Schematic Models for Distribution Systems Design. II: Continuum Approach

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Abstract: This is the second of two papers dealing with the development and use of schematic models for water distribution systems. The first paper covered methods for step-wise combination of elements into equivalent ones. Herein, a different and novel approach, which views the system as a nonlinear horizontal continuum, is proposed. An areally distributed conductance, a function of pipe properties and areal density, is the link between a potential function, which is related to the heads and the flow field. The model can be used to obtain the flow field of an existing system for a prescribed potential, or to optimize the conductance of a planned system for a given distribution of demands and specified boundary conditions. The method is demonstrated by a case study, for the city of Jerusalem, Israel.

Introduction

This paper, together with the preceding one (Hamberg and Shamir 1988), presents an approach and methodology suitable, during the preliminary phases of an engineering investigation, for analysis and design of water distribution systems.

Three phases may be distinguished in the design work: (1) Preliminary planning; (2) design; and (3) construction drawings and specifications. In this division of work, the preliminary planning and design are, respectively, analogous to the architectural planning and structural design of a building.

Computer programs for steady-state network analysis, also called network solvers, and for the simulation of its operation over time are used quite extensively (AWWA 1984; Cesario et al. 1984). Methods and programs for optimal design are also beginning to become available (e.g., Shamir 1979). These programs operate on a model of the distribution network in which its components—pipes, pumps, reservoirs, valves, and special control devices—are identified and modeled. The model can be, and usually is, schematized, but this schematization mostly amounts to no more than dropping small-diameter pipes and replacing pipes or pumps that are in parallel or in series by an equivalent pipe or pump.

These models and computer programs have been developed specifically for carrying out a detailed analysis of a planned or existing network. The writers have previously advocated the use of schematized models for those stages of analysis in which the detailed flow and pressure information at

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every point in the system is not important (Shamir 1973; Shamir and Howard 1977).

The principal reason for using small, simplified models is that the planners' and engineers' time and effort should be spent examining a broad range of alternatives and not on the tasks of constructing a detailed model, collecting data, calibrating, checking for errors, and analyzing voluminous computer outputs to determine what the results of various runs mean for the planning process. This is best done with simple models, which capture the main features of the proposed systems performance without having the extensive details.

Thus, the motivation for the writers' work is not computational efficiency but better planning. The writers propose to provide tools for the planning phase, a phase that has received too little attention from the developers of methodologies. The procedure proposed by Deb (1976) is an exception; he suggested a method of sizing pipes in a network, assuming that the pressure surface over the area covered by the network has a parabolic shape (whose parameters are adjusted iteratively).

PRINCIPLES OF CONTINUUM APPROACH

One may compare two network models by considering the overall areal patterns of performance, neglecting insignificant local phenomena.

In a specific network model, the flows are associated with the links (pipes and pumps), the demands and heads with the nodes. Therefore, to compare one network model to another the demands, heads, and flows have to be released from this direct association with network elements, and considered continuous variables over the area covered by the network. Demands are, in reality, spread over the area; only for modeling are they lumped and assigned to nodes. For heads, it is quite common to produce a map of constant-head lines that describes a continuous surface. This is convenient, although not physically precise, since a head exists only along the links of the network.

For flows, the continuous, areally distributed variable, which is *flow per unit width* has to be defined. Straight line cuts in any arbitrary direction through the map of the system are taken. Each cut is sufficiently long to pass through several pipes, but at the same time short enough so that the average flow per unit width across the cut is meaningful and describes the areal flow pattern in the network. The flow unit width, which will be referred to simply as "flow," is computed by dividing the normal component of the flow through the cut by its length. [The concept and definition of a cut is somewhat analogous to that of the "representative elementary volume" (REV) in flow through porous media (Bear 1972)].

In contrast with the step-wise combination approach discussed in Hamberg and Shamir (1988), which looks in detail at individual elements and assembles them into a combined model, the writers propose to look at the system as a whole, i.e., as a *continuum*. This can be viewed, pictorially, as standing back from the system map far enough to see only the overall flow pattern and hydraulic head map, without taking interest in the individual system elements.

Continuum Properties

The writers will use the definition of the conductance in equation 3 of Hamberg and Shamir (1988), i.e., $d = D^{\alpha}$, where D = diameter and $\alpha \approx$ 2.63 or 2.67, according to the flow formula used. The conductance of a system of parallel pipes is the sum of the conductance of each pipe, according to equation 7 of Hamberg and Shamir (1988).

A conductance per unit width is now defined by

$$\bar{d} = \frac{d}{b} \qquad (1)$$

i.e., the conductance per unit width perpendicular to the direction of flow. d may be viewed as the local conductance of closely spaced (not necessarily equal) pipes, and $\bar{d}$ is its local value per unit width. For such a system of parallel pipes, the flow per unit width is defined by

$$q = \frac{Q}{b} \qquad (2)$$

where Q = discharge that passes through the width b. $\bar{d}$ and q are continuous variables over the area, defined for adequately small widths b.

From the flow equation, generally defined by equation 1 of Hamberg and Shamir (1988), the loss of head H along L is now obtained:

$$\frac{\Delta H}{L} = \beta\left(\frac{qb}{\bar{d}b}\right)^{\gamma} = \beta\left(\frac{q}{\bar{d}}\right)^{\gamma} \qquad (3)$$

in which γ = 2.0 or 1.852, according to the flow formula used, and β is a numerical coefficient that includes smoothness coefficient (e.g., C of Hazen-Williams) and unit conversions. For the Hazen-Williams formula, $\beta = 1.131 \times 10^9 C^{-1.852}$.

The flow per unit width (which, hence, will also be referred to simply as the "flow") is expressed as

$$q^{\gamma} = \left(\frac{\bar{d}^{\gamma}}{\beta}\right)\frac{\Delta H}{L} = K\frac{\Delta H}{L} = -K\,\nabla H \qquad (4)$$

Eq. 4 looks like Darcy's law for flow through a porous medium, except that it is nonlinear, i.e., it relates q^{γ} to the gradient of the head by a constant K, which is a property of the medium.

Since a system generally has pipes in different directions, not only parallel, and there are cross connections between them, K in Eq. 4 must depend not only β and $\bar{d}$ but also on the "tortuosity" of the network (as it is called in the study of flow through porous media). The tortuosity, t, is defined here as

$$t = \frac{L(\Delta H)}{L} \qquad (5)$$

where $L(\Delta H)$ is the straight-line distance between the points at which the head difference ΔH is defined; L is the actual length of (a typical) pipe in the system between these two points; t is typically in the range of 1.0 to 0.7

($\cong 1/\sqrt{2}$, the ratio of the distance along the sides of a square to the distance along its diagonal), as actual pipes seldom bend backwards.

$$\text{Thus} \quad K = t\,\frac{\bar{d}^{\gamma}}{\beta} \qquad (6)$$

The properties q, $\bar{d}$, and K are useful only when the width of a cut through the system, b, is selected so that it averages over the very local phenomena, but at the same time is sufficiently short so that the averaging does not "smear out" details that are of interest for the particular study. Thus, the cuts are similar to the notion of REV used in porous media (Bear 1972) in that they are small enough to be "local" yet large enough to integrate over the minute details.

K at a selected point in a given system for a specified direction of flow is obtained by taking a cut of length b perpendicular to this direction. The conductance of each pipe cut, d, is related to its diameter by $d = D^{\alpha}$, where d is the average of d over the width b. $\bar{d}$ and t (from Eq. 5) are combined to yield K, according to eq. 6. The K-field over the system may be (and usually is) anisotropic and nonhomogeneous.

Flow Field

Simple flow geometries, such as parallel flow and radial flow, can be solved analytically from Eq. 4 for a given K-field and appropriate boundary conditions on H, q, or both. Contrary to porous media, in which the flow equation is linear, superposition is not possible here. For further analysis, the discussion shall be restricted to $\gamma = 2$.

The continuity equation is, as in flow in porous media:

$$\text{div } q = 0 \qquad (7)$$

but contrary to flow in porous media, here

$$\text{div } (q\,|q|) \neq 0 \qquad (8)$$

and the Laplace equation $\nabla^2 H = 0$ does *not* hold.

One way of still obtaining solutions by superposition of simple cases is to use the transformation

$$K\,\nabla H = (k\,\nabla h)^2 \qquad (9)$$

where, for a homogeneous and isotropic domain, k may take any value, even 1.

The transformed variable, h, now satisfies $q = k\,\nabla h$, and thus by Eq. 7, also saisfies

$$\nabla^2 h = 0 \qquad (10)$$

and solutions of combined cases can be obtained by superposition. The boundary conditions on H are transformed to boundary conditions on h, and those for flows remain unchanged. The solutions for h have to be transformed back into heads, H, by integrating the function $(\nabla h)^2$, according to Eq. 9. This integration is normally done numerically, bearing in mind that $(\nabla h)^2$ should be read as $\nabla h\,|\nabla h|$.

Another possibility is an iterative procedure. At iteration j, the flow field is given by q_j, and $(K_q)_j$ is defined as

$$(K_q)_j = \frac{K}{q_j} \quad \text{(11)}$$

Then solve:

$$\nabla^2 H_j = 0 \quad \text{(12)}$$

and obtain the next flow distribution from

$$q_{j+1} = -(K_q)_j \, \nabla H_j \quad \text{(13)}$$

Substitute q_{j+1} for q_j in Eq. 11 to define $(K_q)_{j+1}$ and then solve Eq. 12 for H_{j+1} and Eq. 13 for q_{j+2}. Repeat until the process converges. The process may be initiated with q_0 uniform over the area.

A third method for obtaining the flow field and head map would be to use a numerical solution of the nonlinear continuum problem (Eq. 4) using, e.g., a finite element approach.

The three methods would, however, defeat the writers' main purpose, to a lesser or greater degree, since one numerical solution (in the continuum) would be substituted for another (a network solver). The methods were used only during the investigation, for study and comparison.

The first two methods were used in two case studies: Minneapolis, Minnesota, and Jerusalem, Israel. The latter case study will be described in later sections.

These methods may turn out to be quite laborious for analyzing an existing system, because one has to measure the K-field. Still, if the model is to be used many times, to analyze a variety of planning options (e.g., new sources, storage, major pipelines), then the effort invested once is worthwhile. The continuum approach is even better suited to the design of a new system, as explained next.

Design of System by Continuum Model

Eq. 1 defines the conductance per unit width, which, for a number of pipes intersected by the cut of length b perpendicular to the direction of flow, is given by

$$\bar{d} = \frac{\sum_{i=1}^{M} d_i}{b} = \frac{\sum_{i=1}^{M} D_i^{\alpha}}{b} \quad \text{(14)}$$

in which α = 2.63 or 2.67.

If the conductance map of the system is designed, then it can be translated into actual pipes, with specified diameters and spacings, as will be explained later.

The investment and energy-loss costs per unit length of pipe are, respectively:

$$I = C_D + C_D D^p \quad \text{(15)}$$

and $$E = C_h F Q \left(\frac{\Delta H}{L}\right) = C_h F Q \left(\frac{\beta Q^{\gamma}}{D^{\alpha\gamma}}\right) \quad \text{(16)}$$

in which C_0, C_D, p, and C_h are coefficients; and F is a factor that includes conversion of costs over the planning horizon into present value, conversion of units from flow times head to power, then by summation over time to energy, and then to cost.

These cost equations can be expressed in terms of the conductance and combined to yield the total cost function:

$$T = I + E = C_D + C_D d^{p/\alpha} + C_h F \frac{\beta Q^{\gamma+1}}{d^\gamma} \quad (17)$$

Eqs. 16 and 17 do not include a term for the energy needed for the static lift, as it is not dependent on the diameter, and therefore does not influence the optimal diameter of a single segment with constant flow. Other cases will be referred to later.

The optimal conductance, d^*, of a single pipe is determined by differentiating Eq. 17 with respect to d, setting it equal to zero, and solving. The result is

$$d^* = \left(\frac{C_h F\beta}{C_D} \cdot \frac{\alpha\gamma}{p}\right)^{[\alpha/(\alpha\gamma+p)]} Q^{[\alpha(\gamma+1)/(\alpha\gamma+p)]} \quad (18)$$

Using coefficient values of the Manning formula [table 1 of Hamberg and Shamir (1988)] and taking the cost exponent from actual data to be $p = 1.2$, the optimal conductance is

$$d^* = 1.839\left(\frac{C_h F\beta}{C_D}\right)^{0.408} Q^{1.225} \quad (19)$$

This result is for the case in which the amount pumped is that which passes through the pipe, i.e., the pump is close to the pipe. Actually the energy cost associated with the pipe is proportional to the pumping discharge, QP, and the head loss in the pipe itself, which depends on the flow in it, Q. When the pump is far from the pipe under consideration, QP does not necessarily equal Q and can sometimes be considered independent of Q in the actual pipe under consideration. The optimal conductance for such a case is derived through a similar analysis of the one herein, resulting in:

$$d^* = 1.839\left(\frac{C_h F\beta}{C_D}\right)^{0.408} QP^{0.408} Q^{0.82} \quad (20)$$

Eqs. 19 and 20 indicate that the optimal conductance, d^*, is approximately proportional to the discharge. The range of exponents is from 0.82 for a pipe far from the pump to 1.225 for a pipe at the immediate discharge of the pump. Since most pipes of a network are somewhere midway between these conditions, the power 1.0 is a reasonable approximation. The resulting linear equation is very convenient for the analysis.

These results for a simple pipe can now be extended to the continuum:

$$q = K_C \bar{d}^* \quad (21)$$

where q and $\bar{d}^*$ are, respectively, the discharge and optimal conductance, both per unit width. The constant K_c depends, as can be seen from Eqs. 19 and 20, on parameters and exponents of the pipe flow formula and on coefficients of the cost equation. The exact value of K_c also depends on the value of QP/Q at each point, or on the validity of Eq. 19. QP affects K_c by a relatively low exponent (see Eq. 20), so that a rough estimate of QP suffices.

This linear relation is analogous to the Darcy equation for flow in porous media. This suggests that a potential function, W, can be defined analogous to the head in groundwater flow, such that

$$\bar{d}^* = \nabla W \qquad (22)$$

$$\nabla^2 W = 0 \qquad (23)$$

Since W satisfies the Laplace equation Eq. 23, it can now be solved by the many procedures developed for groundwater flow. Boundary conditions on the flows in the W-q problem remain unchanged from the original H-q problem in the continuum.

This enables the analytical solution of a large variety of problems (most of them for homogeneous and isotropic domains), by superposition (see an example later), or by numerical methods, e.g., finite element, all with existing procedures and even computer programs.

Conversion from Conductance to Pipes

The conversion from optimal conductance map to actual pipes is not unique. Many pipe configurations have the same conductance. In practice, existing pipes, distribution needs, alignment of roads, and local phenomena limit the number of feasible configurations.

Some criteria were developed to further guide the conversion from conductance to pipes (if this is already needed at the preliminary planning stage).

1. A few main pipes (others having only the minimal diameter needed for distribution) are normally better than many equal pipes (unless either p of Eq. 15 or the minimal diameter is rather high).
2. The advisable maximum distance B between adjacent main pipes depends on their length L and on the general configuration of sources, consumption, etc. A list of guidelines for various cases is outside the scope of this paper. For example, suppose one has to design a water system supplying a rectangular domain out of four sources located at one end of the area, and distributing it equally along the domain (of length L). Assuming the diameters of all pipes are determined by mathematical optimization, the distance between the distributing pipes should be 0.44 L.
3. The angle between the general alignments of any two main pipes in a network should be close to 90° (Fig. 1). Bhave and Lam (1983) suggested 120° as a first guess in an iterative procedure, but 90° was found to be a better guess, very close to the optimum of many typical cases.

Improvements in Existing System

Most studies deal with improvements to existing systems, rather than with the design of a completely new one. If the existing density of pipes is m_0 per unit width and their combined conductance is d_0, then the optimal new density, m, should be

$$m^{\gamma+1} = \frac{C_h \beta F \gamma}{C_D} \left(\frac{m_0}{d_D}\right)^{(\alpha\gamma + p/\alpha)} Q^{\gamma+1} \qquad (24)$$

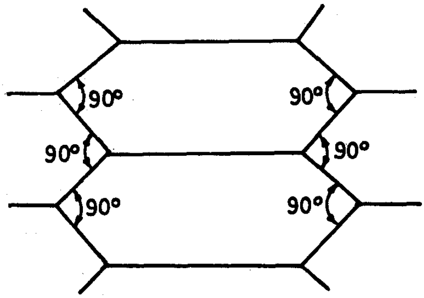

FIG. 1. Desired General Configuration of Pipes

The value $m - m_0$ means how many new pipes of an equal conductance (d_0/m_0 each) should actually be installed per unit width. Eq. 24 was derived in a way similar to Eq. 18, by differentiating a modified version of Eq. 17 with respect to $m - m_0$.

Design of System with No Pumping

If only the investment costs of pipes is considered in a system that is fed by gravity and so incurs no energy costs, then the ratio of optimal conductances of any two pipes out of a number of pipes in series is obtained by equating to zero the derivative of Eq. 15 considering a constraint on the total head loss ΔH along the whole system of pipes in series (this is done by the method of Lagrange multipliers).

$$\frac{d_j}{d_s} = \left(\frac{Q_j}{Q_s}\right)^{[\alpha\gamma/(\alpha\gamma + p)]} \qquad (25)$$

For M pipes in series

$$\Delta H = \beta \sum_{i=1}^{m} \left(\frac{Q_i}{d_i}\right)^{\gamma} L_i \qquad (26)$$

If the cost exponent is $p = 1.2$, and the Manning formula is used, then for each pipe j, out of the m pipes in series, from Eqs. 25 and 26, one gets

$$d_j^{2.0} = \frac{\beta}{\Delta H} Q_j^{1.633} \sum_{i=1}^{m} (Q_i^{0.367} L_i) \qquad (27)$$

This means that, again, d^* of pipes in series is related to the discharge by a power of 0.8. For parallel pipes, d is linear with Q, as for a single pipe ($M = 1$).

A linear approximation for a continuum, derived from Eq. 4 is:

$$q = \sqrt{\frac{\nabla H}{\beta}}\, \bar{d} = K_h \bar{d} \qquad (28)$$

Superposition of Simple Cases

Because the system equations (e.g., Eq. 23) are linear (for the design of a continuum), it is possible to use superposition and combine solutions of simple cases to yield the solution of a combined situation. The elementary solutions are for such cases as source or sink in an infinite field, parallel flow, a continuous line source, etc.

For example, consider the flow field due to S sources placed anywhere in the field. It is similar to the flow field to groundwater wells (Bear 1972). At any point x in the field, the potential function W is given by

$$W(X) = W_0 + \frac{1}{K_C} \sum_{s=1}^{S} \frac{Q_s}{2\pi} \ln \frac{r_s}{r_0} \qquad (29)$$

where Q_s = discharge of source (s); r_s = distance from point x to this source; and r_0 = distance from point x to a reference point at which $W = W_0$. Usually the reference is taken as $W_0 = 0$. Once $W(x)$ has been computed, $\vec{d}^* = \nabla W$ is solved for the optimal conductance, $\vec{d}^*$.

Distributed Demands

If the consumptions are distributed over the area, as they usually are, then the potential function, W is the solution of the Poisson equation:

$$\nabla^2 W = q' \qquad (30)$$

where q' is the consumption per unit area.

For simple cases, such as a source with distributed consumptions over an infinite radial domain, there exists a simple analytical solution. The way to solve for W in a real case is to consider the distributed consumption as part of one of the basic cases that are solved separately and then superimposed. One of the basic cases includes distributed consumption and is formulated as a Poisson equation, while all other basic cases are solved as Laplace equations. Boundary conditions for sources and sinks (Laplace cases) are satisfied, e.g., by reflecting them through the boundaries (the image method). The boundary conditions are satisfied for the Poisson case too by assuming the entire consumption concentrated at the "center" of the domain, as a single sink, which is then reflected through the boundaries.

Unsteady Flows

Since consumptions vary over time so do the flows throughout the system, and this must be taken into consideration in the design.

It is of interest to note here that the accepted practice today is to base the design on very few flow conditions—usually only one, such as maximum day plus fire flows or peak hour. Space here does not allow a discussion of this practice, but the writers believe that this practice is in many cases wasteful. For the preliminary design work, which the writers consider here one can use the equivalent steady discharge, as defined in Hamberg and Shamir (1988). (In that paper, equation 17 was defined for a discharge, which varies linearly with time; and similar expressions have been developed for other forms).

For the continuum approach, there is a more accurate way of considering unsteady flows. Define two new variables, δ and Ω, by

$$\delta^2 = \frac{\partial(\bar{d}^2)}{\partial t} \qquad (31)$$

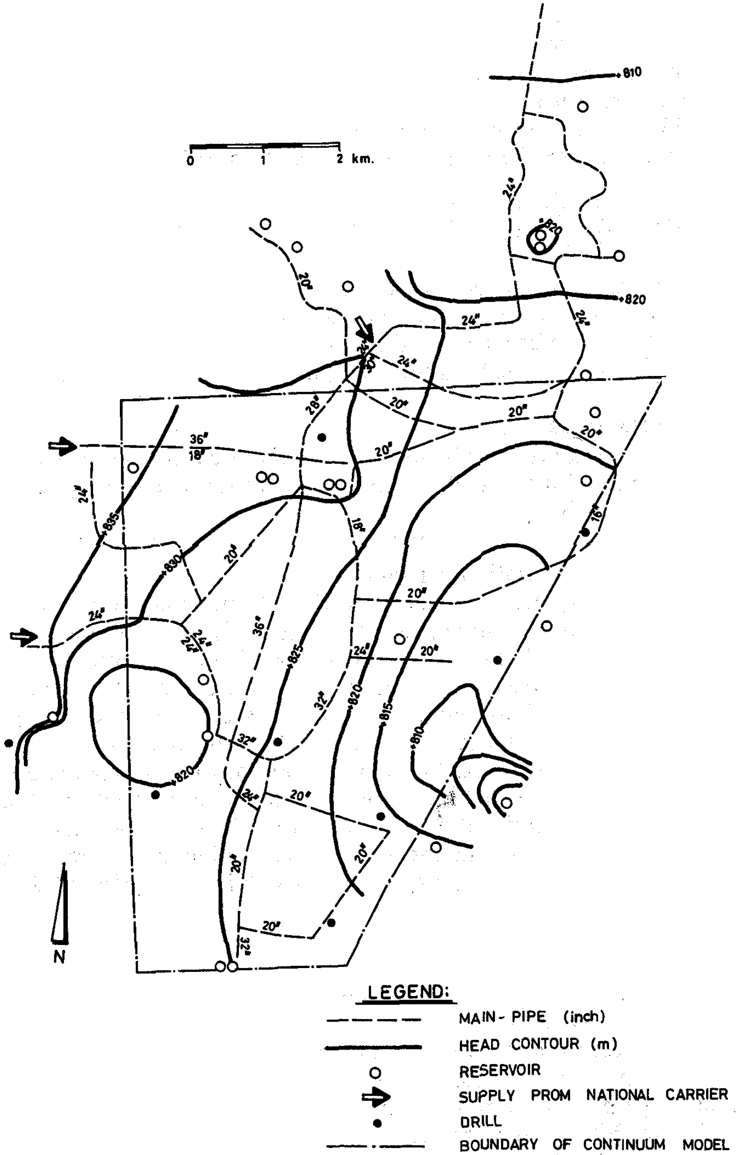

FIG. 2. Water Supply for Jerusalem, According to Master Plan for Year 2000 (Peak Hour)

and $\nabla\Omega = \delta$ .... (32)

For steady flow, a relation between the flow, q, and the optimal conductance per unit width, $\bar{d}^*$, is given by Eq. 21.

K_c of Eq. 21 includes a term of $\sqrt{F}$, according to Eqs. 19 and 20. F is a linear function of T, the duration of flow, throughout the year, as it sums

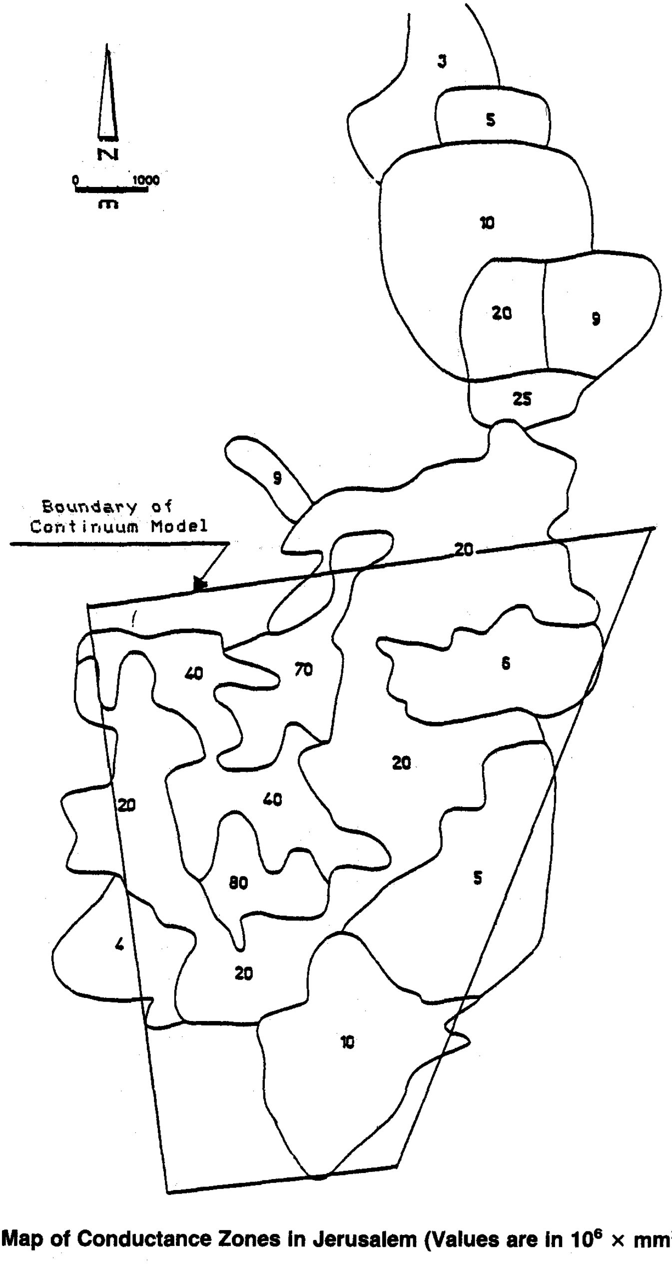

FIG. 3. Map of Conductance Zones in Jerusalem (Values are in $10^6 \times mm^{2.63}/km$)

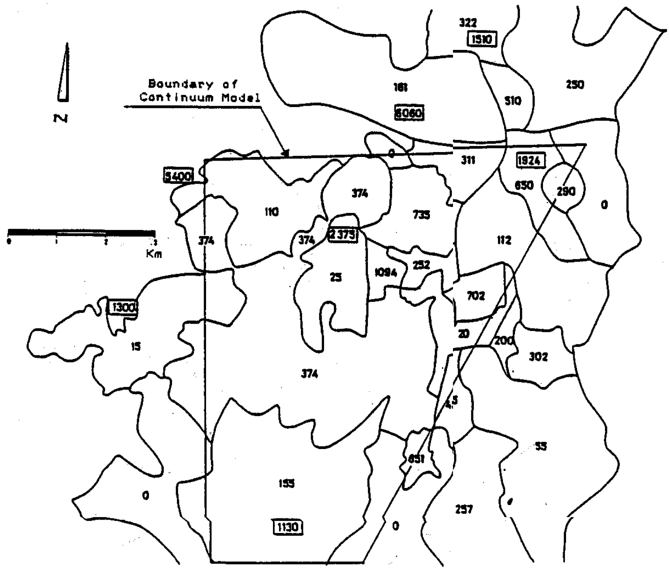

FIG. 4. Map of Water Consumption in Jerusalem: Consumption in $m^3/hr/km^2$ = 374; Source Flow in m^3/hr for Peak Hour at Year 2000, e.g., $\boxed{2,375}$

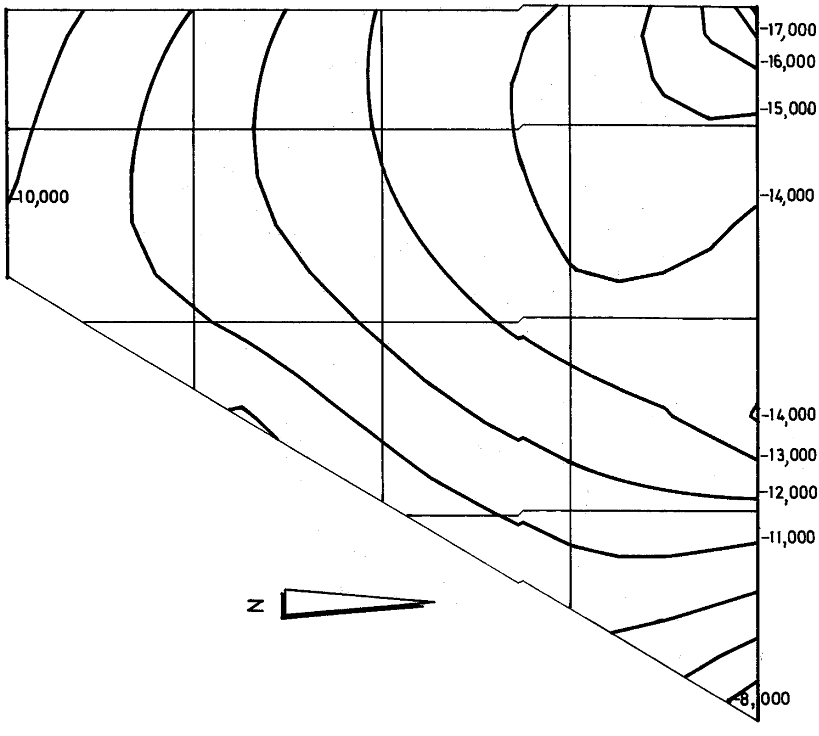

FIG. 5. Design of Jerusalem as Continuum: $k_c W$ Lines (Basic Case)

FIG. 6. Design of Jerusalem as Continuum: Flow Arrows (Basic Case)

up discharge into water quantity or power into energy (for unsteady flow it is rather an integral of QP over t).

One can rewrite (for a steady flow) Eq. 21 as

$$(\bar{d}^*)^2 = \frac{T}{K_{ct}} q^2 \quad \text{(33)}$$

Differentiating with respect to t and taking the square root will lead to:

$$q = \sqrt{K_{ct}}\, \delta \quad \text{(34)}$$

where $\sqrt{K_{ct}}$ can be evaluated from K_c eliminating T. Eq. 34 is also valid for unsteady flows.

Comparing these results with Eqs. 21 and 22 δ is analogous to $\bar{d}^*$, there, and Ω to W. Thus, Ω can be obtained by potential flow methods, as was W, and δ is derived by the same methods as was $\bar{d}^*$.

For n_t time periods, of durations T_i, one calculates δ_i for each. Then the conductance is

$$\bar{d}^{*2} = \sum_{i=1}^{n_t} \delta_i^2 T_i \quad \text{(35)}$$

These equations refer to the linear approximation. If the exact value of the pumping discharge is to be considered (analogous to Eq. 20), then the

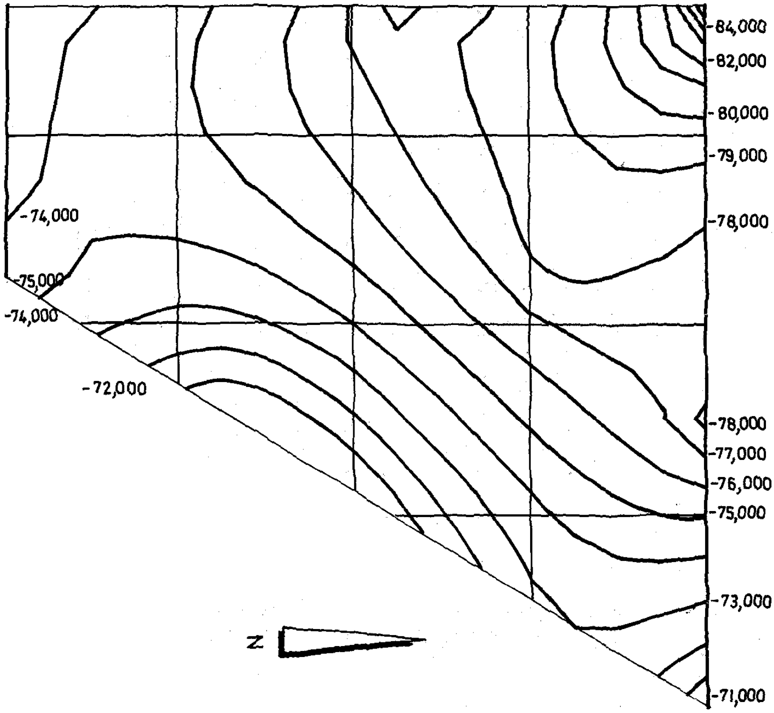

FIG. 7. Design of Jerusalem as Continuum: $k_c W$ Lines for Constant Peak Consumption (Conductance Values are in $10^6 \times mm^{2.63}/km$)

power 2 on the variable d in Eqs. 31–35 should be replaced by $(\gamma + p/\alpha)$. If $p = 1.2$, then for the Manning formula the power is 2.45, and for the Hazen-Williams formula it is 2.31.

Case Study: Jerusalem

Two case studies were used throughout this study, to test the various procedures: Minneapolis, Minnesota, and Jerusalem, Israel. Both cities have extensive distribution systems. Minneapolis was studied about 10 years ago for capacity expansion (Howard 1974) and the engineering report was used as a source of data. A master plan for the distribution system of Jerusalem was completed more recently (Levy and Gero), and this case served for much of the testing. Only some results for Jerusalem are presented here.

Fig. 2 is an outline map of Jerusalem's water supply area. For lack of space, not much detail is provided, and by necessity the maps and figures will lack sufficient detail. They are presented to give an impression of the results.

The water system of Jerusalem supplied $34 \times 10^6 m^3$ in 1980, and is planned to supply $80 \times 10^6 m^3$—more than double—in 2000, to a population of 775,000. The existing sources are three connections to the National Water Carrier (about 85% of the water) and a few local wells. Any

FIG. 8. Design of Jerusalem as Continuum: Optimal Conductance Map, Basic Case (Conductance Values are in $10^6 \times mm^{2.63}/km$)

additional supplies would have to come from the national water carrier through a new connection, which will probably come into the system from the northwest side.

The area is hilly, with pronounced topographical changes (altitudes between 700 and 830 m, changing rapidly over the area) and has variable population densities and water demands. The water system is divided into three pressure zones. The largest is at medium head, and it supplies the other two by gravity or pumping. The two other zones are supplied by local reservoirs. The main reservoirs are located in the central pressure zone. Shown in Fig. 2 is also a trapezoidal area, which covers the main parts of the system, and will be used for the continuum model, as described later. It covers an area of about 38 km^2.

The head contours in Fig. 2 are for the designed peak hour in the year 2000, computed by a standard network solver for the system as planned by engineers in the master plan (Levy and Gero 1982).

Fig. 3 shows the conductance map of this system, derived by the methods described in this paper. The writers took the planned system map, introduced many (a few hundred) cuts, each across the local pipe pattern, computed the local conductance—usually in two orthogonal directions,

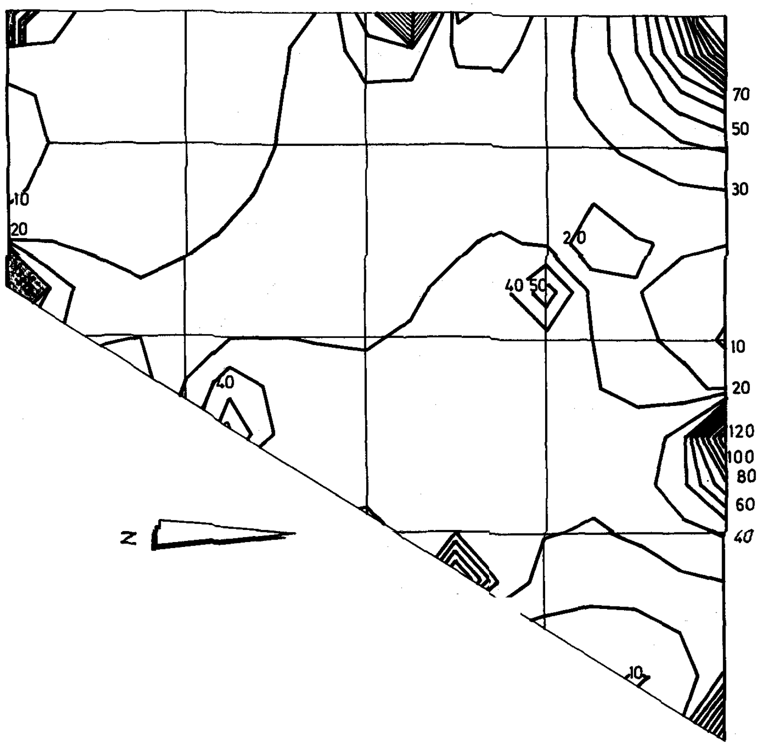

FIG. 9. Design of Jerusalem as Continuum: Optimal Conductance Map for Constant Peak Consumption (Conductance Values are in $10^6 \times mm^{2.63}/km$)

where the network has pipes in these directions—and then identified zones that can be said to have a uniform conductance in one direction. As the results were rather close for both directions, the writers considered the system to be isotropic. The average conductance values are shown in Fig. 3 as a conductance map.

Fig. 4 shows the consumption zones and main sources. For the analytical computation of the continuum solution these were schematized into 10 source/sink points plus a uniformly distributed demand over the entire area. These sources and sinks were used as the basis for computing a distribution of (K_cW), which is shown in Fig. 5. The writers calculated K_cW, rather than W, because we used later (while calculating d^*) a correction where K_c depends slightly on the discharge q and the pumping discharge q_p. The variation of demands over the day was used to compute the equivalent steady-state flows, and then these were used in Eqs. 21–23 to generate the map. Lines of equal (K_cW) are perpendicular to the flow direction. There is a general similarity between the map of heads for the system planned by conventional methods (Fig. 2) and the equipotential map of the continuum model (Fig. 5). Fig. 6 shows the direction and magnitude (length of the arrows) of the flows that correspond to the equipotential map of Fig. 5.

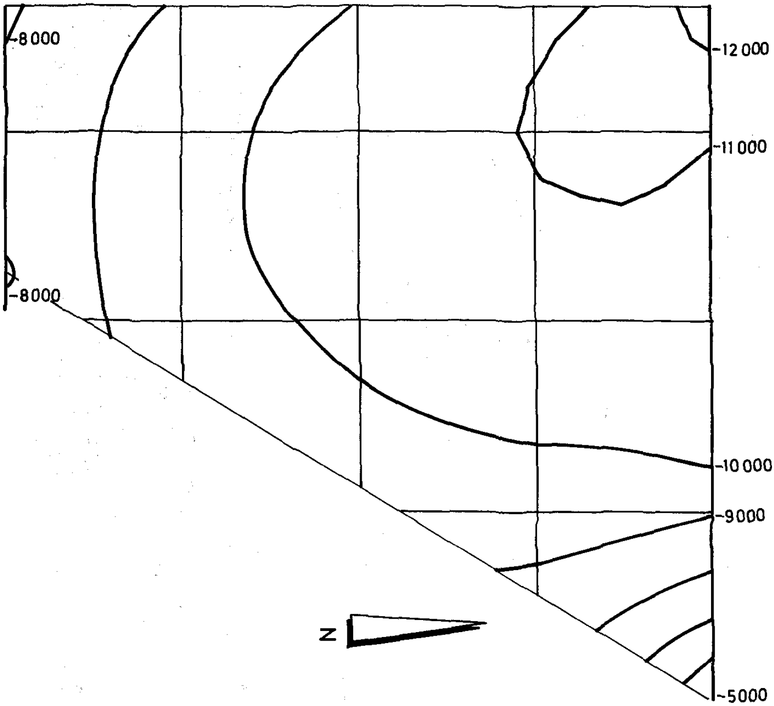

FIG. 10. Design of Jerusalem as Continuum: k_cW Lines for Off-Peak Hour (Night) in Winter

Fig. 7 shows the map of (K_cW) for peak hour flows. It is much more similar to Fig. 2, as expected, since this map is also for the peak hour.

The optimal conductance corresponding to the temporally averaged analysis (Figs. 5 and 6) is shown in Fig. 8, as computed by Eq. 22. The map is of equiconductance contours. For example, the area between the lines of 15 and 22.5 can be interpreted as having an average conductance of 19 $mm^{2.63}/km$ (see Eq. 14).

Fig. 9 shows the optimal conductance map corresponding to the peak hour flows (Fig. 7). This conductance map shows a greater similarity to the system as planned by the conventional method for the peak hour only (Fig. 3), as would be expected. Most of the differences between the conductance in Figs. 3 and 9 are of a local nature.

Fig. 10 shows the map of $K_c\Omega$ for low flows (winter night). This is combined with the peak hour results (Fig. 7) according to the procedure of Eqs. 40 and 37 to yield the conductance map in Fig. 11, which is reasonably similar to the results for the temporally equivalent flow method (Fig. 8).

For the purpose of this study, the writers also solved the continuum model with a finite element computer program. Fig. 12 shows a grid of 190 elements, which correspond to Jerusalem's traffic zones. Each has a consumption value given, and the (K_cW)-field is solved with the following boundary conditions: no flow through the boundaries (except for point

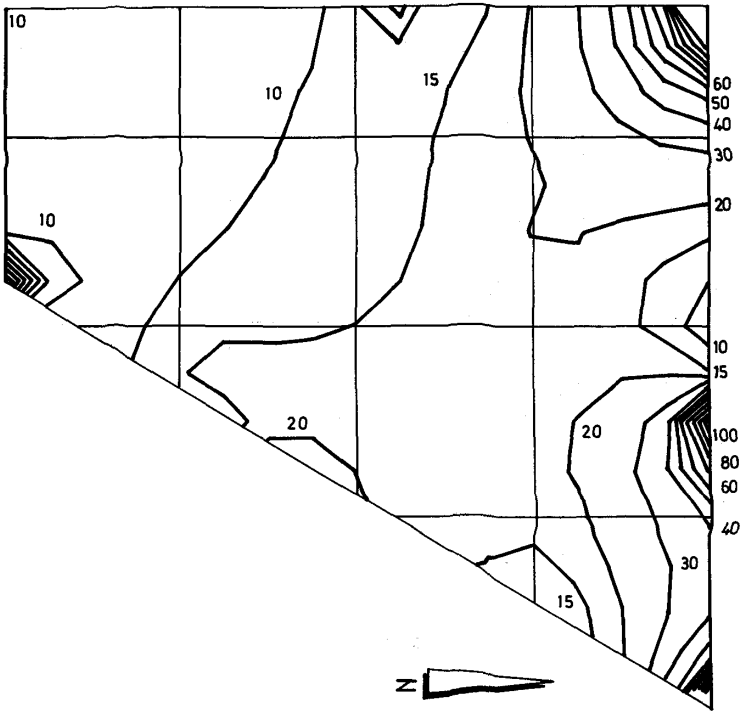

FIG. 11. Design of Jerusalem as Unsteady Continuum: Conductance $\bar{d}^*$ Map (Conductance Values are in $10^6 \times mm^{2.63}/km$)

sources at the connections to the National Water Carrier, at wells and at the reservoirs).

The results are shown in Fig. 13. The trapezoidal area covered by the analytical continuum model is also shown, so that the results within it can be compared to those in Figs. 2, 5, and 7.

The amount of work spent in modeling the Jerusalem system as depicted here was several days (eliminating trials due to the development of the method) compared to weeks for a more conventional planning analysis. This is due to the use of rather raw material and does not take into account the easier interpretation of the results and the savings in repeated calculations with different data as sensitivity tests.

Conclusions

The writers recommend the use of simple models in the preliminary design phase of water distribution networks. The model must be able to replicate the behavior of the system in a general sense, but does not necessarily contain the values of all the heads and flows in all network elements.

The step-wise combination approach, discussed in Hamberg and Shamir (1988), enables a schematization of a network model. Instead of accepting some arbitrary rule for leaving out "small details," such as all pipes below

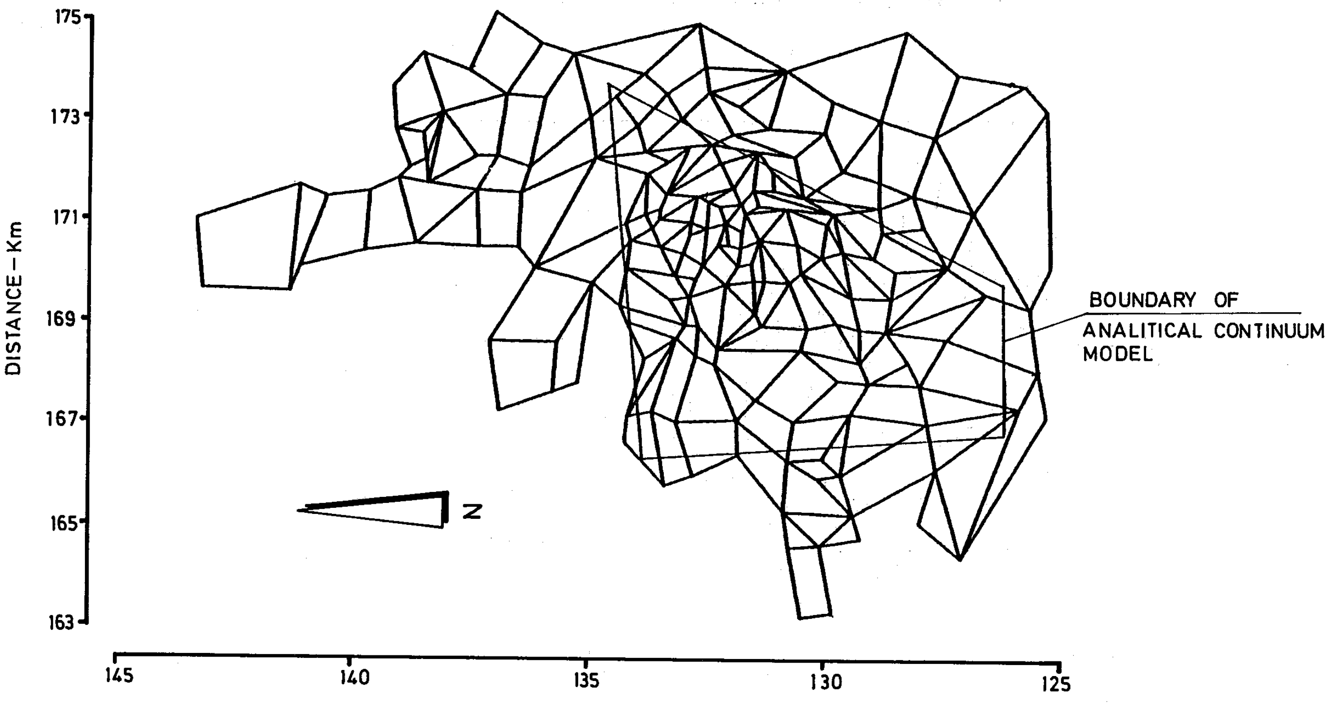

FIG. 12. Jerusalem's Element Map for Calculation by Finite Element Method

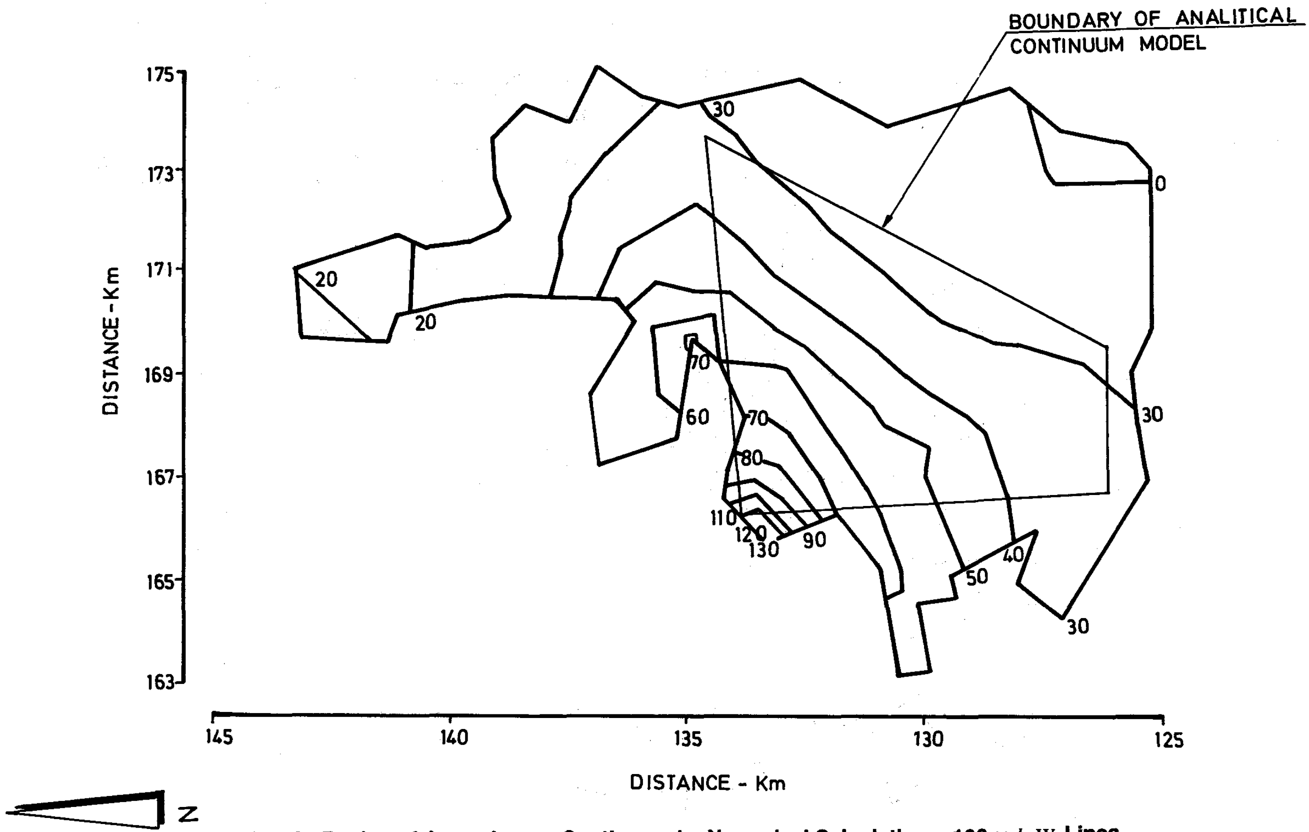

FIG. 13. Design of Jerusalem as Continuum by Numerical Calculations: 100 × $k_c W$ Lines

some diameter, one can schematize the model systematically, thereby reducing its size and increasing its efficiency. Principally, this approach should always be used when a network model is being developed, but especially for the preliminary design phase. Large reduction of model size by this method is rather tedious and therefore especially suits models that are apt to be used many times.

The continuum approach, discussed herein, is a much more drastic reduction in model complexity, and is, to the best of the writers' knowledge, a completely novel idea. It can be used in two ways:

1. To analyze the performance of an existing system under various anticipated conditions.
2. To determine the optimal pipe conductivities when the demands are imposed and a set of boundary conditions is given.

The procedures presented in this paper and the experience with two case studies, one of which was discussed therein, demonstrate that simple schematic models are viable and useful tools, which supplement the arsenal of currently available aids for engineering-economic analysis of water distribution systems.

Acknowledgments

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Appendix II. Notation

The following symbols are used in this paper:

b	=	width;
C	=	smoothness coefficient in Hazen-Williams formula;
C_0, C_d	=	cost coefficients;
D	=	diameter;
d, $\vec{d}$	=	conductivity and conductivity per unit width;
E	=	energy cost;
F	=	cost coefficient;
H, h	=	head;
I	=	investment cost;
K, K_c, K_h, K_{ct}	=	constants;
L, l	=	length;
M	=	number of pipes;
m	=	density of pipes;
n	=	friction coefficient in Manning formula;
p	=	exponent in pipe investment cost formula;
Q, q	=	discharge, discharge per unit width;
q'	=	consumption per unit area;
r	=	distance;
T, t	=	time;
t	=	tortuosity;
W	=	potential (whose gradient is $\vec{d}$);
α, β, γ	=	coefficient and exponent in flow formula;
δ	=	variable changing over time, analogous to $\vec{d}$; and
Ω	=	potential (whose gradient is δ).

Subscripts

e	=	equivalent;
i, j, s	=	index of consumption or pipe; and
$*$	=	optimal.