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OPTIMAL OPERATION OF MULTI-QUALITY WATER SUPPLY SYSTEMS-II: THE Q-H MODEL

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This paper describes the second in a series of three models for optimal operation of multi-quality water supply systems. This second model, which is termed the Q-H (flow-head) model, seeks to determine the optimal operation of pumps and valves, and does not consider water quality aspects. However, the model belongs to the group of three models for multi-quality systems because it is one of the two building blocks (the other is the flow-quality Q-C) of a full-flow-quality-head (Q-C-H) model. This Q-H model is based on continuous representations of the head-flow and power-flow functions of the pumping stations, which in turn results in a continuous non-convex optimization model. For a given flow distribution in the network, Q_0, the Q_0-H model is solved for the optimal operation of pumps and valves. The flow distribution is then modified by changing the circular flows, using a projected gradient method combined with the Complex Method which employs the results of the Q_0-H solution, such that the locally optimal solution at the next point has a better value of the objective function. The process is continued until one of the termination criteria is satisfied. The circular flows thus serve as decision variables in an external problem, while in the internal problem the decisions are the operation of pumps and valves. The method is demonstrated by application to a sample problem.

Keywords: Water supply systems; water quality; optimal operation of water systems; network analysis; hydraulic analysis

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INTRODUCTION

Optimal operation of a water supply system, without consideration of water quality, has been addressed by a number of researchers and practitioners. Carpentier and Cohen [2] proposed a method based on Dynamic Programming (DP), combined with a decomposition-coordination and aggregation–disaggregation approach. They developed the approach and associated computer programs for use in a real water supply system in France. However, to the best of the authors' knowledge, at this time the system is not operational. Zessler and Shamir [14] used another variant of DP known as Progressive Optimality (PO), which can be used when the objective function is convex. Their iterative process leads to the global optimum for a convex objective function. Jowitt and Germanopoulos [9] and Olshansky and Gal [12], used a Linear Programming (LP) model to optimize the operation of a water distribution system. In their model, the time horizon (usually 24 hours) is divided into a number of periods (usually 5–8), such that demands can be taken as constant and energy tariff remains fixed during the period.

The approach taken in this paper differs from the methods discussed above. It is an improvement of existing methods for identifying the optimal operation of single quality water supply systems. However, it is also a component in a full approach, called the Q-C-H model, for optimal operation of a multi-quality water supply system which is described in a companion paper [6].

Mathematical Representation of the Network

A water system can be described or represented as a graph, consisting of arcs connected at nodes. See Cohen *et al.* [5] for a topological definition of the network.

A path is defined between each consumer node and a node at which the head is fixed independently of the flow distribution, *e.g.*, a reservoir or well. The paths of the consumers to fixed head nodes are represented by the path matrix, P_a, whose components are: $(P_a)_{ij} = 1$ if path i includes pipe j and the positive directions of both coincide. $(P_a)_{ij} = -1$ if path i includes pipe j and their positive directions are opposite. $(P_a)_{ij} = 0$ if path i does not include pipe j. The placement of pump-stations, booster pumps, and control valves on pipes is specified, respectively, by the matrices B_p, B_b and B_v, whose components are $(B_k)_{ij}=1$ if element i is on pipe j, and $(B_k)_{ij}=0$ otherwise, for $k=p$, b, v. The cyclic matrices related to pump-stations, boosters and control valves are:

$$L_k = L_a B_k^T \quad k = p, b, v \tag{1}$$

Similary, the path matrices related to pump-stations, boosters and control valves are:

$$P_k = P_a B_k^T \quad k = p, b, v \tag{2}$$

Decision Variables

The decision variables of the Q-H problem are as follows.

Γ are the pumps which operate and their series-parallel interconnections within pumping stations. Each operating set is called "a configuration". $\mathbf{m}_v$ are the opening ratios of the valves. $\mathbf{q}_s$ are the discharges to be supplied from the sources. $\mathbf{h}_a$ are the head losses in pipes, $\mathbf{h}_v$ are the head losses in valves, $\mathbf{h}_b$ are the pumping heads of the boosters and $\mathbf{h}_p$ are the pumping heads of the pump stations.

The flow distribution in the network is determined as part of the solution. An initial flow distribution, which satisfies water continuity at all nodes, is specified. This initial solution is modified in the solution process, in a way which retains continuity. Continuity is maintained by considering the circular flows in loops and pseudo-loops, $\mathbf{q}$, as decision variables, since when these flows are modified, the continuity at nodes is maintained. As a consequence of this definition, the water flow continuity equations can be omitted, thereby reducing the size of the optimization model.

The vector $\mathbf{q}$ is of order nl, the number of loops, with the ith component of the vector being the flow in the positive direction of loop i. Since the number of loops is considerably smaller than the number of pipes, using the circular flows in the loops rather than the flows in the pipes as the decision variables in this manner results in an even smaller model. The relationship between the flows in all pipes of the network, $\mathbf{q}_a$, and the circular flows is defined by:

$$\mathbf{q}_a = \mathbf{q}_a^0 + L_a^T \mathbf{q} \tag{3}$$

where $\mathbf{q}_a^0$ is the initial pipe discharges, which satisfy the water flow continuity equations, and $(\cdot)^T$ denotes the transpose. The discharges to be supplied from the sources are defined by:

$$\mathbf{q}_s = \hat{A}\mathbf{q}_a = \hat{A}[\mathbf{q}_a^0 + L_a^T\mathbf{q}] \tag{4}$$

where $\hat{A}$ is a submatrix of A, obtained from the rows of A which are related to source nodes. The discharges through the valves, boosters, and pump stations are:

$$\mathbf{q}_k = B_k^T\mathbf{q}_a = B_k^T[\mathbf{q}_a^0 + L_a^T\mathbf{q}] \quad k = p, b, v \tag{5}$$

Objective Function

The objective of the optimization is to minimize the total cost over a time period t. This total cost is made of the following components.

Water Supply Cost

This cost represents the cost of water supplied from the sources. In general, the specific cost (per unit volume) varies with discharge. Thus, the specific cost of water at the sources will be given by a vector of functions, denoted by $\mathbf{w}_s(\mathbf{q}_s)$, the dimension of which is the number of sources. When the unit cost at a source is fixed, the value in this vector is a constant. The total supply cost from the sources, ϕ_s, for the entire period t, is given by:

$$\phi_s = t\mathbf{w}_s(\mathbf{q}_s)^T\mathbf{q}_s \tag{6}$$

Pumping Cost at Boosters

The hydraulic characteristic of every booster includes the relationship between input power and the discharge. This relationship can be expressed by a polynomial function whose coefficients are obtained from regression analysis. Usually, this relationship is expressed by a linear, or at most a quadratic, function.

For generality, denote the power-discharge relationship at booster i by $N_b^i(q_b^i)$. The total pumping cost at boosters, ϕ_b, for the time period is then described by:

$$\phi_b = tk_e\alpha_b \mathbf{N}_b(\mathbf{q}_b)^T \mathbf{1}_b \tag{7}$$

where k_e is the cost per unit energy, α_b is a constant which depends on the units used (for SI units $\alpha_b = 104$), and $\mathbf{1}_b$ is a vector of order n_b whose elements are all 1.

Pumping Cost at Pump Stations

The power used at a pump station in a particular situation depends on the discharge, the head and the efficiency of the operating configuration. The relationships among these parameters are addressed in the model in the constraint set. The total pumping cost at the pump station, ϕ_p, is:

$$\phi_p = tk_e\alpha_p \mathbf{1}_p^T H_p E^{-1} \mathrm{Q}_p \mathbf{1}_p \tag{8}$$

where α_p is a constant whose value depends on the units (for SI units $\alpha_p = 104$). H_p is the matrix of pumping heads at the n_p pump stations; it is square and diagonal with components $\mathbf{h}_p$. Q_p is the matrix of discharges of the n_p pump stations; it is square and diagonal with components $\mathbf{q}_p$. E is the efficiency matrix of the n_p pump stations; it is square and diagonal with components $\boldsymbol{\eta}_p$ (the pumping efficiency vector of the n_p pump stations).

Constraints

The constraints within the model include the physical laws, which define the system, the hydraulic characteristics of the various hydraulic components of the network, limits on the heads at consumer nodes, and limits on the discharges in various components of the network. The mathematical representation of these constraints is described below.

Physical Laws

Continuity of mass is maintained by the flow decision variables, as explained previously. Continuity of the energy grade line is achieved by specifying Kirchoff's second law in the following form.

$$L_p \mathbf{h}_p + L_b \mathbf{h}_b - L_a \mathbf{h}_a - L_v \mathbf{h}_v = \mathbf{b}_p \tag{9}$$

for each of the paths in the network. b_p is the head difference between the ends of the path: $b_p = 0$ when the path is a loop, and has some known value when the path is between two nodes (*e.g.*, reservoirs) at which the heads are fixed.

Hydraulic Characteristics

Pipes

The head loss in a pipe is given by the Hazen-Williams equation:

$$h_a = \alpha_a q_a^{1.852} \mathrm{Chw}^{-1.852} d_a^{-4.87} l_a \tag{10}$$

where α_a is a constant whose value depends on the units (for the SI units used $\alpha_a = 10.708$), q_a is the discharge (m^3/s), d_a is the diameter (m), Chw is the Hazen-Williams friction factor, and l_a is the pipe length (m).

Combining all the constants in Eq. (10) through:

$$k_a = \alpha_a \mathrm{Chw}^{-1.852} d_a^{-4.87} l_a \tag{11}$$

results in:

$$h_a = k_a q_a^{1.852}$$

with the head losses in the all pipes, including those with negative flows being given by:

$$\mathbf{h}_a = K_a \mathrm{Q}_a |\mathbf{q}_a|^{0.852} \tag{12}$$

where K_a is square diagonal matrix whose components are $\mathbf{k}_a$, and Q_a is a square diagonal matrix whose components are $\mathbf{q}_a$.

Valves

The head loss through a valve depends on the discharge through the valve and the opening ratio, m_v of the valve. This relationship is generally specified in a table or diagram. However, the data can be fitted to the general formula:

$$h_v = k_v q_v^{\alpha_v} m_v^{-\beta_v} \tag{13}$$

where k_v, α_v and β_v are obtained by regression of given data. For a linear valve $\beta_v = 2$. α_v is approximately 2 and $k_v \cong c_v^{-\alpha_v}$ where c_v is the value coefficient.

The head losses in all valves, again also considering negative flows, are given by:

$$\mathbf{h}_v = K_v Q_v M_v |\mathbf{q}_v|^{\alpha} v^{-1} \tag{14}$$

where K_v is square diagonal matrix, whose components are $\mathbf{k}_v$, Q_v is a square diagonal matrix, whose components are $\mathbf{q}_v$, and M_v is a square diagonal matrix, whose components are the opening ratios of the valves raised to the power $-\beta_v$.

Booster Pumps

The relationship between the head and the discharge through the pump, can be approximated by a quadratic or cubic polynomial of the form:

$$h_b = \alpha_0 + \alpha_1 q_b + a_2 q_b^2 + \alpha_3 q_b^3 \tag{15}$$

where the constants α_0, α_1, α_2 and α_3 are obtained by regression. The relationships at all boosters of the network are given by:

$$\mathbf{h}_b = \mathbf{a}_0 + A_1 \mathbf{q}_b + A_2 \mathbf{q}_b^2 + A_3 \mathbf{q}_b^3 \tag{16}$$

where $\mathbf{a}_0$ is a vector whose components are the coefficients, α_0 and A_i $(i = 1, 2, 3)$ are square diagonal matrices whose components are the coefficients α_1, α_2 and α_3, respectively.

Pump Stations

The relationship between the head and the discharge at a pump station depends on the particular pump configuration which is in operation at the time. The head-discharge curve for a given configuration is obtained by using the serial and parallel connection rules of pumps, and is described by the general expression:

$$\mathbf{h}_p = \zeta_h(\Gamma, \mathbf{q}_p) \tag{17}$$

The efficiency-discharge curve is obtained in a similar fashion and is expressed in the general form:

$$\boldsymbol{\eta}_p = \zeta_\eta(\Gamma, \mathbf{q}_p) \tag{18}$$

where ζ_h and ζ_η are functions, from those of the individual pumps constructed for each of the pump configurations, Γ, to reflect the way in which they are interconnected to form the configuration.

Limits on Discharges

Boosters

The discharges through the boosters are limited through the following equation:

$$\mathbf{0} \leq \mathbf{q}_b \leq \mathbf{q}_b'' \tag{19}$$

where $\mathbf{q}_b''$ are the maximum discharges through the boosters.

Valves

Limits on the discharges through the valves are imposed to prevent cavitation, and if the valve is a one directional valve, to prevent reversal of flow.

$$\mathbf{q}_v' \leq \mathbf{q}_v \leq \mathbf{q}_v'' \tag{20}$$

For a one directional valve $q_v' = 0$, and for a bi-directional valve $q_v' = -q_v''$.

Pump Stations

The maximum discharge through each pump-station depends on the configuration in operation. The value of the maximum discharge is obtained from the head-discharge relationship where the head equals zero.

$$\mathbf{0} \leq \mathbf{q}_p \leq \mathbf{q}_p''(\Gamma) \tag{21}$$

Sources

The upper limit on a source is either the supply capability of the source or some other restriction. A lower limit is used to prevent reverse flow or to prevent overflow if reverse flow into a reservoir occurs. The constraints over the all sources are:

$$\mathbf{q}_s' \leq \mathbf{q}_s \leq \mathbf{q}_s'' \tag{22}$$

where $\mathbf{q}_s'$ and $\mathbf{q}_s''$ are the minimum and maximum discharge of the sources, respectively.

Head Constraints

The heads at consumer nodes must be within bounds. The head at each consumer node is expressed by the energy equation along a path between a node with known head and the consumer node. The resulting head constraints for all consumers are:

$$\mathbf{h}_c' - \mathbf{z}_p \leq P_p\mathbf{h}_p + P_b\mathbf{h}_p - P_a\mathbf{h}_a - P_v\mathbf{h}_v \leq \mathbf{h}_c'' - \mathbf{z}_p \tag{23}$$

where $\mathbf{h}_c'$ and $\mathbf{h}_c''$ are the minimum and maximum head vectors, respectively, and $\mathbf{z}_p$ is a vector whose components are the heads at the origin nodes of the paths.

Operating Constraints

Valves

Restrictions on the opening ratio of each valve are expressed by:

$$\mathbf{m}_v' \leq \mathbf{m}_v \leq \mathbf{m}_v'' \tag{24}$$

Usually $m_v' = 0$ (closed) and $m_v'' = 1$ (fully open).

Pump Stations

Each pump station has a finite number of discrete configurations (D_i) which can be operated. The actual configuration at work (Γ) should be selected from these given configurations (D_i).

$$\Gamma \in \{D_1, D_2, D_3, \ldots, D_n\} \tag{25}$$

Alternative Formulation of the Optimization Problem

The optimization problem presented above has a non-linear objective function and non-linear constraints. If the selection of operating configurations is performed through logical variables, the result is a non-linear integer programming problem, which is difficult to solve. A formulation which avoids the use of integer variables is developed by using continuous smooth functions for the head-discharge and efficiency-discharge relations of the pumping stations (Cohen [4]).

The Operating Surface (Function) of Pump Stations

Consider a system which has a pump, a control valve and a by-pass, as depicted schematically in Figure 1a. Consider also the hydraulic curves of the pump and of the system as described schematically in Figure 1b.

If the valve is fully open and the by-pass is closed, the pump will deliver a discharge of point A (in Fig. 1b) which is determined by the intersection of the pump curve with the system curve. By closing the control valve partially, it is possible to modify the system curve and move to a point such as B. If the flow through the by-pass is increased,

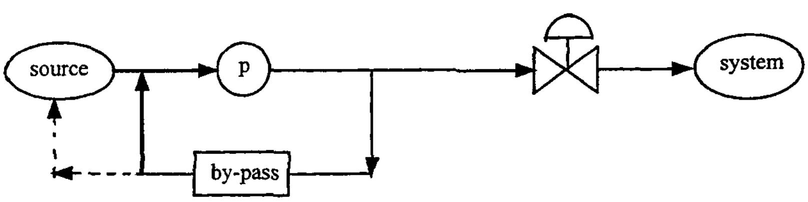

FIGURE 1a A schematic description of a system consisting of a pump, control valve and a by-pass.

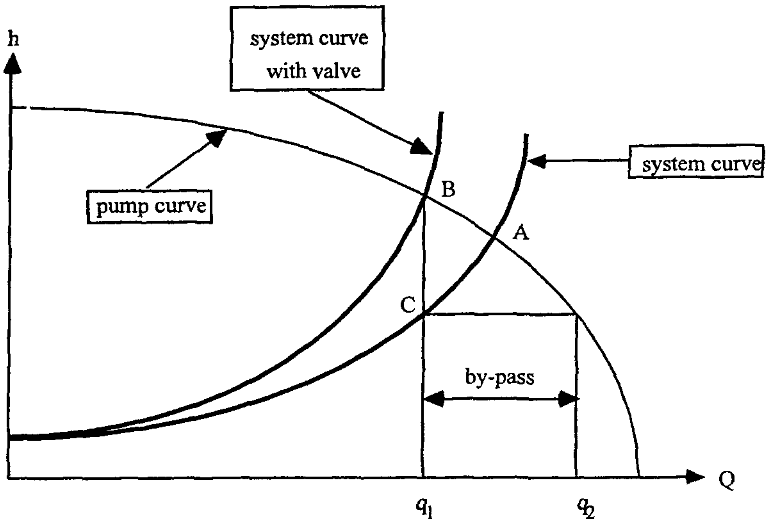

FIGURE 1b Operating point of a system consisting of a pump, control valve and a by-pass.

a point C can be reached: q_1 is the net flow delivered to the system, q_2 flows through the pump and q_2-q_1 is recirculated through the by-pass. The control valve and by-pass therefore enable the operating range of the pump to be expanded from a single point to an entire range.

It may seem at first that choking the control valve and recirculating flow is an inefficient way to operate. However, because the efficiency of the water supply system itself changes with flow, the overall operating cost may be reduced by such controls. The optimization procedure described below is able to handle this feature automatically.

Consider a pump station with n_c discrete configurations. Denote the curves of configuration i by $y_h^i(q)$ (head-discharge), $y_\eta^i(q)$ (efficiency-discharge) and $y_p^i(q)$ (power-discharge). The h-q space of the pump station consists of n_c curves of $y_h^i(q)$ {where $i=1, 2,\ldots,n_c$}. The space is discrete, with each of the discrete points being associated with one of the n_c configurations. Points at which two or more curves intersect are assigned to the configuration which uses the least power. A point (q_i, h_i) is also omitted if there exists another point, (q_j, h_j), which uses less power, and either $h_j=h_i$ and $q_j > q_i$ or $q_i=q_j$ and $h_j > h_i$. After all such points have been omitted, the h-q

space contains points which best generate the range of available head discharge combinations at greatest efficiency. Next, a continuous function is fitted between the discrete points identified as described above, with the thorough use of a control valve, in the following manner. Consider a point (q^*, h^*) which does not belong to the set of discrete points. Find the point which has the same discharge and the next highest head, (q^*, h_j). Denote the configuration associated with this point by j. The point (q^*, h^*) is obtained by operating configuration j, and dissipating the head difference $(h_j - h^*)$ by the control valve. Hence, the power required for point (q^*, h^*) is $y_p^j(q = q^*)$; the power used when configuration j is operated at the discharge q^*. The efficiency associated with the point (q^*, h^*) is $\eta = \eta_j h^*/h_j$, where η_j is the efficiency of configuration j at the discharge q^*.

The same process can be applied for every other point which is not on one of the discrete h-q curves. The result is a continuous function bounded by an envelope curve, $h_p'(q)$, which limits the head that can be supplied by the pump station as a function of the discharge. Each point below this envelope can be defined with its efficiency, the configuration which provides it most efficiently, and the head difference which the control valve has to dissipate.

Pump stations usually consist of different pumps. Consequently, the envelope curve for a pump station generally includes several non-smooth points. This situation is illustrated in Figure 2 which describes a pump station consisting of 3 pumps; two of type a and one of b, where all the pumps are in parallel. The envelope curve for the pump station is obtained from the parallel connections of the 3 pumps. Due to existence of the two types of pumps, the envelope curve contains the non-smooth point A. This envelope curve can be smoothed as follows.

Assume a pump station with n_i non-smooth points q^i $\{i = 1, 2, \ldots, n_i\}$ with each interval between two such points being derived from a different envelope function. Denote the function for the interval $(q^i,\ q^{i+1})$ by $g^i(q)$. The smoothed envelope function is given:

$$h_p'(q) = \frac{\sum_{i=1}^{n_i} g_i(q) \exp(\varpi\{\xi_i\})}{\sum_{i=1}^{n_i} \exp(\varpi\{\xi_i\})} \qquad (26)$$

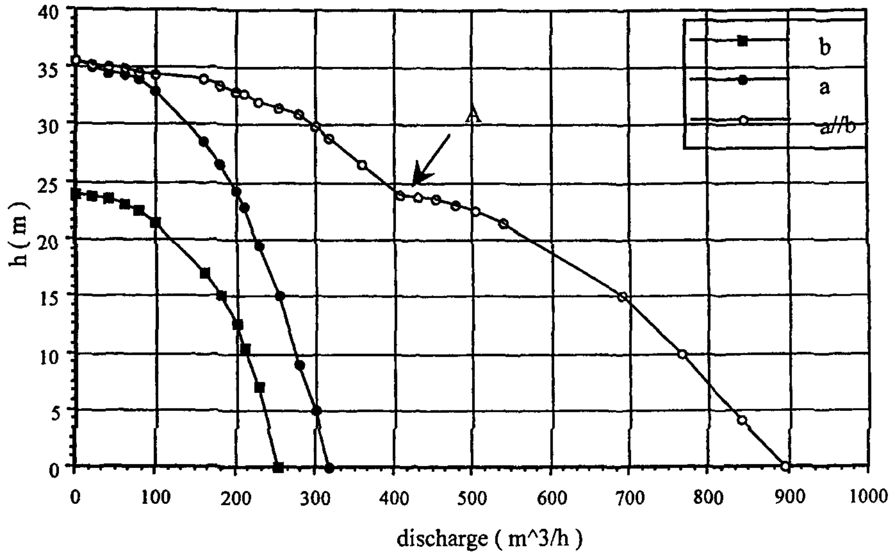

FIGURE 2 Non-smooth point in the head-flow envelope of a pump station.

where

$$\xi_i = (q - q^i)(q^{i+1} - q) \tag{27}$$

$$\varpi\{x\} = k_p \frac{x}{\sqrt{x^2 + \varepsilon_o}} \tag{28}$$

k_p is a gain coefficient, and ε_o is a small number to prevent division by zero.

The ($q=0$, $h=0$) point also has to be included. Inclusion of this point is achieved by bringing the envelope curve down to a point for which $q < 1$ (an arbitrary value, considered small enough). Thus, the smoothed envelope curve, including the operating condition at zero flow, is:

$$h_p''(q) = h_p'(q_p) \frac{\exp(\varpi\{q-1\}) + q^{\alpha} \exp(-\varpi\{q-1\})}{\exp(\varpi\{q-1\}) + \exp(-\varpi\{q-1\})} \tag{29}$$

where α is a coefficient which determines how steeply the curve is brought down to zero.

For a given q the efficiency as a function of the head is defined by:

$$\eta = \sum_{i=1}^{n} \eta_i \frac{h}{h_i} [\exp\{\varpi(\xi_i - 1)\} + \exp\{\varpi(\xi_i - 0.5)\}] \tag{30}$$

in which h_i is the head obtained by configuration i when the discharge is q, and ζ_i is defined by:

$$\zeta_i = \sum_{j=1}^{i} \frac{\exp\{\varpi(h - h_j)\}}{[\exp\{\varpi(h - h_j)\} + \exp\{-\varpi(h - h_j)\}} \tag{31}$$

In this manner the discrete optimization problem initially formulated is transformed into a continuous and smooth model, which can be used to formulate a continuous non-linear optimization.

Q_0-H PROBLEM

The Q_0-H problem is defined as follows: for a given water flow distribution, find the optimal operation of the pumps, boosters, and control valves such that this flow distribution is realized at minimal cost.

When the flow distribution is specified, the supply cost and the pumping cost of boosters are given directly by Eqs. (6)–(8). h_a and h_b are also known and computed from Eqs. (12) and (16). Denote these known values of h_a and h_b by h_a^*, h_b^*. The maximum head added by each pump station, denoted by h_p'', can be computed from Eqs. (29) and (30) with the known flow. In accordance with the formulation of the operating surfaces of the pumping stations, the pumping head of each pump station is continuous, and lies in the range $[0, h_p'']$.

Each valve causes a head loss, denoted by $(h_v^*)^o$, for the fully open position. The value of $(h_v^*)^o$ is computed from Eq. (14) where $m_v = 1$ and q_v^* = discharge through the valve corresponding to the given flow distribution.

Decision Variables

The decision variables of the Q_0-H problem are:

$\mathbf{h}_p$ – pumping heads at the n_p pump stations.
$\mathbf{h}_v'$ – the additional head losses in the n_v control valves.

The total head loss in the valves are determined from:

$$\mathbf{h}_v = \mathbf{h}_v' + (\mathbf{h}_v^*)^o \quad \text{where } \mathbf{h}_v' \geq \mathbf{0} \tag{32}$$

As there is a possibility that no feasible solution exists for the initial flow distribution, in order to assure a mathematical solution, a pair of artificial variables each of which is penalized in the objective function by θ^+ and θ^-, respectively, are added to the constraints described by Eqs. (9) and (23). Denote the artificial variables added to the second Kirchoff's law constraints (Eq. (9)) by $\mathbf{x}_L^+$ and $\mathbf{x}_L^-$, and denote, the artificial variables added to the head constraints specified in Eq. (23) by $\mathbf{x}_p^+$ and $\mathbf{x}_p^-$.

Introduction of the known values, $\mathbf{h}_a^*$, $\mathbf{h}_b^*$ and $\mathbf{h}_p''$, Eq. (32) for the valve head losses, and the artificial variables, and rearranging the constraints yields the following model for the Q_0-H problem:

$$\min f_o = t k_e \alpha_p 1_p^T Q_p H_p E^{-1} 1_p + \theta^+(\mathbf{x}_L^+ + \mathbf{x}_p^+) + \theta^-(\mathbf{x}_L^- + \mathbf{x}_p^-) \tag{33}$$

subject to

$$L_p\mathbf{h}_p - L_v\mathbf{h}'_v + \mathbf{x}_L^+ + \mathbf{x}_L^- = \mathbf{b}_p + L_a\mathbf{h}_a^* + L_v(\mathbf{h}_v^*)^o - L_b\mathbf{h}_b^* \tag{34}$$

$$\mathbf{h}'_c - \mathbf{z}_p + P_a\mathbf{h}_a^* + P_v(\mathbf{h}_v^*)^o - P_b\mathbf{h}_b^* \le P_p\mathbf{h}_p - P_v\mathbf{h}'_v + \mathbf{x}_p^+ + \mathbf{x}_p^-$$
$$\le \mathbf{h}''_c - \mathbf{z}_p + P_a\mathbf{h}_a^* + P_v(\mathbf{h}_v^*)^0 - P_b\mathbf{h}_b \tag{35}$$

$$\mathbf{h}_p \le \mathbf{h}''_p \tag{36}$$

$$\mathbf{h}_p, \mathbf{h}'_v, \mathbf{x}_L^+, \mathbf{x}_L^-, \mathbf{x}_p^+, \mathbf{x}_p^- \ge \mathbf{0} \tag{37}$$

This formulation has a nonlinear objective function and linear constraints. The nonlinearity in the objective function is caused by the dependency of the efficiency on $\mathbf{h}_p$. Usually, this nonlinearity is mild over a wide range of $\mathbf{h}_p$. For given η_p, all the multipliers of $\mathbf{h}_p$ in the first term of Eq. (33) are therefore combined into a constant, resulting in an LP problem with the following objective function:

$$\min f_0 = \rho^T\mathbf{h}_p + \theta^+(\mathbf{x}_L^+ + \mathbf{x}_p^+) + \theta^-(\mathbf{x}_L^- + \mathbf{x}_p^-) \tag{38}$$

where $\rho^T = t\,k_e\alpha_p Q_p^{-1}E^{-1}1_p$. The solution of Q-H$_0$ can be obtained by the following algorithm:

Algorithm Q_0-H

Initialize the iteration counter: $k = 0$.

Step a Assume an initial pumping efficiency η_p^k (say $\eta_p^k = 0.75$).
Step b Compute vector ρ from $\rho_0^T = \rho_0^T E^{-1}$.
Step c Solve the LP problem, and obtain f^k, h_p^k, $(h'_v)^k$.
Step d Compute $\boldsymbol{\eta}_p$ from Eqs. (29) and (30) where $\mathbf{q}_p = \mathbf{q}_p^*$ and $\mathbf{h}_p = \mathbf{h}_p^k$.
Step e If $\|\boldsymbol{\eta}_p - \boldsymbol{\eta}_p^k\| \le \varepsilon_\eta$ or/and $|f^k - f^{k-1}| \le \varepsilon_f$. Stop otherwise set $\boldsymbol{\eta}_p^{k+1} := \boldsymbol{\eta}_p$. Set $k := k+1$ and go back to Step b.

The repeated computation of the LP can be time consuming. However, since the changes between successive iterations occur only in the objective function coefficients, the optimal solution of the last iteration is *at least feasible.* Therefore, computation time is reduced substantially if each iteration starts with the basis of the previous optimal solution. Using this approach, the new iteration needed no more than three inner LP iterations and in many cases it needed only one iteration [3, 4]. The convergence of the above algorithm has not been proven mathematically. However, the algorithm has been applied over a wide range of cases and is found to always converge in at most 3 iterations [3, 4].

The optimal operation of the pump-stations and the control valves are then determined, respectively, by the OPV and OPP algorithms described below:

OPV Algorithm

Step a Compute $\mathbf{h}_v$ from $\mathbf{h}_v = \mathbf{h}'_v + \mathbf{h}^0_v$, where $\mathbf{h}'_v$ is obtained from the optimal solution of the Q-H_0 problem.

Step b Substituting q^i_v and h^i_v, for $i = 1, 2, \ldots, n_v$, into Eq. (13) yields the optimal opening ratio for all control valves through:

$$m^i_v = \left[\frac{k^i_v q_v^{\alpha^i_v}}{h^i_v}\right]^{\beta^i_v} \quad i = 1, 2, \ldots, n_v \tag{39}$$

OPP Algorithm

Step a Denote the configuration in pump station i which has the closest head-discharge discrete curve to the point $(\mathbf{q}^i_p, \mathbf{h}^i_p)$ by j, where $\mathbf{q}^i_p$ is the discharge and $\mathbf{h}^i_p$ is obtained from the solution of the Q_0-H problem.

Step b From the head-discharge curve $h_{ij}(q_p)$ of configuration j at pump station i, compute the head h_{ij} where $q_p = q^i_p$.

Step c Define the by-pass discharge range $[q^i_p, (q''_c)_i]$ where $(q''_c)_i$ is the discharge of the configuration j when the head equals h^i_p.

Step d Compute the minimal power of configuration j in the range $[q^i_p, (q''_c)_i]$ for all pumping stations. This value is determined as follows: The power-discharge curve of a pump is either linear or concave-quadratic. If the curve is linear the minimum is achieved by operating configuration j with the control valve dissipating $[h^j - h_p^i]$, where h^j is the head obtained from configuration j when the discharge is q_p^i. If the power-discharge curve of configuration j is quadratic, then the minimal power is obtained by operating either the control valve dissipating $[h_{ij} - h_p^i]$, or with a by-pass flow $[(q_c'')_i - q_p^i]$. Therefore the minimal power is determined by comparing the power used by configuration j when the discharge is q_p^i and q_c''.

Q-H PROBLEM IN THE SUBSPACE q

It was shown in the previous section that, for a given flow distribution ($\mathbf{q}$), all the decision variables are determined uniquely. Consequently, the Q-H problem can be considered as an inside–outside optimization problem, where the inside problem is defined for a given $\mathbf{q}$, and the outside problem determines the optimal flow distribution.

The objective function for the outside problem of determining the optimal $\mathbf{q}$ consists of the supply cost, the pumping cost of boosters, and the optimal solution of the Q-H_0 problem for given $\mathbf{q}$, denoted for brevity by $\phi_p(\mathbf{q})$. According to Eqs. (4) and (5), the constraints imposed on $\mathbf{q}$ are derived from the discharge limits defined in constraints (19)–(22).

The resulting model for the Q-H problem is:

$$\min f = t\,\mathbf{w}_s(\mathbf{q}_s)^T \mathbf{q}_s + t\,k_e \alpha_b \mathbf{N}_b(\mathbf{q}_b)^T \mathbf{1}_b + \phi_p(\mathbf{q}) \tag{40}$$

subject to:

$$\mathbf{q}_s' - \hat{A}\mathbf{q}_a^0 \le \hat{A}L^T\mathbf{q} \le \mathbf{q}_s'' - \hat{A}\mathbf{q}_a^0 \tag{41}$$

$$\mathbf{q}_p' - L_p^T\mathbf{q}_p^0 \le L_p^T\mathbf{q} \le \mathbf{q}_p'' - L_p^T\mathbf{q}_p^0 \tag{42}$$

$$\mathbf{q}_b' - L_b^T\mathbf{q}_b^0 \le L_b^T\mathbf{q} \le \mathbf{q}_b'' - L_b^T\mathbf{q}_b^0 \tag{43}$$

$$\mathbf{q}_v' - L_v^T\mathbf{q}_v^0 \le L_v^T\mathbf{q} \le \mathbf{q}_v'' - L_v^T\mathbf{q}_v^0 \tag{44}$$

Solution Method

The problem formulated above in subspace $\mathbf{q}$ has a nonlinear objective function and linear constraints and can be solved by the projected gradient method [13] using an analytical formulation for the gradient, based on the LPG method [1]. The algorithm contains the following steps: (1) computation of the objective function f, defined by Eq. (40) for a given $\mathbf{q}$; (2) computation of the gradient of f with respect to $\mathbf{q}$; (3) computation of the projected gradient; (4) movement along the projected gradient direction to achieve an improved point; and (5) use of the Complex method, (Box, 1969 cited in Ref. [8]), when the algorithm gets "stuck" in a non-smooth point which is not optimal.

Computation of the Gradient

The gradient of the objective function, f, is the summation of the gradient components with respect to supply cost, pumping cost at boosters and pump-stations. The gradient with respect to supply cost, $\nabla_q \phi_s$, is derived from Eq. (8):

$$\nabla_q \phi_s = t L_a \hat{A}^T [F_s \mathbf{q}_s + \mathbf{w}_s] \tag{45}$$

where F_s is the Jacobian matrix of $\mathbf{w}_s$ with respect to $\mathbf{q}_s$. This matrix is square diagonal, with components given by:

$$(F_s)_{ii} = d\mathbf{w}_s^i / d\mathbf{q}_s^i \tag{46}$$

If $\mathbf{w}_s$ is constant $F_s = 0$ and Eq. (45) is reduced to $\nabla_q \phi_s = t L_a \hat{A}^T \mathbf{w}_s$.

Similarly, the gradient with respect to pumping cost at boosters, $\nabla_q \phi_b$, is derived from Eq. (7):

$$\nabla_q \phi_b = \alpha_b t\, k_e L_b F_b \mathbf{1}_b \tag{47}$$

where F_b is the Jacobian matrix of $\mathbf{N}_b$ with respect to $\mathbf{q}_b$. This matrix is square diagonal, with components given by:

$$(F_b)_{ii} = d\,\mathbf{N}_b^i / d\mathbf{q}_b^i \tag{48}$$

The Gradient of $\phi_p(q)$

Recall the definition given earlier in this paper wherein the optimal solution of the Q_0-H problem, in which the objective function, the coefficient matrix and the right hand side are all functions of the flow distribution, $\mathbf{q}$ is designated by min $f_0 = \phi_p(\mathbf{q})$. Computation of $\nabla_q \phi_p$ follows the approach first proposed by Alperovits and Shamir [1], and subsequently developed in a matrix formulation by Kessler and Shamir [10]. Eiger *et al.* [7] showed that the outer problem, *i.e.*, minimization of $\phi_p(\mathbf{q})$ in the space of $\mathbf{q}$, is non-smooth, and developed a method for global optimization of a single quality network.

The objective function of a linear program at a feasible basis is given by:

$$f_0 = \mathbf{c}_B^T B^{-1}\mathbf{b} + (\mathbf{c}_N^T - \mathbf{c}_B^T B^{-1} R)\mathbf{x}_N$$

where $\mathbf{c}_B$ and $\mathbf{c}_N$ are the coefficients of the basic and of the nonbasic variables, respectively. B and R are the basis and nonbasis matrices, respectively, and $\mathbf{b}$ is the right hand side vector. Using the chain rule with respect to $\mathbf{q}$ yields:

$$\begin{aligned}\nabla_q f_0 = {} & \nabla_q(\mathbf{c}_B^T B^{-1}\mathbf{b}) + \nabla_q(\mathbf{x}_N)^T(\mathbf{c}_N^T - \mathbf{c}_B^T B^{-1} R) \\ & + \nabla_q^T(\mathbf{c}_N^T - \mathbf{c}_B^T B^{-1} R)\mathbf{x}_N\end{aligned}$$

At the optimal solution $\mathbf{x}_N = 0$, hence the third term vanishes. This means that $\nabla_q(\mathbf{x}_N) = 0$, unless the optimal solution is nonunique and there is a nonbasic variable which can enter the basis set without changing the objective function value. But, this is the case only if the alternate variable has a zero value in the vector $\mathbf{c}_N^T - \mathbf{c}_B^T B^{-1} R$. So even if $\nabla_q(\mathbf{x}_N)$ includes a non-zero element the multiplication is zero. Thus the second term of the chain rule equation as above, can also be omitted, and the gradient can be computed by:

$$\nabla_q f_0 = \nabla_q(\mathbf{c}_B^T B^{-1}\mathbf{b}) \tag{49}$$

Using the chain rule on Eq. (49) yields:

$$\nabla_q f_0 = [\nabla_q \mathbf{c}_B] B^{-1}\mathbf{b} + [\nabla_q \mathbf{b}] B^{-1}\mathbf{c}_B + [\nabla_q B^{-1}]\mathbf{c}_B^T\mathbf{b} \tag{50}$$

where $[\nabla_q \mathbf{b}]$, $[\nabla_q \mathbf{c}_B]$ and $[\nabla_q B^{-1}]$ are the Jacobian matrices with respect to $\mathbf{q}$, of $\mathbf{b}$, $\mathbf{c}_B$ and B^{-1}, respectively. In this paper, B is independent of $\mathbf{q}$, and the third term in Eq. (50) can be dropped.

Computation of the remaining two terms proceeds as follows. In accordance with the relationships between the dual and primal LP problem, the dual vector π is:

$$\pi = B^{-T}\mathbf{c}_B \tag{51}$$

Using the chain rule, Eq. (5), and the special structure of the constraints (34–37) yields:

$$[\nabla_q \mathbf{b}]B^{-1}\mathbf{c}_B = [L_a S_a L_a^T - L_b S_b L_b^T + L_v S_v L_v^T]\pi_L + \\ + [L_a S_a P_a^T - L_b S_b P_b^T + L_v S_v P_v^T]\pi_p + L_p S_m \pi_m \tag{52}$$

where π_L, π_p and π_m are the dual variables of the constraints (34), (35) and (36), respectively, and:

$$L_i = B_i^T L_a \quad i = b,p,v \tag{53}$$

$$P_i = B_i^{-T} P_a \quad i = b,p,v \tag{54}$$

S_a, S_b, S_v and S_m are square diagonal matrices of order n_a, n_b, n_v and n_p, respectively, with components as defined by:

$$(S_a)_{ii} = 1.852 k_a^i (q_a^i)^{0.852} \tag{55}$$

$$(S_v)_{ii} = \alpha_v^i k_v^i (q_v^i)^{\alpha_v - 1} \quad \text{for } m = 1 \tag{56}$$

$$(S_b)_{ii} = d\mathbf{h}_b^i / d\mathbf{q}_b^i \tag{57}$$

$$(S_m)_{ii} = (d\mathbf{h}_p^o)_i / d\mathbf{q}_p^i \tag{58}$$

$[\nabla_q \mathbf{c}_B]\, B^{-1}\mathbf{b}$ is computed as follows. In the objective function f_0, the coefficients of valves are zero, and the coefficients of the artificial variables are independent of the flow. Therefore, $[\nabla_q \mathbf{c}_B]$ is merely affected by the coefficients of the vector $\mathbf{h}_p$. For simplicity, assume that all the components of $\mathbf{h}_p$ are included in the basis of the optimal solution of Q_0-H (later it will be shown that the computation is identical when only part are in the basis). $B^{-1}\mathbf{b}$ represents the values of the basic variables including the $\mathbf{h}_p$'s value in the basis. Using the chain rule, the relationship between $\mathbf{q}_p$ and $\mathbf{q}$ defined in Eq. (5), and in Eq. (38) which defines the vector $\mathbf{c}_B$, yields:

$$[\nabla_q \mathbf{c}_B]B^{-1}\mathbf{b} = \alpha_p k_e t[I_p - Q_p E^{-1}(S_{\eta 1} + S_m S_{\eta 2})]E^{-1}\mathbf{h}_p \tag{59}$$

in which I_p is a unit matrix of order n_p and $S_{\eta 1}$ and $S_{\eta 2}$ are the Jacobian matrices of the pumping efficiency with respect to $\mathbf{q}_p$ and $\mathbf{h}_p$, respectively. Both are square diagonal matrices of order n_p with components defined by:

$$(S_{\eta 1})_{ii} = d\boldsymbol{\eta}_p^i / d\mathbf{q}_p^i \tag{60}$$

$$(S_{\eta 2})_{ii} = d\boldsymbol{\eta}_p^i / d\mathbf{h}_p^i \tag{61}$$

$h_p^i = 0$ if it is not included in the basis. Hence Eq. (59) also covers the cases in which part of the vector $\mathbf{h}_p$ is included in the basic solution.

The gradient of f with respect to $\mathbf{q}$ is the summation of Eqs. (45), (47) and (49) which is given in the summation of Eqs. (52) and (59):

$$\begin{aligned}\nabla_q f = {} & [L_a S_a L_a^T - L_b S_b L_b^T + L_v S_v L_v^T]\pi_L + [L_a S_a P_a^T - L_b S_b P_b^T + L_v S_v P_v^T]\pi_p \\ & + L_p S_m \pi_m + \alpha_p k_e t[I_p - Q_p E^{-1}(S_{\eta 1} + S_m S_{\eta 2})]E^{-1}\mathbf{h}_p \\ & + \alpha_b t\, k_e L_b F_b \mathbf{1}_b + t\, L_a \hat{A}^T [F_s \mathbf{q}_s + \mathbf{w}_s]\end{aligned} \tag{62}$$

Equation (62) includes many matrix multiplications. However, most of the matrices are diagonal and thus the multiplication is simple.

Computation of the Projected Gradient

The computation of the projected gradient at a current point $\mathbf{q}^k$ is based upon the method of Rosen [13] using the modification described by Cohen *et al.* [4], [5]. For the current problem the projected gradient is computed by:

$$\mathbf{s}_q = -[\nabla_q f + H_a^T \lambda] \tag{63}$$

where H_a is the coefficient matrix whose rows are the active constraints of the formulation described by Eqs. (41)–(44) at the current point. The vector λ is computed by the following steps:

Step a Compute ω from:

$$H_L\omega = -H_a^T\nabla_q f \tag{64}$$

Step b Compute λ from:

$$H_U\lambda = \omega \tag{65}$$

where H_L and H_U are the LU decomposition matrices of $H_aH_a^T$. Since H_L and H_U are triangular matrices, ω and λ can be obtained by forward and back substitution, respectively. As long as the active set does not change, no recomputation of H_L and H_U is needed, and the vector λ is computed for each new $\nabla_q f$ by Steps *a* and *b*. If a constraint is added to the active set, the matrices H_L and H_U are updated as described by Cohen *et al.* [4], [5].

At this point, it has been shown that the Q-H problem can be formulated as a optimization problem in the subspace **q** with a nonlinear objective function and linear constraints. For a given **q** the optimal operation of the pump stations and of the control valves are determined in what is known as an inner optimization by solving the Q_0-H problem. The gradient of the objective function of the Q-H problem can be computed analytically as detailed above. Eiger *et al.* [7] showed that the objective function of this outer optimization problem may include non-smooth points. In the present problem non-smooth points occur as a result of the inclusion of ϕ_p. The objective function of the outer optimization, which determines the optimal flow distribution **q**, is neither convex nor concave, with linear constraints. Therefore, even the application of a non-smooth optimization method cannot guarantee a solution.

The Complex Method was developed by Nelder and Mead [11] for unconstrained problems. Box (cited in Jacoby *et al.* [8]) extended the techniques to constrained problems with inequality constraints. The method does not use gradients, so it is applicable to non-smooth problems. The efficiency of the method is improved when the feasible region is convex, since the search process is guaranteed to remain in the feasible domain. Convexity occurs in this problem since the constraints are linear. The efficiency of the Complex Method is reduced substantially as the number of decision variables increases.

The projected gradient and the complex methods are therefore combined in the technique. The process is performed by the modified projected gradient when the projected gradient is effective (*i.e.*, it provides the descent direction). When the process approaches a non-smooth (kink) point the gradient may indicate an inappropriate descent direction, since the gradient is only one of the members of the subdifferential set. In this situation the one dimensional process terminates with a null step where the Kuhn-Tucker conditions are not satisfied. At this point the Complex Method using a search based only on function values is used to reach a better point.

Algorithm

Step 0 (Initialize): Assume an initial water flow distribution, $\mathbf{q}_a^0$, which satisfies continuity at all nodes.

Initialize the iteration counter, $k=0$. Initialize the active constraints counter, $n_a=0$.

Define initial circular discharges, $\mathbf{q}^0=0$.

Step 1 Compute the supply cost and the pumping cost at boosters from Eqs. (6) and (7).

Step 2 Solve the Q-H_0 problem for the given water flow distribution.

Step 3 Compute the gradient $\nabla_q f$ from Eq. (62).

Step 4 Compute the projected gradient.

If the active set is empty then $\mathbf{s}_q=-\nabla_q f$: goto Step 5, otherwise: compute the vector λ from:

$H_L\omega = -H_a^T\nabla_q f$

$H_u\lambda=\omega$

and the projected gradient is obtained from $\mathbf{s}_q = -[\nabla_q f + H_a^T\lambda]$, and goto Step 5.

Step 5 If $||\mathbf{s}_q|| \leq \varepsilon$ goto Step 10, otherwise compute the direction $\mathbf{d}_q=-\mathbf{s}_q/||\mathbf{s}_q||$.

Step 6 Compute optimal step length, α^*, along $\mathbf{d}_q$ from:

$$f(\mathbf{q}^k + \alpha^*\mathbf{d}_q) = \min\{f(\mathbf{q}^k + \alpha\mathbf{d}_q)|0 \leq \alpha \leq \alpha_{\max}\}$$

using a nonexact search method described by Jacoby *et al.* [8]. $\alpha_{\max}$ is the maximal step length along $\mathbf{d}_q$ which does not violate the nonactive constraints of Eqs. (41)–(44).

Step 7 Updating.
If $\alpha^* \leq \varepsilon_q$ or/and $|f^k - f^{k-1}| \leq \varepsilon_f$ then goto Step 8, otherwise: $\mathbf{q}^{k+1} = \mathbf{q}^k + \alpha^* \mathbf{d}_q$, $k := k+1$, if $\alpha^* = \alpha_{\max}$ add to the active set the constraint which becomes active, and update the matrices H_L and H_U. Go back to Step 1.

Step 8 If the Kuhn-Tucker conditions are not satisfied, omit constraint j from the active set according to: $\lambda_j = \max_i\{\lambda_i^+, -\lambda_i^-\}$ if constraint j is active at its upper bound, $\lambda_j = \min_i\{-\lambda_i^+, +\lambda_i^i\}$ if constraint j is active at its lower bound, in which λ_i^+, λ_i^- is the component vector of λ which is associated with constraint i with respect to its upper bound and lower bound, respectively. Go to Step 1.
If the Kuhn-Tucker conditions are satisfied go to Step 9.

Step 9 Complex search.
Use the Complex Method. If an improved point is found, go back to Step 0 with the improved point as the initial flow distribution, otherwise, go to Step 10.

Step 10 Compute the optimal operation of the valves using the OPV algorithm and the optimal operation of the pump station using the OPP algorithm.

EXAMPLE

The network to which the approach is applied is located in Arava Valley in Southern Israel and is shown schematically in Figure 3. The network consists of 9 pipes, 9 nodes, 2 pump stations, one booster, and 2 control valves, and is fed from two constant head reservoirs at nodes 8 and 9. It delivers to consumers at nodes 4, 5, 6 and 7 and is located in a region 200 to 250 m below sea level. The network is operated for 2000 hours and the cost energy is 0.22 NIS/kWh (1 US$ ≅ 3.6 NIS (New Israeli Shekels)). There are two loops: one closed loop of pipes 3, 7, 8 and 9, and one pseudo-loop between the nodes 8 and 9 which includes pipes 1, 2 and 3. The initial flows, the positive directions the pipe flows and of the circular flows are shown in Figure 3.

The data for the sources, consumption, and pipes are given in Tables I, II and III, respectively. All pipes have Chw = 120.

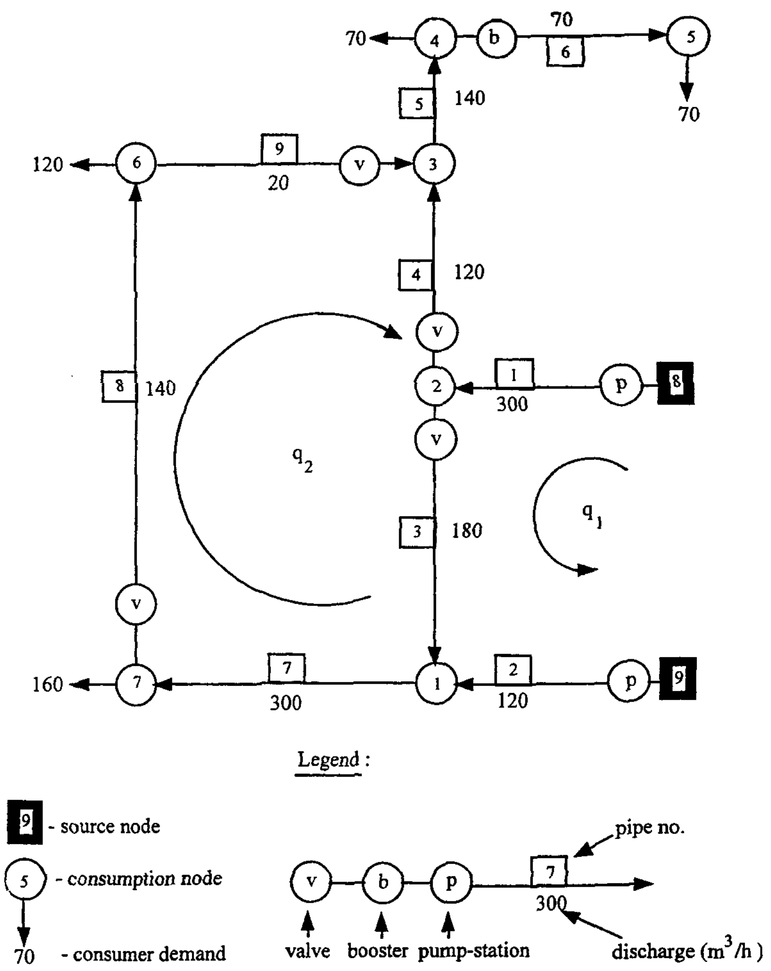

FIGURE 3 Schematic description of the network.

TABLE I Data for sources

Node	*Specific cost*	*Elevation*	*Maximum discharge*
–	NIS/m^3	m	m^3/h
8	0.638	−252.5	325
9	0.256	−255	700

TABLE II Data for consumption nodes

Node	*Consumption*	*Elevation**	*Minimum pressure head*	*Maximum pressure head*
–	m^3/h	m	m	m
4	70	−258	40	80
5	70	−217	40	80
6	120	−242	35	80
7	160	−246	40	80

* Elevations are negative because the system is in the Jordan Valley, below sea level.

TABLE III Data for pipes

Pipe	1	2	3	4	5	6	7	8	9
length (m)	400	1300	3700	1000	4300	2600	800	5000	3500
diameter (mm)	250	250	300	250	300	250	250	250	250
initial flow (m^3/h)	300	120	180	120	140	70	300	140	20

The booster on pipe 6, has the following head-discharge function:

$$h_b = 34.738 - 0.02426q_b + 2.237 * 10^{-4}q_b^2 - 1.603 * 10^{-6}q_b^3$$

and power-discharge function:

$$N_b = 16.29 + 0.011q_b$$

in which q_b is the discharge in m^3/h, h_b is the head in m, and N_b is the power in kW. The two control valves on pipes 3 and 8 are identical with a head loss-discharge – opening ratio function of:

$$h_v = 10^{-6}q_v^2m_v^{-1.5}$$

in which h_v is the head loss in m, q_v is the discharge in m^3/h and m_v is the opening ratio. The pump station on pipe 1 (pump station A) consists of 3 pumps of type a and one pump of type b. The pump station on pipe 2 (pump station B) consists of 2 pumps of type c and one pump of type d. The head-discharge and efficiency-discharge functions for these pumps are given in Figures 4 and 5. Only parallel connections between the pumps are possible.

The optimal solution is $f^* = 355467.12$ NIS, and the optimal circular flows are $q_1^* = -190.39$, $q_2^* = -79.83\,m^3/h$ (these are the changes

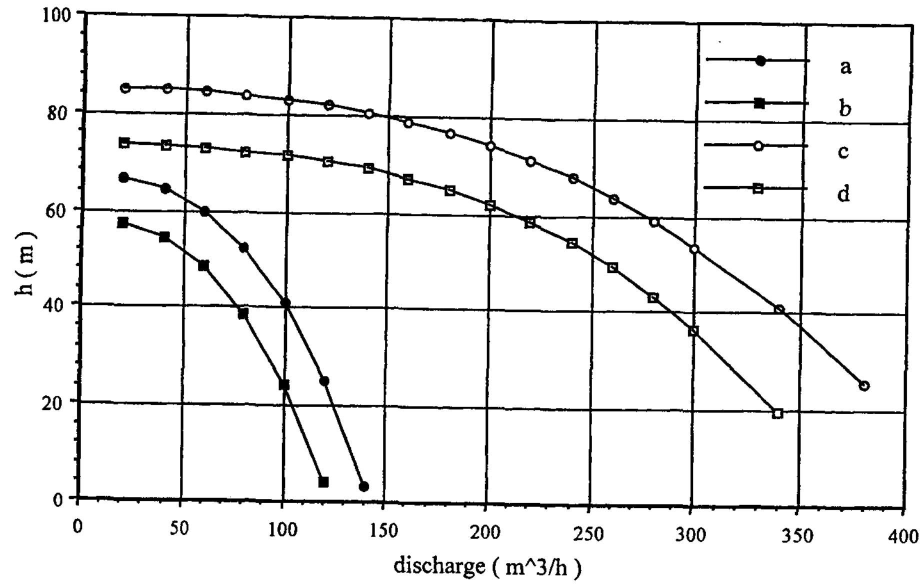

FIGURE 4 Head-discharge functions for the pumps in the pump stations.

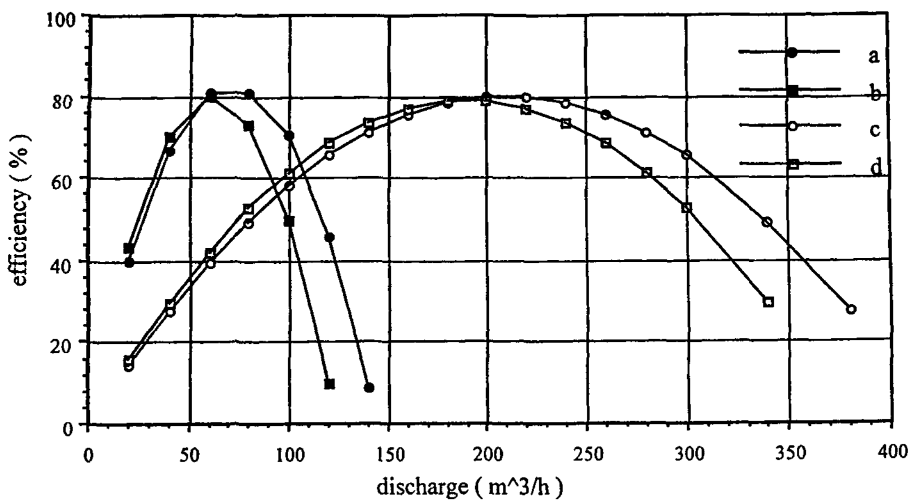

FIGURE 5 Efficiency-discharge functions for the pumps in the pump stations.

relative to the initial flow distribution in Fig. 3). The optimal flow, head and power at the booster are 70 m^3/h, 33.59 m and 17 kW, respectively. The optimal opening ratio of the valves placed on pipes 3 and 8 are 100% and 30.1%, respectively.

The optimal operating conditions of the pump station and the pressure heads at the consumer nodes are summarized in Tables IV and V, respectively.

The solution was obtained after 17 iterations in 2.17 sec on the IBM-3081computer.

The values of the objective function during the solution process are shown in Figure 6.

The values for iterations 0 and 1 are omitted, because their large values are due to penalties. The solution at Iteration 2 is already feasible with an objective value of 414000 NIS. The optimal solution is only 14% better than this first feasible solution. In fact it can be seen in Figure 6 that the process converges rapidly to the optimal region such that from the fifth iteration on improvements in the value of the solution are negligible. The process does not terminate earlier than iteration 17 due to a very strict convergence criterion.

The method was also applied to the water distribution system of Central Arava Region, which supplies an area in the southern region of Israel. This system has 38 nodes, 39 pipes, 11 sources (wells), 9 pumping stations consist of 28 pumps, 14 control valves, 14

TABLE IV Optimal operation of pump stations

pump station	*configuration*	*head (m)*	*flow (m^3/h)*	*head loss* (m)*	*by-pass (m^3/h)*
A	a//a	56.68	109.61	5.69	0.390
B	c//d	79.26	310.39	0.88	2.003

* The head loss is due the control valve.

TABLE V Pressure heads at the consumer nodes

Node	*Elevation (m)*	*Demand (m^3/h)*	*Pressure head (m)*		
			min.	*max.*	*opt.*
4	−258	70	40	80	49.68
5	−217	70	40	80	40.0
6	−242	120	35	80	36.98
7	−246	160	40	80	46.48

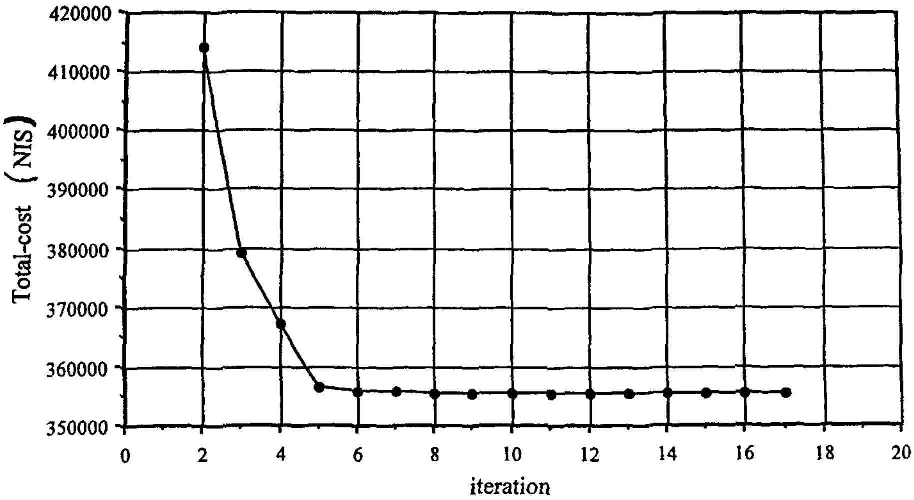

FIGURE 6 The values of the objective function during the solution process.

aggregated consumers each with a prescribed range of pressure. Four nodes are supply to domestic consumers and the others to agriculture. The solution for this larger system was obtained after 19 iterations in 23.61 sec on an IBM-3081computer.

SUMMARY

This paper presents a new approach for the optimal operation of water distribution systems under hydraulic constraints. The model is useful in itself, and also as a component in the full Q-C-H model, which is the topic of a companion paper [6].

This new approach has the following characteristics:

- The "performance surface" (Q-H) of each pumping station is represented by a smoothed two dimensional function.
- The objective is minimization of the total operating cost, which includes the cost of water from the sources and the cost of pumping.
- The demands at nodes are to be met with pressures which lie in a prescribed range.
- The problem decomposed into an inner–outer structure where the inner problem is solved by SLP and the outer is solved by the projected gradient combined with the Complex method.

An example network was used to demonstrate the technique. The optimal solution was achieved practically in 5 iterations and formally in 17 iterations, when the operating cost is reduced during the iterations from 414000 to 355476 NIS.

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