OPTIMAL OPERATION OF MULTIQUAHTY NETWORKS. II: UNSTEADY CONDITIONS

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ABSTRACT: A model is developed for optimal operation of a multiquality water-supply network under unsteady conditions, for a time horizon that is divided into a number of time periods. The objective is to minimize total cost, which includes the cost of water at the sources, of treatment, and of the energy to operate the system. The constraints include equations that describe the change in flow and quality over time throughout the system, the physical laws of flow and concentrations, and the requirements for level of service. The equations that describe concentrations in pipes are of a form that allows the flow direction to reverse during the iterative solution process. The model is solved with GAMS/MINOS. An example system is optimized, with two sources, one with a treatment plant, two reservoirs, 6 consumers and 11 pipes, operated over five time periods. The system has been analyzed through a base run and three additional runs.

INTRODUCTION

This paper extends the model reported in the companion paper by Ostfeld and Shamir (1993) to optimal operation of a multiquality water-distribution system under unsteady conditions, which may result from time-varying water quality at supply nodes, or changing flows in the system, or both.

For the unsteady analysis it is necessary to develop equations that describe quality changes with time, along pipes and in reservoirs. In the steady state model the flows through the pipes are fixed and the quality is constant in the supply sources. Therefore reservoirs have no role to play. In the unsteady model, however, the changes of quality with time in the reservoirs and along the pipes are modeled explicitly.

The study of multiquality systems under unsteady conditions has so far been restricted to simulation (Shah 1985; Liou and Kroon 1987; Grayman et al. 1988; Cohen 1992).

The equations that describe the change of quality over time and along pipes will be developed in this paper in a form that makes them suitable for incorporation as constraints in the optimization model. They are cast in a form that remains valid when the flow direction in a pipe is reversed. This new development may be useful in simulation models as well.

The paper consists of three main parts;

1, Development of the equations for modeling quality in pipes and in reservoirs under unsteady conditions, which are then embedded into the constraints of the optimization model.

2. Formulation of the mathematical model for optimal operation over a planning period, which considers flows *(Q),* concentrations (C), and hy-

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draulic heads (H), and is therefore of the Q-C-H type [see Ostfeld and Shamir (1993)].

3. Application to a system having two sources, one with a treatment plant, two reservoirs, 11 pipes, serving six consumers over five successive time intervals. Water quality is described by a single parameter.

MODEL FOR OPTIMAL OPERATION

The model for optimal operation is of the Q-C-H type, and contains constraints that describe the continuity, hydraulic, concentration, and energy laws. It is designed to provide the least cost operating policy over a planning horizon that is in the order of one day, divided into a number of consecutive time periods, each of one to a few hours. Flows and heads are assumed to remain constant during each time period, while the movement of concentration fronts during the period is accounted for.

The nodes in the system are classified as either: *TSN* (source nodes with treatment plants), *NTSN* (source nodes without treatment), and *IN* (internal nodes, at some of which there are consumers). The *NTSN* nodes are partitioned into two types: *WELLS* and *RES* (reservoirs). The sources at *WELLS* are assumed to have an infinite capacity, compared to the amount of water to be extracted over the planning period. The quality and volume of water at *RES* nodes change over time, in response to flows into and out of the reservoir.

The direction of flow in each arc in each time period will be determined in the solution. Certain arcs may allow flow only in one direction, for example, the pipe leading from a well, or a pump, which can operate only in one direction.

The planning horizon, DT, is divided into a number of periods, At_n, $n = 1, \ldots, m$. t_n is the beginning of time period n. Initial conditions are given for $n = 1$, and final conditions for t_m,

MODELING TIME-VARYING QUALITY

Equations that describe the changes in water quality over time and with flow through the system must be simple enough to be embedded in an optimization model, yet sufficiently accurate. Furthermore, we want to allow the flows in pipes and through reservoirs to be in either direction, so that the optimization model can select the directions that result in the best value of the stated objective function. Formulation of the quality equations is more difficult when flow directions are not fixed. In the following sections we develop such equations, which are then used as constraints in the optimization model.

QUALITY CHANGES IN PIPES

The propagation of pollutant concentration in a pipe is governed by the dispersion advection equation. However, Shah (1985) has already shown that in simulating multiquality networks, the spreading of concentration front by dispersion can be usually neglected relative to the "piston flow" by advection, under flow conditions normally encountered in water-distribution systems. Still, even with advection alone, tracking the movement of concentration fronts through pipes and modeling the mixing at nodes is a complex task.

Grayman et al. (1988) have simulated unsteady concentration distributions in a network by dividing each pipe into segments and following the progress of the fronts through the series of cells. Sinai et al. (1987) used a "transportation lag model" to simulate the progress of pollutants through the system. Embedding either of these into an optimization model would make the solution very complicated and costly.

Instead, an approximate equation for modeling water quality in a pipe has been developed, to be used in the constraints of the optimization model.

Consider a pipe of length *L,* in which flow at mean velocity *V* is from node *u* to node *d.* The time for a front introduced at *u* to reach *d* is: / = *LIV.* Denote by *L(At)* the distance traveled by the front during a time period of length Ar. *At* will be the time interval for the optimization model. Four cases can be defined:

1. $L(\Delta t) < L$: the front has not reached node d during Δt.
2. $L(\Delta t) = L$: the front reaches node d at the end of Δt.
3. $L(\Delta t) > L$: the front arrives at node d prior to the end of Δt.
4. $L(\Delta t) >> L$: the front reaches node d instantaneously.

The average concentration in the pipe over *At* is given by an expression that must hold for all four cases. Furthermore, the direction of flow in the pipes is not known in advance. A positive direction is arbitrarily assigned each pipe, but the flow should be allowed to reverse itself by the optimization algorithm. Therefore, the expression for the concentration in a pipe should allow this to happen.

Since the final equation is somewhat complex, we develop it in stages. Let the subscript *n* denote the time period, and t_n to be the beginning of the *nth* time period, whose length is *At,,.* Flow in the pipe during the time period is assumed to be from node *u* to node *d.* Automatic selection of the direction will be discussed next. Denote by *Cup* the concentration of upstream node, and *Cpipe* will be the resultant average concentration in the pipe for the time period.

When $L(At_n)$ < L the average concentration in the pipe for the time period t_a is given by:

$$Cpipe(t_n) = \frac{1}{2}\,[Cup(t_n) + Cpipe(t_{n-1})]\,\frac{L(\Delta t_n)}{L} + Cpipe(t_{n-1})\,\frac{L - L(\Delta t_n)}{L} \quad (1)$$

which is a weighted sum over the relative volumes occupied by the new and

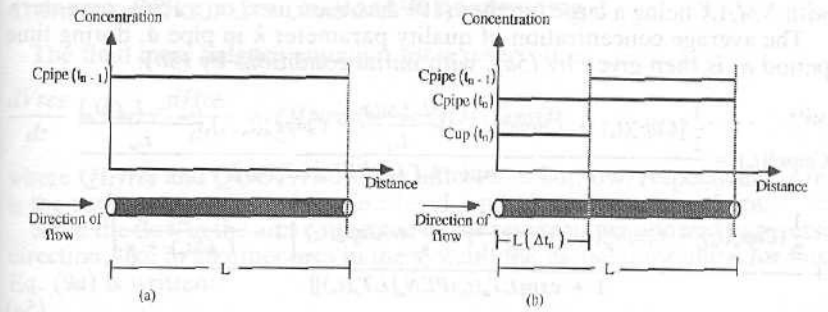


FIG. 1. Concentration Change in Pipe for Case $L(\Delta t_n) \le L$: (*a*) at End of Previous Period, t_{n-1}; (*b*) at End of Current Period, t_n

old fronts. This can be seen in Fig. 1. The validity of this equation can be examined at two extreme cases:

1. When $L(\Delta t_n) = L$, it gives the same result as (1).
2. When $L(\Delta t_n)$ is very large, (2) yields $Cpipe(t_n) \cong Cup(t_n)$, which means that the concentration in the pipe is dominated by that at the upstream node u.

Eqs. (1) and (2) cover all four cases listed above, with compatibility between them at $L(\Delta t_n) = L$. Either (1) or (2) has to be "selected" by the model automatically, depending on the relation between L and $L(\Delta t_n)$. This might have been done by the use of logical variables, but to avoid the added complexity of the model we use instead the following device. Define for each pipe, a, and for each time period, n, the quantity:

$$LT_a(t_n) = L_a - L_a(\Delta t_n) \quad \text{(3)}$$

and then define a penalty function:

$$PEN_a[LT_a(t_n)] = \frac{LN(NMAX)}{1 + |LT_a(t_n)|} \quad \text{(4)}$$

The validity of (2) can be examined at two extreme cases:

with *NMAX* being a large number (10^{22} has been used on a PS/2 computer). The average concentration of quality parameter k in pipe a, during time period n, is then given by (5a), with initial conditions by *(5b):*

$$Cpipe^k_a(t_n) = \frac{\frac{1}{2}\left[Cup^k_a(t_n) + Cpipe^k_a(t_{n-1})\right]\frac{L_a(\Delta t_n)}{L_a} + Cpipe^k_a(t_{n-1})\frac{L_a - L_a(\Delta t_n)}{L_a}}{1 + \exp\{-LT_a(t_n)PEN_a[LT_a(t_n)]\}}$$

$$+ \frac{\frac{1}{2}\left[Cup^k_a(t_n) + Cpipe^k_a(t_{n-1})\right]\frac{L_a}{L_a(\Delta t_n) + \varepsilon} + Cup^k_a(t_n)\left[1 - \frac{L_a}{L_a(\Delta t_n) + \varepsilon}\right]}{1 + \exp\{LT_a(t_n)PEN_a[LT_a(t_n)]\}} \quad \text{(5a)}$$

$$Cpipe^k_a(t_0) = Cpipe^{k,initial}_a \quad \text{(5b)}$$

where e is a small number, to avoid division by zero, if $L_a(\Delta t_n) = 0$. This equation effectively zeros one or the other of the two main terms, depending on the sign of $LT_a(t_n)$. This expression is written for each time period, and for each quality parameter.

To allow for reversal of flow in the pipe, which means that it is not known in advance which node for each pipe and for each time period is the upstream node, a similar expression is used to "select" the upstream node as a function of the direction of flow in the pipe (which, as stated, is to be determined by the model):

$$Cup(t_n) = \frac{C_u(t_n)}{1 + \exp\{-q_a(t_n)PEN_a[q_a(t_n)]\}} + \frac{C_d(t_n)}{1 + \exp\{q_a(t_n)PEN_a[q_a(t_n)]\}} \quad (6a)$$

where $q_a(t_n)$ is the flow in the pipe, and:

$$PEN_a[q_a(t_n)] = \frac{LN(NMAX)}{1 + |q_a(t_n)|} \quad (6b)$$

The indices u and d in (6a) refer to the two ends of the pipe, without fixing in advance which is to ultimately be the upstream node. Eq. (6a) zeros one or the other of the two main terms, depending on the direction of flow, by making its denominator essentially infinite.

The performance of (5) and (6a) can be seen by examining Fig. 1 of the companion paper (Ostfeld and Shamir 1993), which describes the behavior of a similar equation [(6b) there]. It closely approximates a discontinuous function by a smooth one. Similar expressions will be also used later in developing other components of the optimization model.

The validity of (5) is tested by computing the result at extreme conditions:

$$[L_a(\Delta t_n) \to 0] \Rightarrow [q_a(t_n) \to 0] \Rightarrow [LT_a(t_n) \to L_a] \Rightarrow$$
$$[Cpipe_a^k(t_n) \to Cpipe_a^k(t_{n-1})] \quad (7)$$

$$[L_a(\Delta t_n) \to \infty] \Rightarrow [q_a(t_n) \to \infty] \Rightarrow [LT_a(t_n) \to -\infty] \Rightarrow$$
$$[Cpipe_a^k(t_n) \to Cup_a^k(t_n)] \quad (8)$$

Eq. (5) could be modified to account for decay or increase of concentration with time, by multiplying the whole expression by a decay factor, which is a function of the length of the time period, Δt_n.

STORAGE AND QUALITY CHANGES IN RESERVOIRS

The fluid mass balance equation for a reservoir is:

$$\frac{dVres}{dt} = Ares\frac{dHres}{dt} = QINres(t) - QOUTres(t) \quad (9a)$$

where $QINres$ and $QOUTres$ are the inflow and outflow, respectively, $Ares$ is the surface area, $Hres$ the water level, and $Vres$ the water volume.

Since the flow in the arcs connected to the reservoir are allowed to reverse direction, like in all other arcs in the system, the model must allow for this. Eq. (9a) is written:

$$\frac{dVres}{dt} = Ares\frac{dHres}{dt} = \sum_{a\in\xi(res)} q_a(t) \quad (9b)$$

where *i;(res)* is the set of arcs connected to the reservoir, and the flows are taken positive when they enter the reservoir. If the level change in the reservoir is expressed as the difference between the values at the beginning and end of the period, then *(9b)* leads to:

$$Hres(t_n) = Hres(t_{n-1}) + \frac{\Delta t_n}{Ares} \sum_{a \varepsilon \xi(res)} q_a(t_n) \quad \ldots\ldots\ldots\ldots\ldots\ldots (9c)$$

with the initial condition:

$$Hres\{t_0) = Hres^{initial} \quad \ldots\ldots\ldots\ldots\ldots\ldots (9d)$$

The basic mass balance equation for a quality parameter, assuming total and instantaneous mixing, in the reservoir, is:

$$\frac{d(VresCres)}{dt} = Vres(t)\frac{dCres}{dt} + Cres(t)\frac{dVres}{dt} = CINres(t)QINres(t)$$
$$- Cres(t)[QOUTres(t) + \delta res(t)Vres(t)] \quad \ldots\ldots\ldots\ldots\ldots\ldots (10a)$$

where *ClNres* and *Cres* are the concentrations at the inlet and in the reservoir, respectively, and *bres(t)* is a first-order decay coefficient of the concentration. Substituting *(9a)* into (10a), the change in concentration in the reservoir can be expressed as:

$$\frac{dCres}{dt} = \frac{1}{Vres(t)}[CINres(t) - Cres(t)]QINres(t) - Cres(t)\delta res(t) \quad \ldots (10b)$$

The change in concentration is:

$$\frac{dCres}{dt} = \frac{Cres(t_n) - Cres(t_{n-1})}{\Delta t_n} \quad \ldots\ldots\ldots\ldots\ldots\ldots (10c)$$

which finally leads to:

$$Cres(t_n) = \frac{Cres(t_{n-1}) + \dfrac{\Delta t_n}{AresHres(t_n)} \displaystyle\sum_{a \varepsilon \xi(res)} Cpipe_a(t_n) q_a(t_n)}{1 + \Delta t_n \left[\dfrac{\displaystyle\sum_{a \varepsilon \xi(res)} q_a(t_n)}{AresHres(t_n)} + \delta res(t_n) \right]} \quad \ldots\ldots (10d)$$

An initial value is given for the concentration:

$$Cres\{t_n) = Cres^{iniHal} \quad \ldots\ldots\ldots\ldots\ldots\ldots (10e)$$

Eq. *(lOd)* is written for each water quality parameter *k,* and each time step: *n* = 1, . . . , *m,* with an initial condition (lOe), given for each quality parameter.

ADDITIONAL CONSTRAINTS

The hydraulic constraints for each period are the same as those for the steady-state model [(l)-(2e) in Ostfeld and Shamir (1993)], namely:

$$RIM^{RN}\mathbf{q}(t_n) = W(t_n) \quad \ldots\ldots\ldots\ldots\ldots\ldots (11)$$

$$Floop\Delta Hpipe[\mathbf{q}(t_n)] = \Delta Hloop(t_n) \quad \ldots\ldots\ldots\ldots\ldots\ldots (12)$$

$$\Delta Hpipe[\mathbf{q}(t_n)] = -\Delta Hf_a[\mathbf{q}(t_n)] \quad \text{if } a \notin Apumps \text{ ....} \quad (13a)$$

$\Delta Hpipe[\mathbf{q}(t_n)] = \Delta Hp_a[\mathbf{q}(t_n)] - \Delta Hf_a[\mathbf{q}($ *Q]* if « G *Apumps* . . . .

The head losses for pipes are given by:

$$\Delta Hf_a[q_a(t_n)] = \Omega_a f_a \left(\mathrm{R}_a, \frac{\varepsilon_a}{D_a}\right) \frac{L_a}{D_a^5} |[q_a(t_n)]| q_a(t_n) \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad (14)$$

The head gain for pumps, which are not fed from a reservoir, is given by:

$$\Delta Hp_a[q_a(t_n)] = \alpha_0^a[q_a(t_n)]^2 + \beta_0^a[q_a(t_n)] + \gamma_0^a + [x_a(t_n) - y_a(t_n)] \quad \ldots (15a)$$

While for pumps fed from reservoirs, the head equation is:

$$\Delta Hp_a[q_a(t_n)] = \frac{\alpha_0^a[q_a(t_n)]^2 + \beta_0^a[q_a(t_n)] + \gamma_0^a + [x_a(t_n) - y_a(t_n)]}{1 + \exp\{-q_a(t_n)PEN_a[q_a(t_n)]\}} \quad \ldots (15b)$$

where RIM^{RN} = reduced incidence matrix with respect to a reference node *RN,* in an undirected network; q = vector of discharge along arcs; *W* = vector of consumptions; *Floop* — fundamental loop matrix for a given (arbitrarily selected) spanning tree; *AHloop* = head difference for a set of fundamental loops and paths; *AHpipe(q)* = vector of head losses or gains along arcs; $AHf_a(q_a)$ = head loss along arc a, calculated by the Darcy-Weisbach formula; $/_{fl}(R_{fl}$, $e_{fl}/£)_a)$ = friction factor, which depends on the Reynold's number (R_u), and the relative roughness (e_a/D_a), where e_{fl} = a roughness coefficient, and D_a = the internal diameter of the pipe; L_a = arc length; q_a — discharge in arc *a;* O,, = a constant that depends on the units used for L_a, D_a, and q_a; $AHp_a(q_a)$ ~ head gain at pumping station on arc *a;* ag, fig, -yg = coefficients for the *ath* pumping station at is maximum efficiency [see (2e) in Ostfeld and Shamir (1993)]; x_a, y_a = dummy variables, which incur high (artificial) penalties in the objective function [see Ostfeld and Shamir (1993)].

Eq. (11) represents continuity of flows (Kirchoff's law No. 1 for water); (12)-(156) are the laws of energy (Kirchoff's law No. 2). These are the same as in the steady state model except for (156), which is different from the parallel one [(2e) in Ostfeld and Shamir (1993)]. Where a pump is fed from a reservoir, the flow in that arc should be allowed to reverse, and the reservoir can then fill by flow from the network, through a bypass pipe

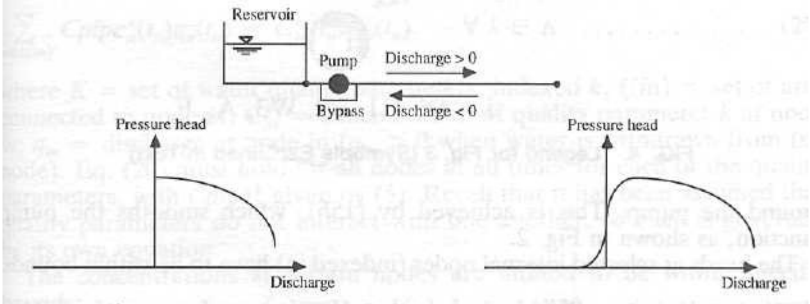


FIG. 2. Smoothing Pump Curve: (a) before Smoothing; (b) after Smoothing

Initial average concentrations in pipes

Pipe number	1	2	3	4	5	6	7	8	9	10	11
Concentration (mg/j)	230	150	80	120	140	200	130	140	225	130	250

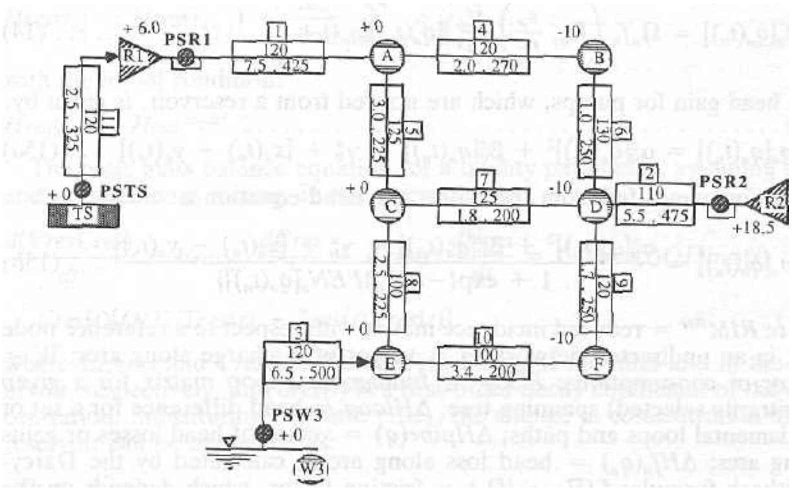
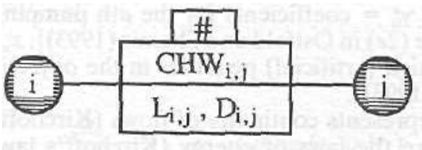


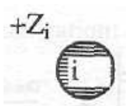
FIG. 3. BASE RUN DATA for Optimal Operation of Multiquaiity Network Example

For pipes :



V { (i,j) : TS, R1,R2, W3, A...F)

For nodes :



i = TS, R1,R2, W3, A...F **FIG.**

4. Legend for Fig. 3 (Symbols Expiained in Text)

around the pump. This is achieved by *(15b),* which smooths the pump function, as shown in Fig. 2.

The heads at selected internal nodes (indexed in) have to be within bounds:

$$H_{in}^{\min}(t_n) \le H_{in}(t_n) = P^{RN}\Delta hpipe[q(t_n)] \le H_{in}^{\max}(t_n) \qquad \text{for some } in \in IN \quad (16)$$

TABLE 1. Pumping Stations Data

Pumping station (1)	Coefficients			Efficiency (η) (5)
	Discharge—Head Curve			
	α_0 (2)	β_0 (3)	γ_0 (4)	
*PSR*1	−0.00012	−0.00123	215	0.75
*PSR*2	−0.0001	−0.001	145	0.75
*PSW*3	−0.00015	−0.0012	172	0.75
PSTS	−0.00012	−0.0123	300	0.75

TABLE 2. Reservoirs Data

servoir (1)	Initial Data		Water Level (m)		Surface area (constant) (m^2) (6)
	er level (m) (2)	Concentration (mg/L) (3)	Minimum (4)	ximum (5)	
R1	10	120	7.3 15	12	1,000
R2	**20**	300		25	1,000

This is a usual requirement at consumer nodes. Reservoir levels are limited to be within given bounds:

$$Hres^{\min}(t_n) \leq Hres(t_n) \leq Hres^{\max}(t_n) \quad (17)$$

The discharges or velocities in arcs may also be constrained within bounds:

$$q_a \exists (q_a^{\min}, q_a^{\max}) \qquad \forall\, a \in A \quad (18)$$

For directed arcs $q^{\text{TM}in}$ = 0 and for undirected $q^{''1'''}$ < 0. The purpose of these constraints is to limit the discharges to be within acceptable levels, for example, those which correspond to velocity values in the range of 0.5-4 m/s. Heads at well sources (indexed well) are known at all times:

$$H_{well} = Head_{well} \qquad \forall\, well \in WELLS \quad (19)$$

Mass conservation of quality parameters at nodes:

$$\sum_{a \in \xi(in)} Cpipe_a^k(t_n) q_a(t_n) = C_{in}^k(t_n) q_{in}(t_n) \qquad \forall\, k \in K \quad (20)$$

where K = set of water quality parameters, indexed *k; %{in)* = set of arcs connected to node *in;* Cf_n — concentration of quality parameter k at node *in;* q_{in} = discharge at node in ($q_{in} > 0$ when water is withdrawn from the node). Eq. (20) must hold for all nodes at all times for each of the quality parameters, with *Cpipe'l* given by (5). Recall that it has been assumed that quality parameters do not interact with one another, so each is governed by its own equation.

The concentrations at certain npdes are limited to be within specific bounds:

$$C_{in}^{k,\min}(t_n) \leq C_{in}^k(t_n) \leq C_{in}^{k,\max}(t_n) \qquad \text{for some } in \in IN \quad (21)$$

TABLE 3. Consumer Demands

Time Interval (h) (1)	CONSUMER A Quantity (m³/h) (2)	A Threshold pressure (m) (3)	A Maximum concentration (mg/L) (4)	B Quantity (m³/h) (5)	B Threshold pressure (m) (6)	B Maximum concentration (mg/L) (7)	C Quantity (m³/h) (8)	C Threshold pressure (m) (9)	C Maximum concentration (mg/L) (10)	D Quantity (m³/h) (11)	D Threshold pressure (m) (12)	D Maximum concentration (mg/L) (13)	E Quantity (m³/h) (14)	E Threshold pressure (m) (15)	E Maximum concentration (mg/L) (16)	F Quantity (m³/h) (17)	F Threshold pressure (m) (18)	F Maximum concentration (mg/L) (19)
0–4	150	35	325	180	22	315	180	30	400	200	27	331	200	35	322	110	20	451
4–10	230	30	243	170	25	248	120	35	251	200	20	132	350	35	147	190	25	123
10–16	160	30	243	100	20	321	280	35	231	180	25	156	200	30	176	170	25	191
16–20	150	35	154	220	25	100	180	35	241	260	25	319	200	35	421	160	20	134
20–24	100	30	122	130	25	131	180	35	156	208	25	191	202	35	133	105	25	182

TABLE 4. Energy Charge during Operating and First-Order Decay Coefficients in Reservoirs

Time interval (h) (1)	Energy charge (dollars/m^3) (2)	First-Order Decay Coefficients	
		Reservoir *R*1 (3)	Reservoir *R2* (4)
0-4 4-10 10-16 16-20 20-24	0.05 0.15 0.18 0.25 0.35	0.00125 0.00127 0.00128 0.00126 0.00124	0.00123 0.00128 0.00126 0.00122 0.00121

TABLE 5. Source Data

Source (1)	Head (m) (2)	Inlet concentralion (before treatment) (mg/L) (3)	Coefficient of treatment cost (m^3/dollars) (4)	Minimum concentration possible at outlet (mg/L) (5)	Water cost (before treatment) (dolfars/m^3) (6)
TS ***W3***	5 0	300 80	4.0	20	0.05 0.5

which is normally the case at consumer nodes.

The concentration in sources without treatment plants is fixed for each quality parameter *k:*

$$PUO = C^{k,const}_{well} \quad \forall\ well \in WELLS \quad \text{.........} (22)$$

OBJECTIVE FUNCTION

The objective function is the cost of operating the system over the planning horizon. It contains two types of terms: (1) The cost of water, its purchase or production at the sources plus the cost of treatment; and (2) the cost of energy to operate the pumps.

The cost of treating water at treatment source nodes *(TSN,* indexed *tsn),* with respect to water quality parameter *k,* is given by:

$$Cpipe^k_a[TC^k_{tsn}(t_n)] = Cinitial^k_{tsn} \exp[-Ktc^k_{tsn} TC^k_{tsn}(t_n)]$$
$$\forall a \in \xi^-(tsn),\ tsn \in TSN,\ k \in K \quad \text{.........} (23a)$$

where; $\xi^-(tsn)$ = arcs directed away from the node *tsn;* $Cinitial^k_{tsnn}$ = concentration of quality parameter *k,* at the inlet of source node *tsn*; TC^k_{tsn} =

! treatment cost of quality parameter *k,* per unit volume of treated water, at source node *tsn;* Ktc^k_{tsn} — coefficient of the treatment cost of quality parameter *k,* at source node *tsn.* A schematic representation of the outlet quality for a single quality parameter, as a function of treatment cost, is shown in Fig. 2, in Ostfeld and Shamir (1993). The range of plant removal ratios is restricted by:

$$0 \le \frac{Cinitial^k_{tsn} - Cpipe^k_a[TC^k_{tsn}(t_n)]}{Cpipe^k_a[TC^k_{tsn}(t_n)]} \le RR^{k,\max}_{tsn}$$

Time interval (hr)	Type of question and its number		Reservoirs R1	R2	Well source W3	Treatment source TS	Total
T1 0-4	Q	1.1	526.4	226.5	267.2	0	1020
			0.52	0.22	0.26		1.0
		1.2				0MIN	
	C	2.1					
		2.2					
	H	3.1					
		3.2					
T2 4-10	Q	1.1	569.4	325.2	365.1	1079.9	1260
			0.45	0.26	0.29		1.0
		1.2					
	C	2.1			F		
		22				0.926	
	H	3.1					
		3.2					
T3 10- 16	Q	1.1	860.0	- 397.3	627.3	548.3	1090
			0.79	-0.37	0.58		1.0
		1.2					
	C	2.1					
		2.2				0.93 MAX	
	H	3.1			D		
		3.2		1.67			
T4 16-20	Q	1.1	851.4	- 292.5	611.1	1180.6	1170
			0.73	-0.25	0.52		1.0
		1.2					
	C	2.1			B		
		2.2				0.93 MAX	
	H	3.1			D		
		3.2		1.97			
T5 20-24	Q	1.1	499.2	181.5	244.3	0	925
			0.54	0.20	0.26		1.0
		1.2				0MIN	
	C	2.1					
		2.2					
	H	3.1					
		3.2					

Remark : a square is left blank for results which are either not relevant or non - binding.

FIG. 5. BASE RUN Results for Optimal Operation of Multiquality Network Example under Unsteady Conditions

V *a* G £;-(&«), *tsn E TSN, k E K* . . . . (23*b*)

where $RR_{tsn}^{k,max}$ = maximum removal ratio of quality parameter k at node *tsn.*

The cost of water and treatment is:

$$WC = \sum_{n=1}^{m} \left\{ \sum_{well \in WELLS} WC_{well} q_{well}(t_n) \Delta t_n + \sum \left[WC_{tsn}^{initial} + \sum TC_{tsn}^{k}(t_n) \right] q_{tsn}(t_n) \Delta t_n \right\} \quad (23c)$$

where m = the number of periods; WC = water cost; WC_{we} u = fixed charge

Time interval (hr)	Type of question and its number		Reservoirs		Well source	Treatment source	Total
			R1	R2	W3	TS	
T1 0-4	Q	1.1	690.9	324.1	5.0,	1178.7	1020
			0.68	0.32	=0		1.0
		1.2			5 MIN		
	C	2.1					
		2.2				0.93 MAX	
	H	3.1					
		3.2	41.72		-43.8		
T2 4-10	Q	1.1	702.8 .	552.2	5.0	1174.5	1260
			0.56	0.44	=0		1.0
		1.2			5 MIN		
	C	2.1			F		
		2.2				0.93MAX	
	H	3.1					
		3.2		5.52	- 109.1		
T3 10- 16	Q	1.1	1006.2	78.8	5.0	1042.6	1090
			0.92	0.07	= 0		1.0
		1.2			5 MiN		
	C	2.1					
		2.2				0.84	
	H	3.1					
		3.2	189.27		-35.4		
T4 16-20	Q	1.1	693.5	471.5	5.0	693.5	1170
			0.59	0.40	= 0		1.0
		1.2			5 MIN		
	C	2.1			B		
		2.2				0.86	
	H	3.1					
		3.2	15.25		-67.3		
T5 20-24	Q	1.1	662.1	257.9	5.0	0	925
			0.72	0.28	= 0		1.0
		1.2			5 MIN	O MIN	
	C	2.1					
		2.2					
	H	3.1					
		3.2	31.86		- 44.5		

Remark : a square is left blank for results which are either not relevant or non - binding.

FIG. 6. SENSITIVITY ANALYSIS Results, CASE 1, for Optimal Operation of Multiquality **Network Example under Unsteady Conditions**

for unit volume of water at well nodes; q_{well}, q_{tsn}= discharge supplied by source node well and source node *tsn,* respectively; $WC^{initial}_{tsm}$ = fixed charge per unit volume of untreated water at *tsn* nodes. The cost of energy is:

$$EC = \sum_{n=1}^{m} \left\{ \sum_{a \in Apumps} ECCP_a q_a(t_n) k H_{Pa}[q_a(t_n)] kwhc(t_n) te_n + penalty[x_a(t_n) + y_a(t_n)] \right\} \quad (24)$$

where EC = energy cost; $ECCP_a$ = power coefficient of pumping station located on arc *a,* assuming efficiency is constant; *kwhc* = energy charge

Time interval (hr)	Type of question and its number		Reservoirs R1	Reservoirs R2	Well source W3	Treatment source TS	Total
T1 0-4	Q	1.1	526.36	226.46	267.19	0	1020
			0.52	0.22	0.26		1.0
		1.2				0 MIN	
	C	2.1					
		2.2					
	H	3.1					
		3.2					
T2 4-10	Q	1.1	579.5	300.5	380.0	1183.1	1260
			0.46	0.24	0.30		1.0
		1.2					
	C	2.1			D		
		2.2				I0.93MAX	
	H	3.1					
		3.2		5.20			
T3 10-16	Q	1.1	861.2	-398.4	627.2	534.1	1090
			0.79	■0.37	0.58		1.0
		1.2					
	C	2.1					
		2.2				I0.93MAX	
	H	3.1			D		
		3.2		1.49			
T4 16-20	Q	1.1	699.2	= 0	470.8	1179.6	1170
			0.60	= 0	0.40		1.0
		1.2					
	C	2.1			B		
		2.2				0.93 MAX	
	H	1.1					
		3.2					
T5 20-24	Q	1.1	574.0	= 0	351.0	0	925
			0.62	= 0	0.38		1.0
		1.2				0 MIN	
	C	2.1					
		2.2					
	H	3.1					
		3.2					

Remark : a square is left blank for results which are either not relevant or non - binding.

FIG. 7. SENSITIVITY ANALYSIS Results, CASE 2, for Optimal Operation of Mul-tiquality Network Example under Unsteady Conditions

during time of operation; *penalty* = large positive number which incurs high (artificial) penalties in the objective function, on the dummy variables x_a, y_a, and so makes it possible to obtain a mathematical solution to the problem even when the physical system cannot meet all the head constraints [Note that penalty has no connection to the *PEN* terms in the smoothing *(5a)* and (6a)].

OPTIMIZATION MODEL

The optimization problem to be solved, with the initial conditions (56), *(9d),* and (l0e), is:

Minimize(WC + *EC)* ....(25)

Time interval (hr)	Type of question and its number		Reservoirs		Weil source	Treatment source	Total
			Rl	R2	W3	TS	
Tl 0-4	Q	1.1	526.4	226.5	267.2	0	1020
			0.52	0.22	0.26		1.0
		1.2				0 MTN	
	c	2.1					
		2.2					
	H	3.1					
		3.2					
T2 4- 10	Q	1.1	569.4	325.5	365.1	1183.1	1260
			0.45	0.26	0.29		1.0
		1.2					
	c	2.1			F		
		2.2				0,88	
	H	3.1					
		3.2					
T3 10- 16	Q	1.1	547.1	236.5	306.4	0	1090
			0.50	0.22	0.28		1.0
		1.2				0 MIN	
	c	2.1					
		2.2					
	H	3.1					
		3.2					
. T4 16- 20	Q	1.1	554.5	295.5	319,9	1182.3	1170
			0.47	0,25	0.27		1.0
		1.2					
	c	2.1					
		2.2				0.87	
	H	3.1					
		3.2					
T5 20-24	Q	1.1	515.2	138.0	271.7	=0	925
			0 56	0.15	0.29		1.0
		1.2					
	c	2,1			A		
		2.2				0MIN	
	H	3.1					
		3.2					

Remark : a square is left blank for results which are either not relevant or non - binding.

FIG. 8. SENSITIVITY ANALYSIS Results, CASE 3, for Optimal Operation of Multiquality Network Example under Unsteady Conditions

Subject to constraints: *(5a), (9c), (lOd);* (11)—(22), for all the time periods, and all the quality parameters considered.

ASSUMPTIONS

1. The water-quality parameters do not interact with one another.
2. The water-quality parameters have a first order decay in reservoirs, but are conservative in pipes.
3. Complete and instantaneous mixing in reservoirs.
4. Complete and instantaneous mixing at nodes.
5. The power coefficient for pumping stations is computed assuming that the efficiency is constant.

TABLE 6. Cost (Dollars), and Relative Weight, of Objective Function Parts

Run (1)	Energy (2)	Water and treatment (3)	Penalty (4)	Total (5)
Base	4.51×10^3	1.56×10^4	3.64×10^3	2.37×10^4
	0.19	0.66	0.15	1.0
Sensitivity analysis case 1	5.1×10^3	1.2×10^8	5.8×10^5	1.2 x 10*
	=0	=1	=0	=1.0
Sensitivity analysis case 2	1.2×10^5	1.6×10^4	6.7×10^3	1.4×10^5
	0.84	0.11	0.05	1.0
Sensitivity analysis case 3	3.9×10^3	1.1×10^4	0	1.4×10^4
	0.27	0.73	0	1.0

6. The cost of treating one water-quality parameter at a treatment plant is independent of the costs for other parameters.

METHOD OF SOLUTION

The solution is obtained with GAMS (Brooke et al. 1988)/MINOS (Murtagh and Saunders 1982), on a PS/2 mod 80. GAMS is used to build the model, MINOS is used to solve it, by employing a projected augmented Lagrangian algorithm. A detailed description of the optimization technique can be found in Murtagh and Saunders (1982).

EXAMPLE

The optimization model was applied to the system shown in Fig. 3. It contains one source with a treatment plant *(TS),* which discharges into a reservoir *(Rl)* and from there water is pumped by a pumping station *(PSR* 1) into the system. The system is fed from two more sources: reservoir *R2,* through pumping station *PSR2,* and well *W3,* through *PSR3.* Water can flow back into the reservoirs, through the bypass around the pumps. The system supplies six consumers, located at nodes A-F, and contains 11 pipes. The operating horizon is 24 h, divided into five time intervals: 0-4, 4-10, 10-16, 16-20, 20-24 hours. Fig. 4 contains the legend for Fig. 3. For pipes: # = number of pipe, and an arrow denotes a directed arc; CHW_{ti} = friction coefficient of the Hazen Williams head loss formula; $L_{ij}(km)$ = length; $D_{ij}(mm)$ = internal diameter. Note that the Darcy-Weisbach head loss formula may be used here as well, as has been formulated in (14), and used in the steady state case (Ostfeld and Shamir 1993). For nodes: $+Z_i$(m) = elevation of node i, Tables 1-5 contain the remaining data for the network.

RESULTS

Results are shown in Figs. 5-8 and in Tables 6 and 7, for a base run and three additional sets of input data, for the purpose of testing the response of the solution to various conditions. The data in Figs. 5-8 is organized as follows: each of the figures contains five blocks, denoted *T* to *T5,* corresponding to the five time intervals. In each block there are results that contain the following:

TABLE 7. Reservoirs Levels and Concentrations

Run (1)	LEVELS (m)										CONCENTRATIONS (mg/L)									
	*R*1					*R*2					*R*1					*R*2				
	*T*1 (2)	*T*2 (3)	*T*3 (4)	*T*4 (5)	*T*5 (6)	*T*1 (7)	*T*2 (8)	*T*3 (9)	*T*4 (10)	*T*5 (11)	*T*1 (12)	*T*2 (13)	*T*3 (14)	*T*4 (15)	*T*5 (16)	*T*1 (17)	*T*2 (18)	*T*3 (19)	*T*4 (20)	*T*5 (21)
Base	10	7.89	10.96	9.09	10.40	20	19.09	17.14	19.53	20.70	302.3	172.9	134.8	93.4	92.5	118.66	117.29	117.98	116.68	116.38
Sensitivity analysis case 1	10	11.95	14.78	15	15	20	18.7	15.39	14.92	13.03	210.5	137.1	108.6	97.4	96.8	118.65	117.41	116.46	115.81	115.2
Sensitivity analysis case 2	10	7.89	11.52	9.55	11.48	20	19.09	17.29	19.68	19.68	302.3	165.4	131.1	92.4	91.5	118.7	117.3	117.6	117.0	116.5
Sensitivity analysis case 3	10	7.89	11.58	8.29	10.81	20	19.09	17.14	15.72	14.54	302.3	172.9	170.7	120.4	119.04	118.7	117.3	116.3	115.6	115

Discharge *(Q):*

1. The amounts of water supplied from the sources and reservoirs: *Rl, R2, W3,* and *TS* (m^3/h), then converted into their relative contributions to total supply (e.g., 52%, 22%, and 26% in Fig. 5 for 71, for *Rl, R2,* and *W3,* respectively, with *TS* not operating at this time).
2. Flows which reach their bounds are indicated.

Quality (C):

1. Nodes at which the concentration reaches its bound are indicated.

2. The degree of treatment at *TS*.

Heads (H):

1. Nodes at which the head reaches its bound are indicated.
2. Pumping stations at which dummy variables are indicated (e.g., 1.67 m at *R2* at 73). When dummy variables are used, they contribute a penalty to the objective function.

Table 6 lists the values in the objective function, of energy cost, water and treatment cost, and penalty cost [in ($)], for the base run, and the sensitivity analysis cases. Their relative contribution to the total cost is also shown, for each run. Table 7 lists the reservoir levels (m) and concentrations (mg/L), where the levels correspond to the beginning of each time interval, and the concentrations for each of the whole time periods.

Sensitivity Analysis Results

Excluding the base run, three additional runs, which were designed to test the sensitivity of the results to various changes in the data, are shown in Figs. 6-8.

Case 1

A very high cost (5 x 10^6 \$/m^3, compared with 0.5 \$/m^3 in the base run) is assigned to the water in the well source (node *W3).* As a result, treatment is at its maximum in *T1* and T2, and least in *T5.* In *T2* and *T4* threshold concentrations are reached at nodes Fand *B,* respectively. Dummy variables are used. Due to the high cost of water given in well source *W3,* in comparison to the penalty induced on the dummy variables in the objective function, and the cost of treatment, the minimum possible is subtracted from *W3* in all the time periods. Therefore, amounts of over 1,000 (m^3/h) are taken from node *TS,* and treated to the maximum possible level at *T1* and 72.

Case 2

A low efficiency is assigned to the pumping station at reservoir *R2* (0.001, compared with 0.75 in the base run). As a result, treatment is maximum in *T2,* 73, and *TA.* The minimum amount of water possible is taken from *TS* at *T1* and T5. In T2 and *T4* threshold concentrations are reached at nodes *D* and B, respectively. In 73 the pressure is minimum at node *D.* Dummy variables are used. The low efficiency of pumping station *PSR2* increases the energy cost of water supplied from reservoir R 2. Therefore, the reservoir R2 fills in *T3* and no water is supplied from him in *T4* and *T5.*

Case 3

A less restrictive requirement is placed on quality at node *B* in *T4* (400 mg/L, compared to 100 mg/L in the base run). The results show that the minimum is taken from *TS* at 7'1, *T3,* and *T5.* In T2 and T5 threshold concentrations are reached at nodes F and A, respectively. Dummy variables are not used. Due to the less restrictive requirement at node *B* in *T4,* in comparison to the base run, no treatment, and no water need be supplied from node *TS* in 71, 73, and *T5,* dummy variables are not used, and the threshold concentration requirement is met at node *A* in 75.

CONCLUSIONS

A model for optimal operation of a multiquality water-supply system under unsteady conditions has been developed, applied to a sample system, and solved. The results demonstrate that the equations which have been developed to describe water-quality changes in pipes and reservoirs over time, enable the model to select directions of flow, and to follow quality throughout the system. The runs made with the model, under a variety of data and conditions, show that the optimal solution is sensitive to changes in data, and it uses the flexibility afforded by mixing sources, and using treatment to meet consumers demands for quantities, qualities, and hydraulic heads, at minimum cost.

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APPENDIX I. REFERENCES

Brooke, A., Kendrick, D.,and Meeraus, A. (1988). *GAMS: a user's guide.* Scientific Press, U.S.

Cohen, D. (1992). "Optimal operation of multiquality networks," DS thesis, Technion, Haifa, Israel.

Grayman, W. M., Clark, R. M., and Males, R. M. (1988). "Modeling distribution system water quality: dynamic approach." *J. Water Res. Ping, and Mgnit. Div.,* ASCE, 114(3), 295-312.

Liou, C. P., and Kroon, J. R. (1987). "Modeling the propagation of waterborne substances in distribution networks." J. *Am. Water Works Assoc.,* 79(11), 54-58.

Murtagh, B. A., and Saunders, M. A. (1982). "A projected Lagrangian algorithm and its implementation for sparse nonlinear constraints." *Mathematical Programming Study,* 16, 84-117.

Ostfeld, A. (1990). "Management of a regional water supply system with sources of different qualities," MS thesis, Technion, Haifa, Israel.

Ostfeld, A., and Shamir, U. (1993). "Optimal operation of multiquality networks: steady state conditions."J. *Water Res. Ping, and Mgmi. Div.,* ASCE, 119(6),645-662.

Shah, M. (1985). "A dynamic model for dilution in water distribution systems," MS thesis, Technion, Haifa, Israel.

Sinai, G., Shina, G., Kitai, E., and Shah, M. (1987). "Physical and computer models of multiquality networks." *J. Water Res. Ping, and Mgmt. Div.,* ASCE, 113(6), 745-759.

APPENDIX II. NOTATION

The following symbols are used in this paper:

- A = set of arcs, indexed a;
- $Apumps$ = subset of A, on which pumping stations are located;
- $Ares$ = surface area of reservoir;
- $Cinitial^{k}_{tsn}$ = concentration of quality parameter k, at inlet of source node tsn;
- $CINres$ = concentrations at inlet of reservoir;
- C^{k}_{in} = concentration of quality parameter k at node in;
- $C^{k,\min}_{in}$, $C^{k,\max}_{in}$ = maximum and minimum concentrations, respectively, of water-quality parameter k at node in;
- $Cpipe$ = average concentration in pipe;
- $Cpipe^{k}_{a}$ = concentration of quality parameter k in arc a, computed by *(5a)*;
- $Cpipe^{k,initial}_{a}$ = initial average concentration in arc a of water-quality parameter k;
- $Cres$ = concentrations in reservoir;
- $Cres^{initial}$ = initial concentration in reservoir;
- C_u, C_d = concentrations at nodes u and d of pipe, respectively;
- Cup = concentration at upstream node calculated by (6a);
- $C^{k,const}_{well}$ = fixed known concentration of water-quality parameters k at node well;
- C^{k}_{well} = concentration of water-quality parameter k at node *well;*
- D_a = internal diameter of arc a;
- DT = planning horizon;
- d = downstream node of pipe;
- EC = energy cost;
- $ECCP_a$ = power coefficient of pumping station located on arc a ($a \in Apumps \subset A$), assuming efficiency is constant;
- $Floop$ = fundamental loop matrix for given (arbitrarily selected) spanning tree;
- $f_a(R_a, \varepsilon_a/D_a)$ = friction factor of arc a;
- H_{in} = total heads at selected internal nodes;
- $H^{\min}_{in}$, $H^{\max}_{in}$ = minimum and maximum total heads allowed at the selected internal nodes, respectively;
- $Head_{well}$ = fixed known head at node well;
- $Hres$ = water level of reservoir;
- $Hres^{initial}$ = initial water level in reservoir;
- $Hres^{\min}$, $Hres^{\max}$ = minimum and maximum levels allowed in reservoir;
- H_{well} = head at node well;
- IN = set of internal nodes, indexed in;
- K = set of water-quality parameters, indexed k;
- Ktc^{k}_{tsn} = coefficient of treatment cost of quality parameter k, at source node tsn;

$kwhc$ = energy charge during time of operation;
L = pipe length;
$L(\Delta t)$ = distance traveled by water-quality front in pipe, during time period of length Af; length of arc *a;*
L_a = quantity defined for arc *a* in (3);
LT_a = natural logarithm; number of
LN = periods;
m = subscript of set of time periods: 1, . . . , m;
n = maximum number possible in computer being used;
$NMAX$ = set of nontreatment source nodes, indexed *ntsn;*
$NTSN$ = penalty terms used in smoothing (5a) and (6a);
PEN_a = large positive number that incurs high (artificial)
$penalty$ = penalties in objective function, on dummy variables

path matrix, connecting reference node *RN* with
P^{RN} = subset of internal nodes; inflow and outflow of
reservoir, respectively; vector of discharge along
$QINres, QOUTres$ = arcs; flow in arc *a;*
$\mathbf{q}$ = minimum and maximum discharges in arc *a,* respectively;
q_a =
q_a^{min}, q_a^{max} = discharge at node in ($q_{in} > 0$ when water is withdrawn from node);
q_{in} = discharge supplied by source node well and source node *tsn,* respectively; Reynolds number of arc *a;*
q_{well}, q_{tsn} = set of reservoirs, indexed *res;* reduced incidence matrix with respect to reference node *RN;*
$\Re_a$ = maximum removal ratio of quality parameter *k* at
RES = node *tsn;*
RIM^{RN} = treatment cost of quality parameter *k,* per unit volume of treated water, at source node *tsn;* set of
$RR_{tsn}^{k,\max}$ = treatment source nodes, indexed *tsn;* beginning of time period *n;* upstream node of pipe; mean
TC_{tsn}^{k} = velocity in pipe; water volume of reservoir; vector of consumption (except for the reference node
TSN = *RN)* at nodes; water cost;
t_n = fixed charge for unit volume of water at well nodes;
u = fixed charge per unit volume of untreated water at
V = *tsn* nodes;
$Vres$ = set of wells, indexed well;
W = dummy penalty variables;
coefficients for *aih* pumping station at its maximum
WC = efficiency;
WC_{well} = head loss along arc *a,* calculated by Darcy-Weisbach
$WC_{tsn}^{initial}$ = formula;

$WELLS$ =
x_a, y_a =
$\alpha_0^a, \beta_0^a, \gamma_0^a$ =
$\Delta Hf_a(q_a)$ =

$\Delta Hloop$ = head difference for set of fundamental loops and paths;
$\Delta Hpipe(q_a)$ = vector of heads, losses or gains, along arcs; head
$\Delta Hp_a(q_a)$ = gain at pumping station, located on arc *a;* time
Δt = interval for optimization model; length of time
Δt_n = period *n;*
δres = first-order decay coefficient of concentration in reservoir;
ε = small number;
ε_a = roughness coefficient of arc a; relative roughness
ε_a/D_a = of arc *a;* pumping station efficiency; set of arcs
η = connected to reservoir; set of arcs connected to
$\xi(res)$ = node *in;* arcs directed away from node *tsn;* and
$\xi(in)$ = constant dependent upon units used for L_a, D_a, and
$\xi(tsn)$ =
Ω_a =