontinuity, by Eq. (6b)

TABLE 1. Computed Quality in Arc by Eq. (6b)

Discharge in arc q_a (m^3/hr) (1)	Concentration along arc C_{pipe_a} (mg/L) (2)
2,000	LOO
100	100
1	100
0	150
-1	200
-100	200
-2,000	200

are normally imposed only at consumption nodes. The minimum concentration level is usually zero, but there are quality parameters where a minimum level is also imposed, for example, residual chlorine for drinking water. 4. Concentrations in pipes originating at nontreatment source nodes.

$$C_{pipe_a}^k = C_{ntsn}^k \quad \forall a \in \xi^-(ntsn), ntsn \in NTSN, k \in K \quad \dots (8)$$

where $B, \sim(ntsn)$ = set of arcs with flow out of node $ntsn$; C_{ntsn}^k = the concentration of water quality of parameter k at source node $ntsn$.

OBJECTIVE FUNCTION

The objective function includes two terms: (1) Water cost, which is the sum of water cost at source nodes and the treatment cost needed to improve water quality at treatment source nodes; and (2) energy cost needed to operate pumping stations.

COST OF TREATMENT

The resulting quality at the outlet of a treatment source node is approximated here by an exponential function:

$$C_{pipe_a}^k(TC_{tsn}^k) = C_{initial_{tsn}}^k \exp(-Ktc_{tsn}^k TC_{tsn}^k) \quad \forall a \in \xi^-(tsn),$$

$$tsn \in TSN, k \in K \quad \dots \dots \dots (9a)$$

where $\xi^-(tsn)$ = arcs directed away from the node tsn ; $C_{initial_{tsn}}^k$ = concentration of quality parameter k at the inlet of source node tsn ; TC_{tsn}^k = treatment cost of quality parameter k , per unit volume of treated water, at source node tsn ; Ktc_{tsn}^k = coefficient of the treatment cost of quality parameter k at source node tsn . A schematic representation of the outlet quality for a single quality parameter, as a function of treatment cost, is shown in Fig. 2.

REMARKS

1. TC_{tsn}^k are decision variables. When no treatment is needed (i.e., concentration constraints are met without treatment, possibly by dilution) the treatment cost is zero.

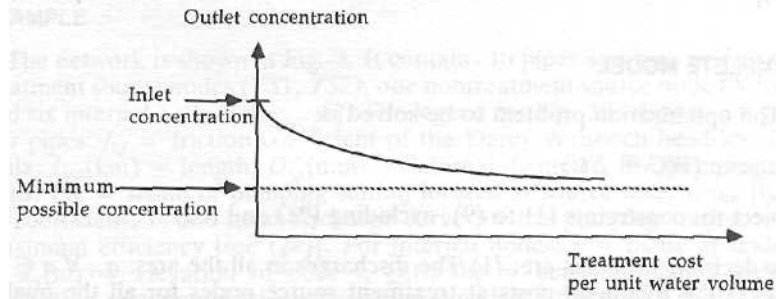


FIG. 2. Cost Function at Treatment Source Node

2. Treatment cost, per unit volume of treated water, increases as the concentration of the quality parameter k in the outflow is reduced. However, the treatment level of any quality parameter is bounded by the maximum removal ratio of the treatment facility, and therefore the treatment cost at any treatment source node is bounded. This is expressed by imposing bounds on the removal ratios at treatment source nodes for every quality parameter:

$$0 \leq \frac{C_{tsn}^{initial,k} - C_{pipe_a}^k(TC_{tsn}^k)}{C_{pipe_a}^k(TC_{tsn}^k)} \leq RR_{tsn}^{k,max} \quad \forall a \in \xi^-(tsn),$$

$$tsn \in TSN, k \in K \dots \dots \dots (9b)$$

where $RR_{tsn}^{k,max}$ = maximum removal ratio of quality parameter k at node tsn .

The cost of water and treatment is:

$$WC = DT \left[\sum_{ntsn \in NTSN} WC_{ntsn} q_{ntsn} + \sum_{tsn \in TSN} \left(WC_{tsn}^{initial} + \sum_{k \in K} TC_{tsn}^k \right) q_{tsn} \right] \dots \dots \dots (10a)$$

where WC = water cost; DT = time unit of operation (e.g., hour, day); WC_{ntsn} = fixed charge for unit volume of water at node $ntsn$; q_{ntsn} , q_{tsn} = discharge supplied by source node $ntsn$ and source node tsn , respectively; $WC_{tsn}^{initial}$ — fixed charge per unit volume of untreated water at node tsn .
The cost of energy is:

$$EC = DTkwhc \left[\sum_{a \in Apumps} ECCP_a q_a \Delta HP_a(q_a) \right] + penalty(x_a + y_a) \dots (10b)$$

where EC — energy cost; $kwhc$ = energy charge during time of operation; $ECCP_a$ = power coefficient of pumping station located on arc a ($a \in Apumps \subset A$), assuming efficiency is constant; $penalty$ = large positive number that incurs high (artificial) penalties in the objective function, on the dummy variables x_a , y_a . The introduction of these artificial variables makes it possible to obtain a mathematical solution to the problem even when the physical system can not meet all the head constraints [Note that penalty has no connection to the PEN terms in the smoothing equation (66)].

COMPLETE MODEL

The optimization problem to be solved is:

minimize ($WC + EC$)

subject to: constraints (1) to (9), excluding (2e) and (6a) (11)

The decision variables are: (1) The discharges in all the arcs: q_a , $\forall a \in A$; and (2) the treatment costs at treatment source nodes for all the quality parameters involved: TC_{tsn}^k , $\forall tsn \in TSN, k \in K$.

ASSUMPTIONS

The following assumptions were used in developing the optimization model:

1. The time of operation (DT) is much longer than the transient phase after a change in the operation, during which a steady-state concentration is reached.
2. Water-quality parameters are conservative.
3. There are no chemical reactions among quality parameters.
4. Complete and instantaneous mixing at nodes.
5. The power coefficient of pumping stations is calculated assuming efficiency is constant.
6. The cost of improving water quality at a treatment source node with respect to a specific water-quality parameter is independent of other treatment costs at the same node, i.e., improving water quality with respect to one quality parameter does not improve water quality with respect to other water-quality parameters in the same treatment plant.

Assumptions 1-3 allow us to employ a steady-state formulation with respect to quality parameters. If assumptions 2 and 3 were not valid, then the change of concentration of quality parameters as a function of time along the pipes and at the sources, should have been added. This is done in the companion paper (Ostfeld and Shamir 1993). In the next example we used only one conservative quality parameter.

The model is nonlinear and nonsmooth. Nonlinearities appear both in the objective function and the constraints. Nonsmoothness is due, for example, to the head-loss formula $|f|2d\lambda$ where the absolute value is used to allow the network to be undirected with respect to flows along arcs. Use of the smoothing approximation equation [(66)], permits the consideration of an undirected multiquality flow model, as it smooths the discontinuity of the concentration equation for arcs.

METHOD OF SOLUTION

The solution is obtained on a PS/2 mod 80 using GAMS/MINOS, which is a software package designed to solve large-scale optimization problems involving sparse linear and nonlinear constraints. GAMS is used to build the model, MINOS is used to solve it, by employing a projected augmented Lagrangian algorithm. A detailed description of the optimization technique can be found in Murtagh and Saunders (1982).

EXAMPLE

The network is shown in Fig. 3. It contains 10 pipes and nine nodes: two treatment source nodes ($TS1$, $TS2$), one nontreatment source node ($NTSY$), and six internal nodes (A . . . F). The legend for Fig. 3 is shown in Fig. 4. For pipes: f_{ij} = friction coefficient of the Darcy-Weisbach head-loss formula; L_{ij} (km) = length; ϕ_{ij} (mm) = internal diameter. For pumping stations: PS_j = name of pumping station located at source node j ; a_{ij} ($3/i$, yl) = coefficients of the head-discharge curve for the pumping station at its maximum efficiency [see (2e)], For internal nodes: i = name of node i ; Z_i (m) = elevation of node i ; Q_i (m^3/h) = consumption at node i ; Hf_i^m (m) = minimum total head required at node i ; C_i^{max} (mg/L) = maximum concentration allowed at node i . For source nodes: Q_i^{max} (m^3/h) = maximum

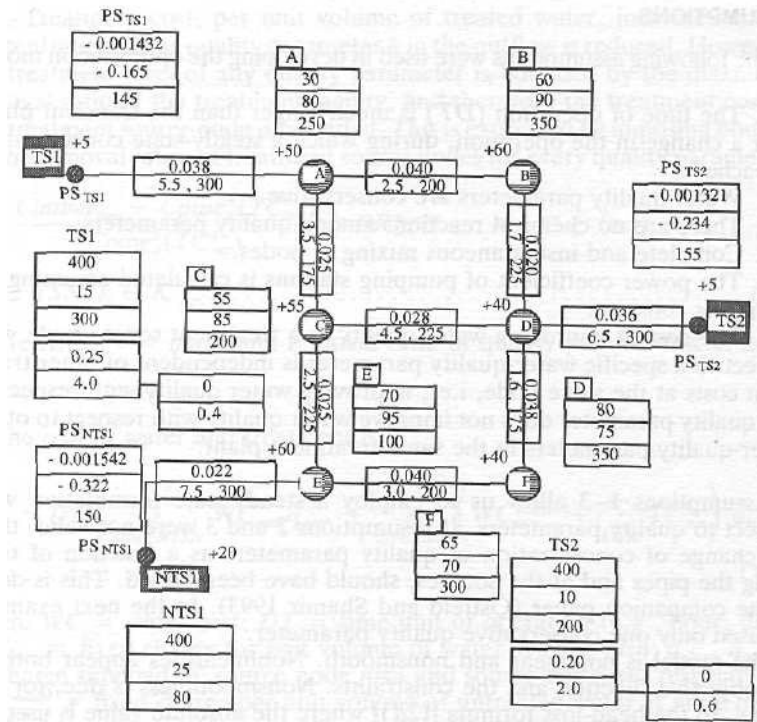


FIG. 3. BASE RUN DATA for Optimal Operation of Multiquality Network Example

discharge allowed from source node i ; $H_i(m)$ = head at source node i ; $C_i(mg/L)$ = inlet concentration to source node i where at source nodes $TS1$, $TS2$ water quality concentration may be improved by additional treatment, and at source node $NTS1$ water quality concentration cannot be changed. The elevations of source nodes $TS1$, $TS2$, and $NTS1$ are: 5, 5, and 20 (m), respectively. For treatment plants: $WC_i^{initial} (\$/m^3)$ = water cost before treatment at treatment source node i ; $ATC_i (\$/m^3)$ = cost coefficient of treatment cost function; $UBTC_j (\$/m^3)$ = upper bound on treatment cost, generated from the maximum removal ratio (zero is the lower bound, which corresponds to no removal). Additional data: water cost at nontreatment source node $NTS1$ equals 0.40 ($\$/m^3$); efficiency, assuming constant at all pumping stations is 0.80; time of operation 2 (h); and energy cost 0.08 ($\$/kw-h$).

Results for the base run are shown in Fig. 5. Legend for these results appears in Fig. 6. For pipes: $Q_v(m^3/h)$ = discharge (the faded arrow means that the flow in the final solution is in the direction opposite to that arbitrarily selected as positive for the pipe); $C_v(mg/L)$ = concentration in the pipe. For pumping stations: PS_i = name of pumping station located at source node i ; HPS_i = head (m); x_i, y_i = dummy variables. For internal nodes: i = name of node; $H_i(m)$ = total head; $C_i(mg/L)$ = concentration. For treatment plants: l = name of treatment source node; $CT_l(mg/L)$ = concentration at the outlet; $ZWC_l (\$/m^3)$ = treatment cost per unit volume of treated water. For the objective function: $EC(\$)$ = energy cost; $WC(\$)$ = water cost (treated and nontreated); PC = cost of operating the dummy

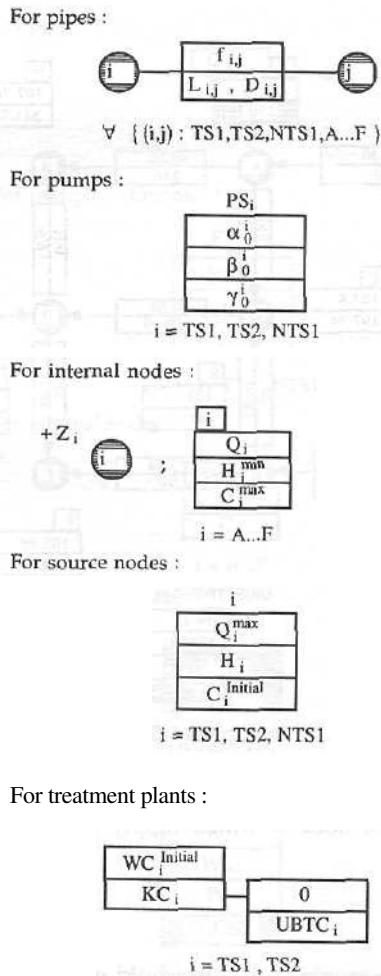


FIG. 4. Legend for Fig. 3 (Symbols Explained in Text)

variables; $TCOST(\$)$ ~ total cost of operating the system for 2 hours. Remark: A netted block denotes a binding constraint. For example the concentration at node A (refer to Fig. 5).

BASE RUN AND SENSITIVITY ANALYSIS RESULTS

The base data and results of the base run are shown in Figs. 3 and 5, respectively. Four more runs were made, to study how the optimal solution changes in response to modification in certain key data. In each case, four aspects of the results were examined.

Discharge (Q)

1. From which sources is water being withdrawn, and what is the relative contribution of each source node to total supply?

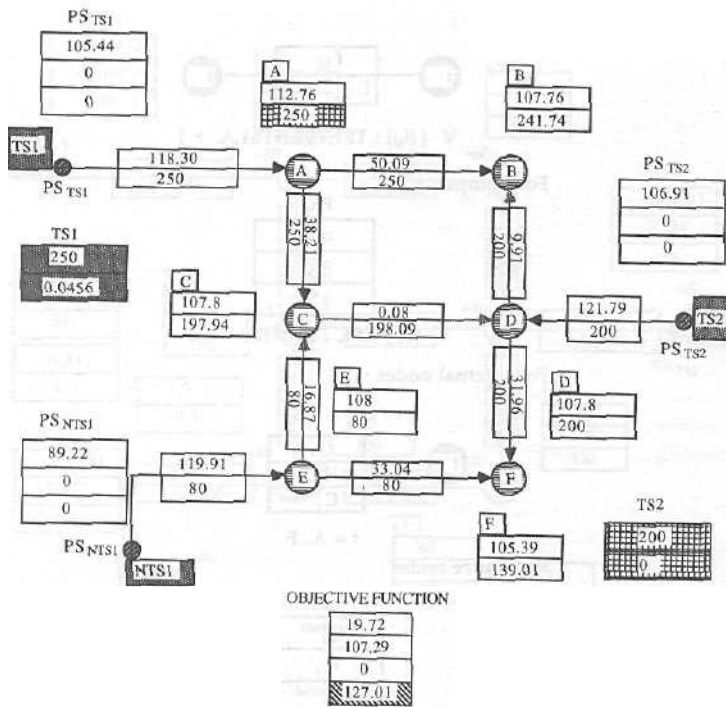


FIG. 5. BASE RUN RESULTS for Optimal Operation of Multiquality Network Example

2. Is there a source node at which supply reaches its bound (maximum or minimum)?

Quality (C)

1. Is there a node at which the threshold concentration requirement of water quality is reached?
2. To what degree is water being treated (removal ratio) at the treatment source nodes, and is there a treatment source node at which treatment reaches its bound (maximum or minimum)?

Head (H)

1. Is there a node at which the head requirement reaches its bound?
2. Are dummy variables being used within the pumping stations?

Objective Function (OF)

What is the cost, and relative weight in the objective function value, of: energy (denoted *EC*), water and treatment (denoted *WC*), and penalty (denoted *PC*). Results of the base run and the four sensitivity runs are shown in Fig. 7.

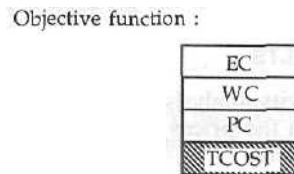
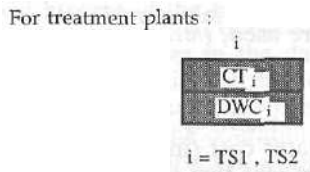
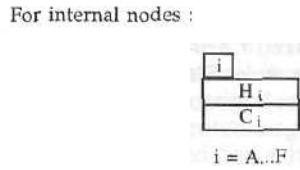
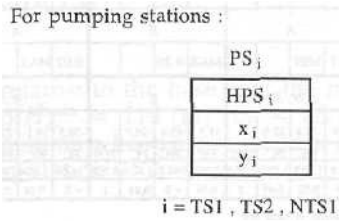
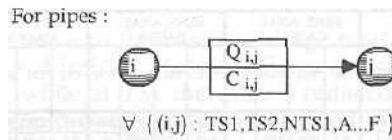


FIG. 6. Legend for Fig. 5 (Symbols Explained in Text)

BASE RUN RESULTS

Discharge (Q)

1. The total supply of 360 (m^3/h) is taken in almost equal parts from the three sources.
2. None of the source nodes reaches its supply limits (maximum or minimum).

Quality (C)

1. At node A the threshold concentration is reached.
2. Removal ratio at treatment source node $TS1$ is 0.17. At treatment source node $TS1$ water is not treated, i.e., treatment is at its minimum level.

TYPE OF QUES.	#	BASE RUN				SENS. ANAL. CASE 1				SENS. ANAL. CASE 2				SENS. ANAL. CASE 3				SENS. ANAL. CASE 4			
		TS1	TS2	NTS1	TOT	TS1	TS2	NTS1	TOT	TS1	TS2	NTS1	TOT	TS1	TS2	NTS1	TOT	TS1	TS2	NTS1	TOT
		Q	1.1	118.3	121.8	119.9	360	178.2	15	166.8	360	163.1	181.9	15	360	35	159.1	165.9	360	115.3	127.5
	1.2	0.33	0.34	0.33	1	0.5	0.04	0.46	1	0.45	0.51	0.04	1	0.1	0.44	0.46	1	0.32	0.35	0.33	1
	2.1					MIN					MIN										
C	2.1	A				A				E				A				C			
	2.2	0.17	MIN			0.17	MIN			MAX	0.20			0.02	MAX			0.18	MIN		
H	3.1					B; E				E				E				C			
	3.2					37.9	-71.4	28.6		31.3	43.8	-74.7		-59.3	14.1	27.7			6.3		
OBJECTIVE FUNCTION		EC	WC	PC	TOT	EC	WC	PC	TOT	EC	WC	PC	TOT	EC	WC	PC	TOT	EC	WC	PC	TOT
		19.7	107.3	0	127.0	12.9	1.556	1.4E6	2.9E6	2.2E6	168.5	1.5E6	3.7E6	14.84	1.4E4	3E6	1E6	19.7	106.8	6.3E4	6.3E4
		0.16	0.84	0	1	=0	0.52	0.48	1	0.59	=0	0.41	1	=0	0.01	0.99	1	=0	0.002	0.998	1

Remark : a square is left blank for results which are either not relevant or non-binding.

FIG. 7. BASE RUN and SENSITIVITY ANALYSIS RESULTS for Optimal Operation of Multiquality Network Example under Steady State Conditions

Head (H)

1. None of the heads reaches its minimum level.
2. No dummy variables are used, i.e., the optimal solution is feasible.

Objective Function (OF)

The total cost is \$127. It is made up of \$19.70 (16%) for energy (EC) and of \$107.30 (84%) for water cost (including treatment at *TS1*). There are no penalty costs, which means that the solution is physically feasible.

SENSITIVITY ANALYSIS RESULTS

The objective of the sensitivity analysis is to examine the response of the optimal solution to changes in the prices of water, treatment, and energy, and to changes in the constraints.

Case 1

Change in data relative to the base run: a very high cost of water in source *TS2* = 5×10^5 (\$/m³) [*br* = 0.20 (\$/m³), meaning that in the base run the corresponding value was 0.20 (\$/m³)]. Results: The minimum possible is taken from *TS2*; the threshold concentration is reached at node *A*; minimum head reached at nodes *B* and *E*; dummy variables are used. Hence, in this case the dummy variables are used because of the high cost of water given in source *TS1*, as compared to the penalty incurred on the valves in the objective function, and not as a result of infeasibilities.

Case 2

Change in data relative to the base run: large energy cost at pumping station *NTS1*, $PC_{NTS1} = 10^3$ (\$ x hr/m⁴) (*br* = 0.000545 [\$ x hr/m⁴]). Results: the minimum amount of water allowed is taken from *NTS1*; at node *E*, the closest to *NTS1*, the threshold concentration and minimum head are reached; dummy variables are used, because of the large energy cost at pumping station *NTS1*.

Case 3

Change in data relative to the base run: large treatment cost $KC_{rsi} = 4 \times 10^{-5}$ ($\text{m}^3/\text{\$}$) ($br = 4$ [$\text{m}^3/\text{\$}$]). Results: The removal ratio at *TS2* reaches its maximum value, while at *TS1* the ratio is reduced to 0.02 ($br = 0.17$); at node *A* the threshold concentration is reached; at *E* the head is at its minimum level; dummy variables are used, because of the high treatment cost of water at source *TS1*.

Case 4

Change in data relative to the base run: the minimum head required at node *C* is raised to $H_j^n = 110$ (m) ($br = 85$ [m]). Results: Thresholds concentration and minimum head are reached at node *C*; a dummy variable operates at pumping station *TS1*.

CONCLUSIONS

A model for optimal operation of multiquality distribution systems under steady state flow and concentration conditions has been formulated and applied to an example network. The optimal solution is obtained with GAMS/MINOS, using the projected augmented Lagrangian method.

The example network was solved for a base case and four additional cases, in each of which some of the base data were changed. Analysis of the results indicates that the optimal operation indeed responds in an explainable manner to these changes in the data. The work reported here demonstrates that optimal operation of multiquality networks can be formulated and solved, albeit for small networks. It should be considered as a first step towards dealing with the operation of such networks. The next phase of this work, which is reported in a companion paper, expands the analysis to unsteady conditions.

ACKNOWLEDGMENT

The research was supported by the Technion V.P.R. Funds: Coleman Cohen Research Fund, and by the New York Metropolitan R. Fund.

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APPENDIX II. NOTATION

The following symbols are used in this paper:

A	= set of arcs, indexed a ;
A_{pumps}	= a subset of A , on which pumping stations are located;
$C_{pipe_a}^k$	= concentration of quality parameter k in arc a , computed by (6b);
C_{in}^k	= concentration of quality parameter k at node, in ;
C_{ii}^k, C_{jj}^k	= concentrations of quality parameter k at nodes ii and jj , respectively;
$C_{in}^{k,min}, C_{in}^{k,max}$	= maximum and minimum concentrations, respectively, of water-quality parameter k at some internal nodes;
C_{ntsn}^k	= concentration of water quality of parameter k at source node $ntsn$;
$C_{initial_{tsn}}^k$	= concentration of quality parameter k , at the inlet of source node tsn ;
DT	= time unit of operation (e.g., hour, serial hours, a day);
D_a	= internal diameter of arc a ;
EC	= energy cost;
$ECCP_a$	= power coefficient of pumping station, located on arc a ;
$Floop$	= fundamental loop matrix for a given (arbitrarily selected) spanning tree;
$f_a(R_a, \epsilon_a/D_a)$	= friction factor of arc a ;
H_{in}	= total head at selected internal nodes;
$H_{in}^{min}, H_{in}^{max}$	= minimum and maximum total heads allowed at selected internal nodes, respectively;
H_{sn}	= head at source node sn ;

- $Head_{sn}$ = known head at source node sn ;
 IN = set of internal nodes, indexed in ;
 ii, jj = end nodes of pipe a ;
 K = set of water-quality parameters, indexed k ;
 Ktc_{tsn}^k = coefficient of treatment cost of quality parameter k , at source node tsn ;
 $kwhc$ = energy charge during time of operation;
 L_a = length of arc a ;
 LN = natural logarithm;
 $NMAX$ = maximum number possible in computer being used;
 $NTSN$ = set of nontreatment source nodes, indexed $ntsn$;
 $PEN_a(q_a)$ = penalty product in equation (6b);
 P^{RN} = path matrix, connecting reference node RN with the subset of internal nodes;
 $penalty$ = large positive number;
 \mathbf{q} = vector of discharge along arcs;
 q_a = discharge in arc a ;
 q_a^j = the j th feasible discharge at a th pumping station;
 q_a^{\min}, q_a^{\max} = minimum and maximum discharges in arc a , respectively;
 q_{in} = discharge at node in ;
 q_{ntsn}, q_{tsn} = discharge supplied by source node $ntsn$ and source node tsn , respectively;
 RIM^{RN} = reduced incidence matrix with respect to reference node RN ;
 $RR_{tsn}^{k, \max}$ = maximum removal ratio of quality parameter k at source node tsn ;
 R_a = Reynolds number of arc a ;
 SN = set of source nodes, indexed sn (union of sets: TSN and $NTSN$);
 TSN = set of treatment source nodes, indexed tsn ;
 TC_{tsn}^k = treatment cost of quality parameter k , per unit volume of treated water, at source node tsn ;
 W = vector of consumptions (except for reference node RN) at nodes;
 WC = water cost;
 WC_{ntsn} = fixed charge for unit volume of water at $ntsn$ nodes;
 $WC_{tsn}^{initial}$ = fixed charge per unit volume of untreated water at node tsn ;
 x_a, y_a = dummy penalty variables;
 $\alpha_0^a, \beta_0^a, \gamma_0^a$ = coefficients for the a th pumping station at its maximum efficiency;
 $\Delta Hf_a(q_a)$ = head loss along arc a , calculated by the Darcy-Weisbach formula;
 $\Delta Hloop$ = head difference for set of fundamental loops and paths;
 $\Delta Hpipe(q)$ = vector of heads, losses or gains, along arcs;
 $\Delta Hp_a(q_a)$ = head gain at pumping station, located on arc a ;

ε_a = roughness coefficient of arc a ;
 ε_a/D_a = relative roughness of arc a ;
 $\eta_a^{j,\max}$ = maximum efficiency of j th feasible discharge at a th pumping station, among all possible combinations of pumps at station;
 $\xi(in)$ = set of arcs connected to node, in ;
 $\xi^-(in), \xi^-(tsn), \xi^-(ntsn)$ = set of arcs with flow out of nodes $in, tsn,$ and $ntsn,$ respectively; and
 Ω_a = constant dependent upon units used for $L_a, D_a,$ and $q_a.$