Optimal Operation of Water Distribution Systems

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Abstract: Optimal operation of a water supply system is solved by progressive optimality (PO), an iterative dynamic programming (DP) method. Given the forecasted demands for the coming 24 hr, the initial and final conditions in the reservoirs, the hydraulic properties of all system components, and the variable energy cost over the day, an optimal schedule of pump operation is found. The algorithm cycles iteratively over the time steps (hours of the day) and network subsystems, and converges to the optimum from any (feasible or infeasible) initial solution. The global optimum is guaranteed only under certain conditions; otherwise a local optimum may be reached. The method is developed and demonstrated on a regional water supply system with eight reservoirs and seven pumping stations.

Introduction

A survey conducted in 1984 (Computer assisted design of water systems committee 1984; Velon et al. 1984) among water utilities in the United States showed that many are considering installation of computer control systems. The motivation is due to one or more of the following factors (Shamir 1985).

1. Operation of water supply systems is in many cases becoming more complex, with rising demands, incorporation of waters from a variety of sources, and aging systems.
2. Retirement of experienced personnel, often not replaced by people of similar capabilities.
3. High operating costs, which justify investments to improve efficiency.
4. Control and computer hardware is rapidly becoming cheaper, more available, and more reliable.
5. As more computer control systems are installed, there is more experience from which to learn.
6. Operators, engineers, and managers feel less threatened by computers and control systems.

To use control systems effectively, one must use software for analysis, simulation, and optimal operation of the water supply network. Programs for analysis and simulation are readily available and used quite widely (Computer-assisted design of water systems committee 1984; Velon et al. 1984). Methods and computer programs for optimal control are less common.

A survey paper in 1985 (Shamir 1985) summarized the state of the art of optimal control of water systems to that time. We know of no new developments since then. The work reported herein was already mentioned and explained without technical detail in that survey. This paper awaited more

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practical experience and refinements of the method.

The optimal control problem is to find the decisions for operating pumps and valves over a prescribed time period (here 24 hr) such that demands are met, pressure conditions are satisfied, and total cost is minimized. This will be formulated precisely and discussed in later sections of this paper.

Dynamic programming (DP) is an obvious solution procedure for such problems, but its practicality is limited by the size (number of reservoirs, whose volumes are the state variables in the DP) of the network. Dreizin et al. (1971) developed and used such models for off-line analysis of the operation in the late 1960s. This has been a standard approach in Israel since then.

Sterling and Coulbeck (1985a, 1985b) tried to extend the applicability of the basic DP approach that was used by Coulbeck in his thesis (1977) to larger systems. When this proved impractical, they also tried a method (Coulbeck 1980; Coulbeck and Sterling 1978) based on linearization of the system functions and simultaneous solution of the optimization problem for all time periods.

Cohen and his coworkers (Carpentier and Cohen 1984; Cohen 1982; Joalland and Cohen 1980) developed a procedure of decomposition-coordination for coping with the "curse of dimensionality" of the DP method. The system is divided into subsystems, each small enough to be solved by DP. At the "cuts" between adjacent subsystems, the vectors of 24-hr values of flows and heads are used for the coordination process. One subsystem sends its neighbor the vector of flows and "prices of heads," the other subsystem uses these vectors to optimize its own operation and returns to the first subsystem vectors of heads and "prices of flows." The first then optimizes its own operation, and so on. The "prices" are duals (shadow prices) of the prescribed boundary values given by the neighboring subsystem. The procedure converges to the global optimum (under certain conditions on the functions) when optimal values do not change from one iteration to the next. The software developed by Cohen and his coworkers is being tested at the water supply system of Le Pecq, about 40 km west of Paris, France.

Fallside and his coworkers (Fallside and Perry 1975; Gray 1978; Marlow and Fallside 1980; Moss 1979; Perry 1975) developed an approach (not based on DP) and it has been applied for the East Worcestershire Water Co. (EWWC) in England since the mid-1970s. The system is divided into subsystems, and a linear continuity equation is written for each. The transfers between subsystems must be assumed in advance, and this is the major deficiency of this approach.

The work reported herein started with the thesis of Zessler (1984) in 1984. The method of progressive optimality (PO) proposed by Howson and Sancho (1975), which is an iterative DP, formed the basis of the procedure. Turgeon (1981) used the same approach for optimal operation of a multi-reservoir hydropower system. PO proceeds by moving along the stage axis (time, in our case), optimizing the decision for two adjacent stages (two time steps in our case) with the remainder of the state trajectory (reservoir volumes) fixed. The overall procedure is guaranteed to converge only if the objective function components are convex in the decisions. In the problem we solve, this condition is not always satisfied. When it is not, the solution method may end at a local optimum. Our approach must thus be considered heuristic for those cases where the objective function is not convex.

Our algorithm (Zessler 1984) was first implemented on a very small home computer (a Sinclair Spectrum), for the same water system used here as the example. Subsequently the program was generalized and rewritten in Pascal on an IBM PC/XT. This paper has been written after we gained considerable experience with the program on a few water systems.

Water Supply System and its Model

Consider the water supply system in Fig. 1. Shown is a schematic model of the Ein Ziv system, which serves an area of about 180 km^2. It supplies annually about 5.5×10^6 m^3 of water, of which 68% is for agriculture and 32% for domestic use. Energy consumption is about 13×10^6 kWh/yr, which averages 2.4 kWh per m^3 supplied. This high energy consumption is due to the very pronounced topography in the northern part of Israel. The main sources are at elevations around +95 m, while the extremities of the system reach elevations of close to +900 m.

Fig. 2 shows in detail one part of the system, that which contains reservoirs V5 (Shomera) and V7 (Shtula) of Fig. 1. The rather complex piping system in this area has been simplified in the model of Fig. 1 to a single line between the Ein Ziv pumping station (P5) and the first reservoir (V5), and similarly from there on to the final reservoir on this line (V7). Consumers have also been aggregated. Each pressure zone—a part of the system between an intake and a discharge reservoir—now has a single pumping station, and a single aggregate consumer located close to the discharge reservoir. Some pressure zones of this system have an even more complex structure than that shown in Fig. 2.

The model of Fig. 1 now contains 7 pumping stations. In each there are several pumps that can be operated in a number of configurations. A configuration is a certain set of pumps operating in series and/or parallel. Each configuration produces a discharge and incurs an energy cost. The discharge

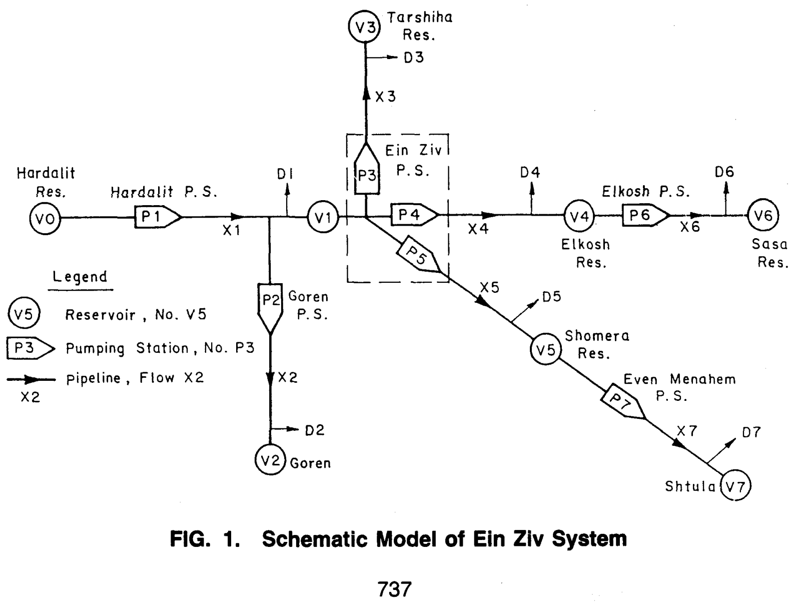

FIG. 1. Schematic Model of Ein Ziv System

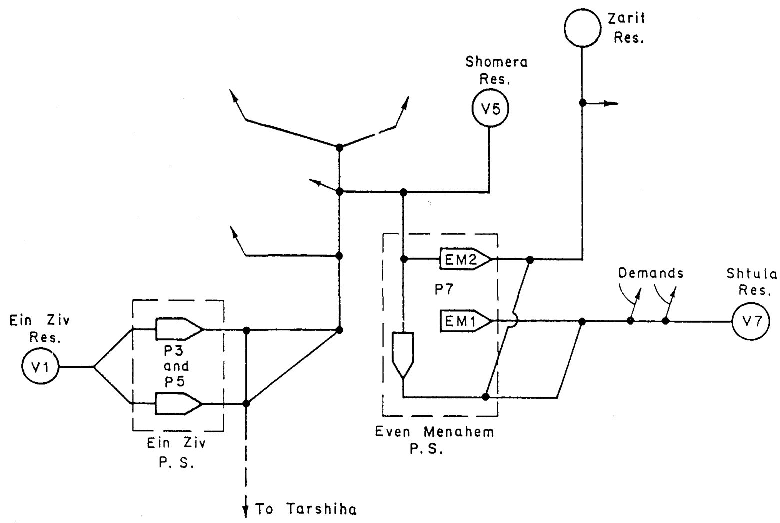

FIG. 2. Shomera and Shtula Sub-Systems

and energy consumption of a configuration depend on the water levels in the two reservoirs between which it operates, and on the discharges drawn by the consumers within its pressure zone.

A network simulator was used to investigate the behavior of the system. The simulator is a program that computes the performance of the system over a specified period of time, usually 24 hr. It receives as data the configuration of the network, physical characteristics of all its elements, the demand pattern over time at each consumer node, and the control routine according to which the system is operated (automatically or manually, as the case may be). The program computes a sequence of solutions, and a new one whenever a demand or a control changes, or when the change in any reservoir level is greater than a prespecified value (to maintain accuracy of the hydraulic solutions).

The level of detail in the network simulator was that of Fig. 2; i.e., a fully detailed representation of the system. Based on this investigation, the schematic model shown in Fig. 1 was developed and the properties of its elements were computed. The full simulation model was used to generate the cost-versus-discharge functions of each pumping station, which are needed for the optimization model to be described later.

Comparing Fig. 2 to pressure zones 5 and 7 in Fig. 1, we observe the following. Four consumers between the Ein Ziv pumping station and the Shomera reservoir have been aggregated into the single consumer D5. Similarly, those between the Even Menachem pumping station and the Shtula reservoir together with pumping to the Zarit reservoir appear as D7 in the model. This means that the Zarit pressure zone is not included explicitly in the model, and its withdrawals from pressure zone 7 are included in D7. This is done to simplify the model, and is justified because, for practical reasons, operation of pump EM2 and the Zarit reservoir are fixed in advance and need not be considered as decision variables in the optimization. Had this not been the case, we would have to include this pressure zone explicitly in the model of Fig. 1.

Returning to Fig. 1, which is the model of the water supply system to be used in the optimization, the following are known.

1. Lengths, diameters, and friction coefficients of the pipes (which have been obtained by matching the performance of the schematic model with that of the full system).
2. Properties of the pumps in each pumping station: head-versus-discharge and efficiency-versus-discharge curves for each pump, and their possible series and/or parallel arrangements into configurations.
3. Geometry of the reservoirs, and their minimum and maximum admissible water levels. These limits may vary over the day.
4. Demand pattern of each (aggregate) consumer over the day.
5. Elevations of all points in the system.
6. Minimum and/or maximum admissible pressures at certain points (for example, at demand points).
7. Variable energy costs over the day.

The time horizon is divided into periods. We usually consider 1 day, divided into 1-hr periods. A longer time period can be handled, and the periods need not be of equal length. The time horizon is selected so that initial and final boundary conditions can be conveniently set. The time periods must be small enough to allow the computed operating policy to follow changes in demands, reservoir levels, and energy tariff. Henceforth we shall deal with operation over 1 day, divided into 24 1-hr intervals. This is done for clarity of the presentation, and we keep in mind that the period could be longer or shorter, and divided into unequal intervals.

Water supply systems of the type considered here often have a 24-hr operating cycle. It can be assumed with a reasonable degree of assurance that all reservoirs will be full at the end of the low-energy-cost period, usually the night, and will again return to the same condition at the same time the following day. This creates a logical 24-hr period for the analysis.

The optimization algorithm uses the following boundary conditions.

1. Known water volumes in all reservoirs at the beginning of the time period. In on-line control systems these are measured in the field and relayed to the control center.
2. Assumed water volumes in all reservoirs at the end of the time period. For example: equal to the initial levels at the beginning of 24 hr; or, full at the end of the next low-energy-cost period, which is then taken as the end of the planning period.

An alternative to 2 would be a monetary value given to the volume in each reservoir at the end of the planning period. This "salvage value" would then allow analysis over a period of noncyclic operation.

Before moving on to the optimization algorithm, it is worth noting that some real systems have the simple structure shown in Fig. 1 and there is then no need to schematize as was done here. This is particularly true for regional supply systems, whose consumers are whole communities.

Optimal Operation Problem

For a system with J pumping stations, operating over a time horizon divided into T intervals, the optimal operation is obtained by solving the following cost minimization problem:

$$\min F = \sum_{t=1}^{T} \sum_{j=1}^{J} \{G_j[X_j(t), V(t), \mathbf{D}(t)]\} \quad (1)$$

subject to

$$0 \leq X_j(t) \leq \bar{X}_j(t) \qquad \forall j,t \quad (2)$$

$$\underline{V}_i(t) \leq V_i(t) \leq \bar{V}_i(t) \qquad \forall i,t \quad (3)$$

$$V_i(t) = f[V_i(t-1); X(t); \mathbf{D}(t)] \qquad \forall i \quad (4)$$

$V(1)$ known initial volumes .... (5)

$V(T + 1)$ assumed known final volumes .... (6)

where $j = 1, \ldots, J$ = index of the pumping stations; $i = 1, \ldots, I$ = index of the reservoirs; $t = 1, \ldots, T$ = index of the time intervals, taken here to be 1 hr each ($t = 1$ is the initial condition, and $t = T$ the final time); $X_j(t)$ = discharge; $V_i(t)$ = volume, at beginning of interval t; $\mathbf{D}(t)$ = vector of demands; G_j = cost in pumping station j, for delivery X_j, which also depends on demands and reservoir volumes; and $\_$, $^-$ = minimum and maximum values, respectively. Eq. 2 sets the limits on the discharge, and Eq. 3 on reservoir volumes. Eq. 4 is a set of continuity equations for the change in reservoir volumes over time.

For low dimensionality of V—say up to three or possibly even four reservoirs—it should be possible to solve the problem by standard discrete dynamic programming. For higher dimensionality this is no longer feasible. An iterative scheme of successive approximations was therefore developed.

Preparing for Optimization

The steps in setting up a model of the system and preparing the data for the optimization are the following:

1. Identify subsystems, each being a portion of the system that operates between two reservoirs, throughout which the presures are governed primarily by the operation of one pumping station and the level in the discharge reservoir.
2. A hydraulic simulator is used to study each subsystem: flows, pressures, and changes in reservoir volumes for each possible pump configuration. This analysis is carried out on a detailed model of the system, as shown, for example, in Fig. 2. The hydraulic analysis is used to construct a schematic model of each subsystem—a pumping station located between two reservoirs, and a single aggregate consumer—and to determine the most economical order of bringing pumps into operation, as will be explained later.
3. Forecasting the demands of the aggregate consumers for the 24 hr ahead. These include transfers of water to secondary pressure zones, whose operation is considered fixed in advance.
4. For each hour, with the forecasted demands, the network simulator is used

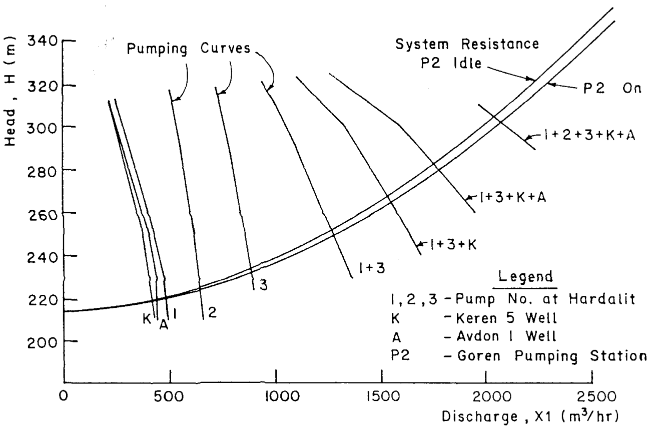

FIG. 3. Hydraulic Analysis of Hardalit Pumping Station (P1)

to construct the energy-cost-versus-discharge function for each pumping station. This function is convex, for any demand level and reservoir volume, because the pumps are introduced in the order of decreasing efficiency. A quadratic is fitted

$$G_j(t) = A_j(t)X_j^2(t) + B_j(t)X_j(t) + C_j(t) \quad \text{(7)}$$

where $A_j(t)$, $B_j(t)$, and $C_j(t)$ = coefficients obtained from fitting the data computed by the simulator. If energy-versus-discharge figures are available from actual measurements, these can obviously be used directly to construct the curves.

Fig. 3 shows the results of the hydraulic investigation of the pressure zone between VO and V1 of Fig. 1. The system resistance increases with flow, due to friction losses. Two curves are shown, corresponding to the cases when P2 is idle or operating. The intersecting pumping curves are for the various pump

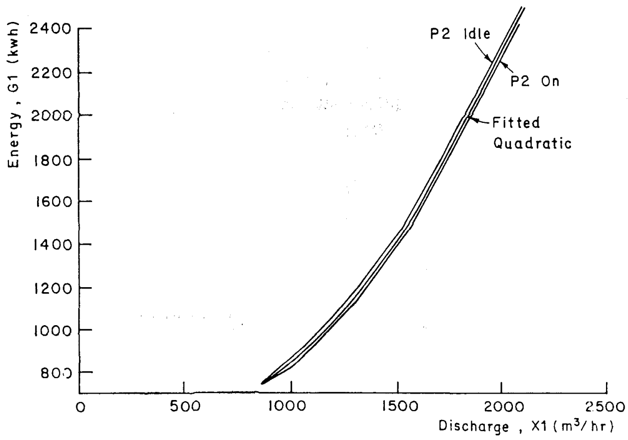

FIG. 4. Cost versus Discharge for Hardalit Pumping Station (P1)

configurations in this pressure zone: three parallel pumps at P1 and two wells that pump into the pipeline between P1 and V1, and have been incorporated into P1 in Fig. 1. The order in which the pump configurations are brought in [K,A,1,2,3, (1 + 3),...(1 + 2 + 3 + K + A) in Fig. 3] corresponds to decreasing efficiency, established from field data and/or from simulation.

Energy-cost-versus-pumping discharge is obtained for data for the intersection points of pumping curves and system resistance: head discharge, pump efficiencies, and (time-dependent) energy cost. The two resulting curves, for P2 operating and idle, and the fitted quadratic, are shown in Fig. 4. The quadratic

$$G1\ (\text{kWh}) = 0.0002857X1^2 + 0.3943X1 + 205.7 \quad (8)$$

with $X1$ in m^3/h is seen to fit the data quite well.

5. For each reservoir, the following are given: the range of admissible volumes (Eq. 3), which can vary over the day or be the same for all times, the continuity equation (Eq. 4), which is a simple mass balance, and the initial and final volumes (Eqs. 5 and 6).

This completes the preparation of data for the optimization.

Optimization of Operation

The optimal solution we seek is a set of discharges, $X_j(t)$, for all pumping stations ($j = 1,\ldots,J$) and times ($t = 1,\ldots,T$). The demands $\mathbf{D}(t)$ are known, and therefore the reservoir volumes $V(t)$ are uniquely determined by the discharges $X(t)$. The volumes are the state variables of the DP optimization model, whose solution may be given by either the values of the pumping station discharges or equivalently by the trajectories of reservoir volumes over time. We use the trajectories of reservoir volumes as a main tool in the optimization.

The optimization is cast as a dynamic programming model. For the example of Fig. 1, this model has seven state variables—too many to solve by conventional DP. We resort to the method of progressive optimality (Howson and Sancho 1975), combined with a spatial decomposition and "local" optimization by an analytic technique. The objective function may be nonconvex for certain values of the coefficients in its components (Eq. 7). In such cases, a global optimum cannot be guaranteed, and the method is considered heuristic.

The system is divided into subsystems; each has two pressure zones in series. For the model of Fig. 1 there are seven subsystems:

S1: VO—P1—V1—P2—V2 .... (9)

S2: VO—P1—V1—P3—V3 .... (10)

S4: VO—P1—V1—P4—V4 .... (11)

S5: VO—P1—V1—P5—V5 .... (12)

S6: V1—P4—V4—P6—V6 .... (13)

S7: V1—P5—V5—p7—V7 .... (14)

A subsystem may also be made of two pressure zones operating in parallel—from two separate intake reservoirs, through two pumping stations, into a

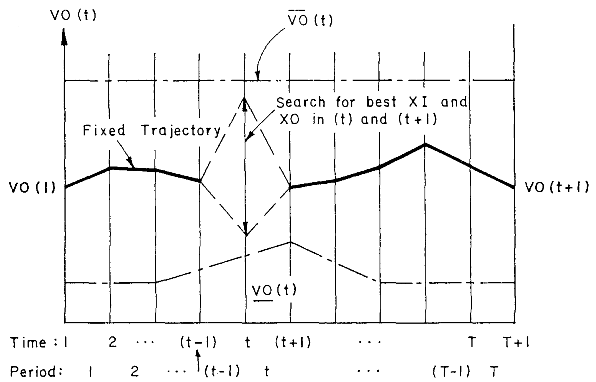

FIG. 5. One Step in Progressive Optimality Procedure: Change in Reservoir VO at Time *t*

common discharge reservoir. The distribution system we use for demonstration here does not have such a configuration, but others may. For clarity of presentation we shall explain the method for the series arrangement only.

Assume that a feasible trajectory of reservoir volumes is given: the volumes satisfy Eqs. 3, 5, and 6, and the corresponding pumping discharges satisfy Eq. 2. How an initial feasible trajectory can be obtained will be explained later. The optimization method adjusts the trajectory iteratively.

The iterative procedure contains two nested loops.

1. Over time, from (t = 2) to (t = T).
2. For each time, over the subsystems.

In iterating over time, the trajectories are kept fixed, except for a pair of adjacent time steps, as shown for one reservoir, in Fig. 5. For this pair of time periods, the inner iterative loop ranges over all subsystems, in order. When dealing with one subsystem for time t, the only thing which is allowed to vary is the volume of its reservoirs at this time. There are four decision variables associated with allowing this volume to change: the discharges of the two pumping stations in the subsystem, one on either side of the central reservoir in which the volume is allowed to change, for the two adjacent time periods (t − 1) and (t).

A "local" optimization problem is solved for each subsystem for the two subsequent time steps. Fig. 6 shows a subsystem and the indices used: C for the central reservoir, I for the pressure zone that provides its inflow, and O for the pressure zone on its outflow side. There are four decision variables—two flows at two time periods—XI (t − 1), XI (t), XO (t − 1), and XO (t).

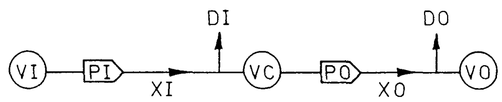

FIG. 6. Notation for Local Optimization

The objective is to minimize the total cost of delivering these four flows, keeping the volumes VI and VO at their fixed values at times $(t-1)$ and $(t+1)$, and three volumes VI, VC and VO within their ranges at time t.

The local optimization problem is

$$\min \{f = GI[XI(t-1)] + GI[XI(t)] + GO[XO(t-1)] + GO[XO(t)]\} \quad (15)$$

subject to

$$VI(t+1) = VI(t-1) - XI(t-1) - XI(t) \quad (16)$$

$$\underline{VI}(t) \leq VI(t-1) - XI(t-1) \leq \overline{VI}(t) \quad (17)$$

$$VO(t+1) = VO(t-1) + XO(t-1) - \mathbf{D}O(t-1) + XO(t) - \mathbf{D}O(t) \quad (18)$$

$$\underline{VO}(t) \leq VO(t-1) + XO(t-1) - \mathbf{D}O(t-1) \leq \overline{VO}(t) \quad (19)$$

$$\underline{VC}(t) \leq VC(t-1) + XI(t-1) - \mathbf{D}I(t-1) - XO(t-1) \leq \overline{VC}(t) \quad (20)$$

$$O \leq XK(n) \leq \overline{XK} \qquad K = I, O;\ n = (t-1), t \quad (21)$$

$VI(t-1)$, $VI(t+1)$, $VO(t-1)$, $VO(t+1)$ are known, fixed at the value they were given last.

Constraints in Eqs. 16 and 18 make VI and VO return to their known values at $(t+1)$. Eqs. 17, 19, and 20 restrict VI, VO, and VC to their admissible range at (t). Eq. 21 limits the flows to their feasible range (Eq. 2).

In this presentation it was assumed, for clarity of presentation, that there are no further inputs or outputs to reservoirs VI and VO. Had there been, they would have been introduced as given values in the appropriate constraints presented. Also, when VC has more than one output (as does, for example, V1 in Fig. 1) all flows not in the subsystem being now considered are not changed are therefore known and can be introduced into the aforementioned appropriate constraints.

All these constraints can be combined, to result in bounds on two out of the four variables, namely $XI(t-1)$ and $XO(t-1)$

$$LI(t-1) \leq XI(t-1) \leq UI(t-1) \quad (22)$$

and

$$LO(t-1) \leq XO(t-1) \leq UO(t-1) \quad (23)$$

where, for example

$$LI(t-1) = \max \{A1, A2, A3, A4\} \quad (24)$$

with

$$A1 = VI(t-1) - VI(t+1) - \overline{XI} \quad (25)$$

$$A2 = VI(t-1) - \overline{VI}(t) \quad (26)$$

$$A3 = \underline{VC}(t) - VC(t-1) + DI(t-1) + XO \quad (27)$$

$$A4 = 0 \quad (28)$$

These values come from Eqs. 16, 17, 20, and 21, with $\overline{XI}$ and $\underline{XO}$ introduced to establish the widest possible ranges for $XI(t-1)$. Similarly

$$UI(t-1) = \min\{B1, B2, B3, B4\} \quad (29)$$

with

$$B1 = VI(t-1) - VI(t+1) - \underline{XI} \quad (30)$$

$$B2 = VI(t-1) - \underline{VI}(t) \quad (31)$$

$$B3 = \overline{VC}(t) - VC(t-1) + \mathbf{D}I(t-1) + \overline{XO} \quad (32)$$

$$B4 = \overline{XI} \quad (33)$$

From Eqs. 16 and 18, $XI(t)$ and $XO(t)$ can be expressed as functions of $XI(t-1)$ and $XO(t-1)$, respectively

$$XI(t) = VI(t-1) - VI(t+1) - XI(t-1) \quad (34)$$

and

$$XO(t) = VO(t+1) - VO(t-1) + \mathbf{D}O(t-1) + \mathbf{D}O(t) - XO(t-1) \quad (35)$$

We are now left with an optimization problem in two variables: $XI(t-1)$ and $XO(t-1)$. The objective is an analytic function: Eq. 15, with the quadratic functions (Eq. 7) used for the G's. $XI(t)$ and $XO(t)$ are given by Eqs. 34 and 35, respectively. There are only upper and lower bounds on both decision variables, Eqs. 22 and 23, and no other constraints.

Solution of the local problem is achieved as follows. Substitute $XI(t)$ and $XO(t)$ from Eqs. 34 and 35 into Eq. 15, using the quadratic expressions (Eq. 7) for the G's, multiplied by the energy cost coefficients for each time period. We shall continue to use the notation AO, BO, CO, AI, BI, CI for the coefficients with the understanding that they are original values in Eq. 15 multiplied by the appropriate energy cost. Eq. 15 is differentiated with respect to the two decision variables, $XI(t-1)$ and $XO(t-1)$. The derivatives are set equal to zero and solved. The results are

$$XI(t-1) = \frac{2AI(t)\cdot S1 - BI(t-1) + BI(t)}{2}\,[AI(t-1) + AI(t)] \quad (36)$$

$$XO(t-1) = \frac{2AO(t)\cdot S2 - BO(t-1) + BO(t)}{2}\,[AO(t-1) + AO(t)] \quad (37)$$

where AI, BI, AO and BO = the coefficients in Eq. 7 for pumping stations PI and PO, respectively. $S1$ is calculated from Eq. 35

$$S1 = VI(t+1) - VI(t-1) \quad (38)$$

and $S2$ from Eq. 34

$$S2 = VO(t-1) - VO(t+1) + \mathbf{D}O(t) + \mathbf{D}O(t-1) \quad (39)$$

When the cost-versus-discharge curves of Eq. 15 are the same in PI and PO for (t) and $(t+1)$, Eqs. 36 and 37 reduce to

$$XI(t-1) = \frac{S1}{2} \quad (40)$$

and

$$XO(t-1) = \frac{S2}{2} \quad (41)$$

The values of $XI(t - 1)$ and $XO(t - 1)$ from Eqs. 36 and 37, are checked against the bounds of Eqs. 22 and 23. If all are satisfied, this is the optimum of the "local" problem. If one or both of the variables violate one of their bounds, they are set to the bound.

Once $XI(t - 1)$ and $XO(t - 1)$ have been determined, $XI(t)$ and $XO(t)$ are computed from Eqs. 34 and 35. They are guaranteed to be within their bounds given by Eq. 2, because these were incorporated into setting the bounds of Eqs. 22 and 23. The local optimization problem can also be solved by a simple search over all (discretized) possible values of $XI(t - 1)$ and $XO(t - 1)$. There will only be a small number of configurations in PI and PO that satisfy the constraints of Eqs. 22 and 23, so the search is easy.

Iterative Process

The steps are the following.

1. Given an initial trajectory of feasible reservoir volumes (Eq. 3).
2. Go over the time period, covering two sequential time steps at each iteration, with a one-step overlap over the previous iteration; i.e., $t = 1$ and 2, 2 and 3, 3 and 4, etc.
3. At each time, go over each subsystem sequentially, holding the reservoirs at its two ends fixed at $(t - 1)$ and $(t + 1)$. Use the "local" optimization to determine the four flows in and out of the reservoir for the two time steps. The flows for time (t) will again be changed the next time the same subsystem is reached, because of the overlap in the iterations over time.
4. The subsystems also have overlap: the central reservoir of one is the intake reservoir for the next subsystem downstream. As the levels are changed in the central reservoir of the first, they become fixed boundary conditions when the next one is treated.
5. The iterations over time are repeated, say from $t = 1$ to $t = T - 1$, until all reservoir volumes, over all times, do not change between iterations by more than some small specified tolerance. The iterative process is guaranteed to converge because the components of the objective function are convex (Howson and Sancho 1975).

Finding Initial Feasible Trajectory

The same procedure is used to find an initial feasible trajectory; i.e. a set of reservoir volumes and discharges that satisfies the constraints of Eqs. 2–6. This is done as follows.

1. Specify any feasible set of reservoir volumes according to Eqs. 3, 5, and 6.
2. Compute the corresponding discharges of all pumping stations for all times. They may not satisfy Eq. 2, and some may even be negative.
3. During the iterative process, impose the bounds of Eq. 2 on every flow which is within its feasible range, so that progressively, over the iterations, all flows end up feasible.

We have found this method to be very effective. If, however, the hydraulic system is such that infeasibilities could arise during such a process, we would have allowed violation of constraints and added a penalty term to the objective function (Eq. 15) of each local optimization where a flow is not feasible.

EXAMPLE

The system shown in Fig. 1 was optimized, for a 24-hr period, with hourly time steps starting at 12 noon and ending the following day at the same time. For simplicity we shall show results for the case where the energy-versus-discharge functions remain constant over time. In reality they are affected by the demand level. There are three energy tariff periods in the day.

TABLE 1. Energy (kWh) versus Discharge (m^3/h) Function Coefficients and Maximum Discharge for Pumping Stations

Pumping station (1)	Energy versus Discharge Coefficients A (2)	B (3)	C (4)	Maximum discharge (m^3/h) (5)
P1	10.4406×10^{-5}	0.512	12.5	2,500
P2	1×10^{-6}	0.125	86	300
P3	1×10^{-6}	1.08	0	950
P4	4.679×10^{-5}	0.86	0	700
P5	1×10^{-6}	0.378	−132	700
P6	1.707×10^{-5}	0.148	0	300
P7	1×10^{-6}	0.125	−40	200

TABLE 2. Relative Energy Cost for Three Time Periods During Day

Period Name (1)	From hour (2)	To hour (3)	Relative energy cost (4)
E1	08	15	2.0
E2	16	22	1.5
E3	23	07	1.0

TABLE 3. Minimum, Maximum, and Initial (Final Equal to Initial) Volumes of Reservoirs

Reservoir (1)	Minimum volume V (m^3) (2)	Maximum volume $\bar{V}$ (m^3) (3)	Initial and final volumes V [Marlow and Fallside (1980)] (4)
V1	100	1,200	600
V2	100	1,000	500
V3	100	6,000	3,000
V4	100	3,000	1,500
V5	100	1,000	500
V6	100	1,000	500
V7	100	1,000	500

TABLE 4. Demand Data

Consumer (1)	Demand (m^3/h) for Period: t = 1–12 (2)	Demand (m^3/h) for Period: t = 13–24 (3)
D1	300	400
D2	100	100
D3	400	490
D4	150	250
D5	200	300
D6	100	150
D7	100	100

Table 1 contains the coefficients of the energy (kWh)-versus-discharge (m^3/hr) function (Eq. 15) for the seven pumping stations and the maximum discharge. As explained previously, these curves were generated by use of a detailed simulator, introducing pump configurations in descending order of efficiency. Table 2 contains the energy cost variation over the day, given in relative terms. The absolute values have no effect on the optimal operating policy, only the relative values. Reservoir data are given in Table 3: mini-

TABLE 5. Reservoir Volumes (m^3)—Optimal Solution

T (1)	V1 (2)	V2 (3)	V3 (4)	V4 (5)	V5 (6)	V6 (7)	V7 (8)
12	600	500	3,000	1,500	500	500	500
13	475	400	2,510	1,350	400	400	440
14	350	300	2,020	1,200	300	300	300
15	225	200	1,530	1,050	200	200	200
16	100	100	1,040	900	100	100	100
17	179	100	906	786	100	100	100
18	258	100	771	671	100	100	100
19	336	100	637	557	100	100	100
20	415	100	503	443	100	100	100
21	494	100	369	329	100	100	100
22	573	100	234	214	100	100	100
23	651	100	100	100	100	100	100
0	712	189	600	322	200	189	189
1	773	278	1,100	544	300	278	278
2	834	367	1,600	767	400	367	367
3	895	456	2,100	989	500	456	456
4	956	544	2,600	1,211	600	544	544
5	1,017	633	3,100	1,433	700	633	633
6	1,078	722	3,600	1,656	800	722	722
7	1,139	811	4,100	1,878	900	811	811
8	1,200	900	4,600	2,100	1,000	900	900
9	1,050	800	4,200	1,950	875	800	800
10	900	700	3,800	1,800	750	700	700
11	750	600	3,400	1,650	625	600	600
12	600	500	3,000	1,500	500	500	500

TABLE 6. Pumping Station Discharges (m^3/h)—Optimal Solution

T (1)	*X*1 (2)	*X*2 (3)	*X*3 (4)	*X*4 (5)	*X*5 (6)	*X*6 (7)	*X*7 (8)
12	625	0	0	0	100	50	0
13	625	0	0	0	100	50	0
14	625	0	0	0	100	50	0
15	625	0	0	0	100	50	0
16	1,620	100	356	136	300	150	100
17	1,620	100	356	136	300	150	100
18	1,620	100	356	136	300	150	100
19	1,620	100	356	136	300	150	100
20	1,620	100	356	136	300	150	100
21	1,620	100	356	136	300	150	100
22	1,620	100	356	136	300	150	100
23	2,500	189	900	561	489	189	189
0	2,500	189	900	561	489	189	189
1	2,500	189	900	561	489	189	189
2	2,500	189	900	561	489	189	189
3	2,500	189	900	561	489	189	189
4	2,500	189	900	561	489	189	189
5	2,500	189	900	561	489	189	189
6	2,500	189	900	561	489	189	189
7	2,500	189	900	561	489	189	189
8	225	0	0	0	75	0	0
9	225	0	0	0	75	0	0
10	225	0	0	0	75	0	0
11	225	0	0	0	75	0	0

mum and maximum volumes (equal for all times, for simplicity of the presentation, but they could easily be varied over time). *VO* is taken as an infinite reservoir. Table 4 contains the demand data. Again, for clarity of the presentation, a simple two-period pattern (whose break point does not coincide with a change in tariff) is used.

The optimal operating policy is given in terms of reservoir levels (Table 5) and pumping station discharges (Table 6). The pump schedule of each station is obtained by "filling" the required flow with pumps in decreasing order of efficiency and arranging the "on" and "off" periods on the time

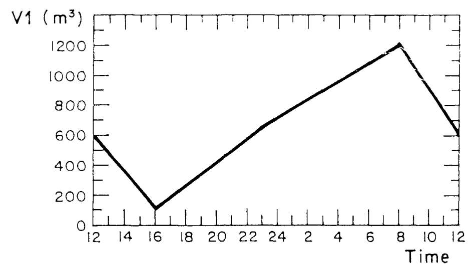

FIG. 7. Ein Ziv Reservoir (V1) Volume Change with Time—Optimal Solution

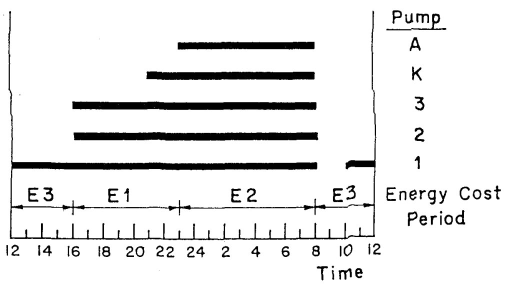

FIG. 8. Hardalit Pumping Station (P1) Operation Over Time—Optimal Solution

axis so as to minimize the number of on/off's.

Figs. 7 and 8 demonstrate graphically the variation of volume in V1 and pump operation in P1 (including two wells adjacent to the station, which has three pumps). All pumps operate during the low-cost period (23–7). During the high-cost period (8–15) only pump 1 operates, and not all of the time. This is necessary to prevent the reservoir V1 from dropping below $\underline{V}_1$ = 100 m^3. During the intermediate-cost period (16–22), the pumps are introduced in decreasing order of efficiency. This is necessary to replenish the volume so it is full at the beginning of the high-cost period (8 o'clock). A high level at the beginning of the high-cost data and a low level at its end are to be expected. The precise trajectory of reservoir levels depends on the demands and pump discharges, and cannot be calculated without the optimization.

Conclusions

The optimization algorithm described in this paper uses the following.

1. Given demand forecasts.
2. An aggregated system model, based on a detailed hydraulic analysis.
3. Given initial states (levels or volumes) and desired final states in all reservoirs.
4. Variable (or fixed) energy costs over the day.
5. Pumping energy curves for each pumping station, also based on a detailed hydraulic analysis. These are converted into cost functions, which must be convex for the iterative process to converge to the global optimum.

The network is divided into subsystems, each being a pumping station between two reservoirs. The time horizon (usually 1 day) is divided into time steps (for example, 1 hr each). The iterative optimization algorithm goes over the time horizon, dealing with two adjacent time steps, with the other decisions fixed. For each pair of time steps, the algorithm ranges over all subsystems, one at a time. The global optimum is reached (under the convexity assumption, which is practically guaranteed in all real systems) from any initial trajectory of reservoir volumes.

The method has been programmed on an IBM PC/XT in Pascal. A full run for a 24-hr operation takes 10–15 minutes. The problem can therefore be run again during the day if the observed conditions (for example, the demands, pump availability due to failures) deviate markedly from those assumed when the initial run was made.

The program was written modularly so it can be applied to water supply systems in which the subsystems appear in any configuration. This ease of implementation was proven in a few applications.

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Appendix. References

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