SCHEMATIC MODELS FOR DISTRIBUTION SYSTEMS DESIGN. I: COMBINATION CONCEPT

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ABSTRACT: This is the first of two papers dealing with schematic models for water distribution systems. Schematic models for use in preliminary design of water distribution systems are developed by two approaches: (1) A step-wise combination of elements; and (b) a nonlinear continuum representation of the system as a whole. The objective is to create models that are equivalent to a detailed network model in terms of the computed distributions of heads and flows over the area, yet are much more efficient, and therefore enhance the engineering-economic analysis. The step-wise combination method is developed beyond existing procedures to consider more complex arrangements of pipes as well as water withdrawals, which vary along the pipes and with time.

INTRODUCTION

This paper [together with its second part, which follows (Hamberg and Shamir 1988)] presents an approach and methodologies that are suitable during the preliminary phases of an engineering investigation for the analysis and design of water distribution systems.

Three phases may be distinguished in the design work: (1) Preliminary planning; (2) design; and (3) construction drawings and specifications. The objective of the first of these phases is to determine the conceptual structure of the system, i.e., how the sources are to be connected into the distribution network, the basic layout of the feeder mains, and the overall flow patterns in the network. In the second phase, the design leads to sizing the individual pipes, pumps, and reservoirs, and to a set of basic operating rules for the system under various loading conditions. These are then turned into detailed engineering drawings and construction specifications in the third phase of the work.

In this division of work, the preliminary planning and design are, respectively, analogous to the architectural planning and structural design of a building. Architectural planning examines primarily the functional aspects of the building's use, while structural design aims at implementing the plan in a safe and economic structure. In reality, these two phases of the work, in the design of buildings as well as of distribution systems, should be done in full coordination, with feedback and iterations. Nevertheless, it is instructive to identify the two phases, and, particularly in the context of this paper, to examine the tools of analysis that are most suitable for use in each.

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Note. Discussion open until August 1, 1988. Separate discussions should be submitted for the individual papers in this symposium. To extend the closing date one month, a written request must be filed with the ASCE Manager of Journals. The manuscript for this paper was submitted for review and possible publication on July 30, 1986. This paper is part of the ***Journal of Water Resources Planning and Management***, Vol. 114, No. 2, March, 1988. ©ASCE, ISSN 0733-9496/88/0002-0129/$1.00 + $.15 per page. Paper No. 22256.

Computer programs for steady-state network analysis, also called network solvers, and for simulation of its operation over time are used quite extensively (AWWA 1984; Cesario et al. 1984). Methods and programs for optimal design are also beginning to become available (Shamir 1979). These programs operate on a model of the distribution network in which its components—pipes, pumps, reservoirs, valves, and special control devices—are identified and modeled. The model can be, and usually is, schematized, but this schematization mostly amounts to no more than dropping small-diameter pipes and replacing pipes or pumps that are in parallel or in series by an equivalent pipe or pump. An attempt to develop a few further procedures for aggregating several elements into an equivalent one has been made by Gupta (1978).

These models and computer programs have been developed specifically for carrying out a detailed analysis of a planned or existing network. The writers have already advocated in the past the use of schematized models for those stages of the analysis which the detailed flow and pressure information at every point in the system is not important (Shamir 1973; Shamir and Howard 1977). Such models have been called "surrogate networks" and "grey boxes." The latter term takes off from the well-known "black box" model, which is expressed as a system response function that converts inputs into outputs without attempting to model the internal workings of the system. A "grey box" is a network model, but a model that is greatly aggregated and simplified when compared to a full ("white box"?) model of all components in the real system.

When the approach to modeling a network is examined for the purpose of aiding planners and engineers in their work, the writers' position is that for different stages of this work it may be beneficial to use different models. The models may differ in degree of detail and in their mathematical formulation and method of solution. Just as an illustration—and this is based on two actual case studies for cities with a population of 600,000 and more—a 20-node model may be used in the preliminary design and a 200-node model in the detailed design. The latter has, say, all pipes 4 or 6 in. and larger. The former has nodes, pipes, pumps, and reservoirs, but they are highly condensed functional representations of whole segments of the real system.

Now what is the advantage of using the smaller model in the preliminary design phase: With the proliferation of cheap accessible computing, why not use a fully detailed model for the preliminary design?

The principal reason for using small, simplified models is that the planners' and engineers' time and effort should be spent examining a broad range of alternatives, not on the tasks of constructing a detailed model, collecting data, calibrating, checking for errors, and analyzing voluminous computer outputs to determine what the results of various runs mean for the planning process. The writers believe this is best done with simple models, which capture the main features of the proposed system's performance without the extensive details.

Thus, the motivation for the writers' work is not computational efficiency but better planning. The writers propose to provide tools for the planning phase, a phase that has received too little attention from the developers of methodologies. The procedure proposed by Deb (1976) is an exception; he suggested a method of sizing pipes in a network, assuming that the pressure surface over the area covered by the network has a parabolic shape (whose parameters are adjusted iteratively).

With this as background, the remainder of the two papers describes specific techniques for formulating and using schematic models of water distribution systems.

Equivalent Models

Schematization of a complex model means finding a simpler one that exhibits the same performance. For water distribution systems the performance examined is in the areal and temporal patterns of heads and flows for various demands imposed on the system (herein called "loadings") and boundary conditions (e.g., fixed reservoir levels, operating condition of sources, and pumps). Thus, two models are said to be equivalent if, for all demand patterns and boundary conditions, the produce the same flow pattern and head distribution.

One way to compare two network models is to identify the most significant points, meaning the sites where one needs to know the heads, flows, or both in order to assess the performance of the system. Those significant points have to appear in both network models, and the models are said to be equivalent if the heads and flows at those points are the same in both models. Another way to compare two network models is to consider the overall areal patterns of performance.

In a specific network model the flows are associated with the links (pipes and pumps), the demands and heads with the nodes. Therefore, to compare one network model to another, the demands, heads and flows have to be released from this direct association with network elements and have to be considered continuous variables over the area covered by the network. Demands are, in reality, spread over the area, and only for modeling are they lumped and assigned to nodes. For heads, it is quite common to produce a map of constant-head lines, which describes a continuous surface. This is convenient, although not physically precise, since a head exists only along the links of the network.

For flows the continuous, areally distributed variable, which is *flow per unit width*, has to be defined; it is to be understood as follows. Straight line cuts in any arbitrary direction are taken through the map of the system. Each cut is sufficiently long to pass through several pipes, but at the same time short enough so that the average flow per unit width across the cut describes the areal flow pattern in the network. The flow per unit width, which will be referred to simply as "flow," is computed by dividing the normal component of the flow through the cut by its length. [The concept and definition of a cut is somewhat analogous to that of the "representative elementary volume" (REV) in flow through porous media (Bear 1972)].

With the continuous representation of heads, demands, and flows, we can now return to the issue of equivalence. Models of networks are said to be equivalent if, for all demand patterns and boundary conditions, they produce the same patterns of heads and flows. Obviously, "same" must be understood in the engineering context as "within acceptable accuracy."

Equivalent models have been developed by two approaches:

1. Step-wise combination of elements into equivalent ones, schematizing progressively.
2. Representing the entire network as a continuum.

The two approaches will next be explained and demonstrated: the first in this paper, and the second in the subsequent one.

Step-Wise Combination of System Elements

The hydraulic gradient along a uniform segment of pipe is

$$\frac{\Delta H}{L} = \beta\left(\frac{Q}{D^{\alpha}}\right)^{\gamma} \quad \text{(1)}$$

where ΔH = head loss between the two ends; L = length; Q = discharge; D = diameter; β is a numerical coefficient, which includes a friction coefficient and unit conversion; and α and γ are exponents. β, α, and γ take on the values shown in Table 1 for two commonly used pipe flow formulas, with ΔH and L in the same unit (e.g., meters), Q in m^3/hr, and D in mm. In Table 1, n and C are the friction and smoothness coefficients, respectively.

Another way to write Eq. 1 is

$$\frac{\Delta H}{L} = \beta \frac{Q|Q|^{\gamma-1}}{D^{\alpha\gamma}} \quad \text{(2)}$$

which automatically takes care of the dependence of the sign of ΔH on the direction of flow.

The conductance of a pipe, d, is now defined:

$$d = D^{\alpha} \quad \text{(3)}$$

and the pipe's resistance, R, is also defined:

$$R = \frac{\beta L}{D^{\alpha\gamma}} = \frac{\beta L}{d^{\gamma}} \quad \text{(4)}$$

These definitions will be used later in the derivation of the equations for combining elements.

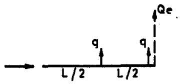
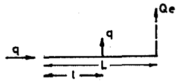
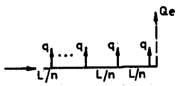
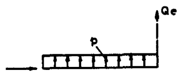
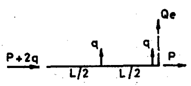
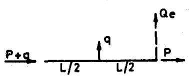
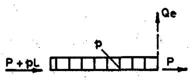
Schematization of Discharge Distribution

The value of a concentrated water withdrawal at the end of a pipe with uniform diameter, which is equivalent to other, more complex, patterns of withdrawals taken along the pipe's length, is sought. Equivalence means the same head loss between the pipe's two ends. Results for several

TABLE 1. Factors in Pipe Flow (Eq. 1)

Formula (1)	β (2)	α (3)	γ (4)
Manning	$7.9 \times 10^9 n^2$	2.67	2.0
Hazen-Williams	$1.131 \times 10^9 C^{-1.852}$	2.63	1.852

TABLE 2. Equivalent Uniform Discharge in Pipe for Several Demand Patterns, Using Manning's Equation

Case (1)	Equivalent uniform discharge (2)
	$Q_e = q\sqrt{2.5} = q^{1.58}$
	$Q_e = q\sqrt{l/L}$
	$Q_e = q\sqrt{[(n+1)(2n+1)/6]}$
	$Q_e = 0.5777\,pL$
	$Q_e + P = \sqrt{1 + 2.5(q/P)^2 + 3(q/P)}\,P$ $Q_e^2 = 2.5q^2$ for $P = 0$ $Q_e = 1.5q$ for $P \gg q$
	$Q_e + P = \sqrt{P^2 + (1/2)\,q^2 + qP}$ $Q_e = q/\sqrt{2} = 0.707q$ for $P = 0$ $Q_e = 0.58q$ for $P = q$ $Q_e = 0.5q$ for $P \gg q$
	$Q_e + P = \sqrt{P^2 + (1/3)\,(pL)^2 + (pL)P}$ $Q_e = L/\sqrt{3} = 0.5777pL$ for $P = 0$ $Q_e = 0.53pL$ for $P = pL$ $Q_e = pL/2 = 0.5pL$ for $P \gg pL$
	$Q_e - P = q\sqrt{(P^2/q^2) + [(n+1)/2]\,\{[(2n+1)/3] + (2P/q)\}}$ $Q_e^2 = [(n+1)(2n+1)/6]\,q^2$ for $P = 0$ $Q_e = [(n+1)/2]q$ for $P \gg q$

important practical cases, derived from Manning's flow formula, appear in Table 2. Somewhat different equations result from use of the Hazen-Williams formula. Col. 1 in Table 2 shows schematically for each case the distribution of withdrawals and flows along a pipe of length L and the location of the equivalent withdrawal, Q_e, at the end of the pipe. The equation for Q_e is given in Col. 2.

These results demonstrate that the common procedure of dividing withdrawals taken along a pipe equally between its two ends is only valid for special cases.

Other withdrawal patterns can be treated by superposition of the simpler cases included in Table 2, or by carrying out the analysis of the particular case. The superposition approach will be discussed later.

Schematization of Structure

For m pipes in series with conductances d_i, an equivalent pipe has a conductance, d_e, given by

$$d_e^{-\gamma} = \sum_{i=1}^{m} d_i^{-\gamma} \quad (5)$$

and length $$L_e = \sum_{i=1}^{m} L_i \quad (6)$$

The equivalent pipe for m pipes in parallel has a conductance given by

$$d_e = \sum_{i=1}^{m} d_i \quad (7)$$

and length $$L_e = L_i = \text{constant} \quad (8)$$

This assumes that all pipes have the same roughness. If not, pipes can be "normalized" to an equivalent diameter of a chosen uniform roughness. The same can be done for parallel pipes of unequal length, to convert them to equivalent diameters for length L_e.

Eqs. 5–8 assume a uniform discharge throughout the pipe length, just as the discharge schematization equations in Table 2 assumed a uniform diameter along the pipe.

Superposition

Take as an example the case of two pipes in series with different resistances (as defined by Eq. 4) $R_1 \neq R_2$, and different discharges, $Q_1 \neq Q_2$, resulting from the loadings (consumptions), q_1 and q_2, as shown in Fig. 1.

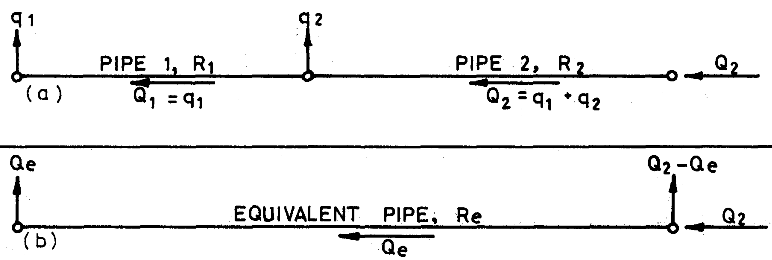


FIG. 1. Equivalent Pipe: (*a*) For Two Pipes in Series with Different Flows; (*b*) Equivalent Pipe

$$Q_e\,|Q_e|\,R_e = Q_1\,|Q_1|\,R_1 + Q_2\,|Q_2|\,R_2 \quad \text{(9a)}$$

where Q_e is an equivalent discharge, which can be chosen arbitrarily. If $Q_e \doteq Q_1$ is selected, then:

$$R_e = R_1 + \frac{Q_2\,|Q_2|}{Q_1\,|Q_1|}\,R_2 = R_1 + R_2\left[1 + 2\left(\frac{q_2}{q_1}\right) + \left(\frac{q_2}{q_1}\right)^2\right]$$

$$\cdot[\text{sign}\,(q_1 + q_2)] \quad \text{(9b)}$$

where the last term determines the appropriate sign of the second term depending on the directions of flows (defining the direction of Q_1 and q_1 as the positive direction). A negative R_e means the head loss is in the opposite direction of Q_e (i.e., Q_1) and one should change the selection of Q_e.

Next, an integral along a pipe in which the flow, Q, and resistance, R, vary from point to point is defined:

$$A_{Q^2} = \int Q^2\, dR \quad \text{(10)}$$

where Q = flow at each point along the pipe, and dR = incremental resistance at that point.

A_{Q^2} is the area under the curve of Q^2 versus R along the system [the two pipes in Fig. 1(*a*)]. $A_{q_i^2}$ is the A that would result should only q_i exist; q_i might be regarded as analogous to a force loaded upon a beam at one point, while Q and H along the whole system result from it as the moments and shear forces result from the loading and the force. $A_{q_1q_2}$ is the A resulting from a loading of $\sqrt{q_1q_2}$ (zero along a segment where only q_1 or q_2 produces discharge).

For the case of Fig. 1(*a*), $A_{q_1^2} = (R_1\, q_1^2) + (R_2\, q_1^2)$, $A_{q_2^2} = (R_1 \times 0) \neq (R_2\, q_2^2)$, and $A_{q_1q_2} = (R_1 \times 0) + (R_2\, q_1\, q_2)$. The equivalent resistance for two loadings, as in Fig. 1(*a*) and Eq. 9, can be expressed as:

$$R_e = \frac{A_{Q_e^2}}{Q_e^2} = \frac{1}{Q_e^2}\,(A_{q_1^2} + A_{q_2^2} \pm 2A_{q_1q_2})\cdot[\text{sign}\,(q_1 + q_2)] \quad \text{(11)}$$

in which q_1 and q_2 may be any two distributions of consumptions (i.e., two loadings) along the system (e.g., equally distributed consumption) and not only concentrated consumption, as in Fig. 1(*a*).

A is negative if q_1 and q_2 are in opposite senses. Sign $(q_1 + q_2)$ in Eq. 11 means that the A at each segment of the system should have the sign of the total discharge at that segment (not the sign of the specific loading for which A is calculated). Should, for example, $q_2 < 0$ and $|q_2| > |q_1|$ then, for Fig. 1(*a*):

$$A_{q_2^2}\ \text{sign}\,(q_1 + q_2) = R_1 \cdot 0 - R_2\, q_2^2 = -R_2\, q_2^2 \quad \text{(12a)}$$

$$A_{q_1^2}\ \text{sign}\,(q_1 + q_2) = R_1 q_1^2 - R_2\, q_1^2 \quad \text{(12b)}$$

$$A_{q_1q_2}\ \text{sign}\,(q_1 + q_2) = R_1 \cdot \bar{0} - R_2\, q_1 q_2 = +R_2\, q_1\, |q_2| \quad \text{(12c)}$$

R increases linearly with the pipe length if the diameter, D, and therefore the conductance, d, remain constant.

In the special (but common) case of a pipe with constant diameter (constant R) with a complicated distribution of consumptions, the superposition is simpler. One may quite easily obtain the equivalent discharge of the sixth case of Table 2 by superposition of the second case and a consumption P at the end of the pipe. Note that even though Q_e and P are at the same point, Q_e at cases 2 and 6 is not the same. While deriving, in a similar way, the seventh case, note that the third term on the right-hand side is twice the result of an integral of Ppx over the length x, divided by L (see Eqs. 10 and 11).

Stepwise Schematization of More Complex Systems

A real system includes more complex combinations than pipes in parallel and in series. Transformations have been developed (Hamberg 1984) for several other "primitives," but only a few will be presented here in detail.

An important transformation is from a "triangle," i.e., a loop made of three pipes [Fig. 2(*a*)] to a "star" (also called a *T*), i.e., three pipes that meet at a central node [Fig. 2(*b*)]. This is a transformation commonly used in the analysis of linear electric systems. Because hydraulic systems are not linear, the transformation is not universal and depends on the ratios of flows in the three pipes. This dependence on the flows is not very pronounced, and in many cases it suffices to know only the ranges of the flows in order to obtain an adequate transformation.

A triangular loop [Fig. 2(*a*)] with any pipe properties, which is part of a larger system, is first converted into a star with three equal pipes plus an additional segment attached to one of them [Fig. 2(*b*)]. Later Fig. 2(*b*) can be transformed into simpler equivalent systems.

The systems shown in Figs. 2(*a*) and (*b*) are said to be equivalent if the flows and heads at points A, B, and C of Fig. 2(*a*) are the same as at points A, B, and C of Fig. 2(*b*).

For the special case of $Q_A = 0$, one gets a conventional combination of pipes in series and in parallel, so that

$$\frac{1}{R} = 2\left(\frac{1}{\sqrt{R_3}} + \frac{1}{\sqrt{R_1 + R_2}}\right)^2 \qquad (13)$$

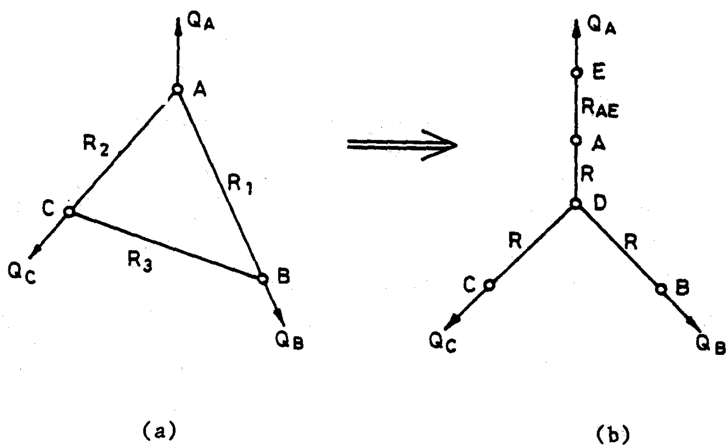


FIG. 2. Transformation of: (*a*) Triangle to (*b*) Star

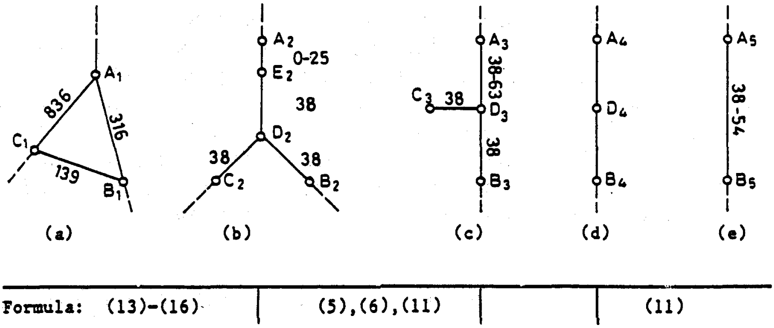


FIG. 3. Steps in Transformation of Triangle into Equivalent Pipe Segment (Dashed Line Represents Connection to Rest of System): (*a*) Any Triangle; (*b*) Equilateral Star Plus "Remainder"; (*c*) Main Line Plus Branch; (*d*) Two Pipes in Series; (*e*) Single Pipe

If the flow ratios can take any value, then the transformation of a triangle with R_1, R_2, and R_3 into an equilateral star with three equal resistances, R would correspond to something in between the results for three extreme cases, each with one of the three discharges having a value of zero. Eq. 14 assumes the transformation is calculated as the average of the results for $Q_A = 0$, $Q_B = 0$, and $Q_C = 0$, each calculated analogous to Eq. 13.

$$\frac{1}{R} = \frac{2}{3} \sum_{i=1}^{3} \left[\frac{1}{\sqrt{R_i}} + \frac{1}{\sqrt{\left(\sum_{j=1}^{3} R_j\right) - R_i}} \right]^2 \qquad (14)$$

For a specific value (or range of values) of flow ratio, one may need to add a "remainder" segment to the equilateral star [see AE in Fig. 2(*b*)]. Such a remainder is not needed in the three special cases of which Eq. 14 is the average.

Should $Q_A = Q_C$ in Fig. 2(*a*), and $R_1 > R_3$, then

$$R = 0.2 \frac{R_3(2R_1 + G)^2}{(R_3 + R_1 + G)^2} \qquad (15a)$$

$$R_{AE} = \frac{R_2(R_3 - R_1)^2}{(R_3 + R_1 + G)^2} \qquad (15b)$$

where G is defined by

$$G^2 = 4R_1R_3 + R_1R_2 - R_2R_3 \qquad (16)$$

Fig. 3 shows a progression of steps in the transformation of a triangle into an equivalent single-pipe segment. The figures noted near the pipe segments of Fig. 3 represent examples for values of R, where one assumes $Q_A/Q_C = 0$ to 1 (i.e., one knows only the direction of flows and $Q_c > Q_a$). The values of R in Fig. 3(*a*) correspond to $D_1 = 300$ mm, $D_2 = 250$ mm, $D_3 = 350$ mm (12, 10, and 14 in., respectively), considering all $L_i = 500$ m and all $n_i = 0.010$.

The conversion between Figs. 3(*a*) and (*b*) should be performed by Eq. 14, in general, when nothing is known about the discharges. If a range of flows can be estimated in advance, as in the example, then a more accurate equation can be used, like Eq. 13, 15, or 16, which were used for the two extreme cases of the example.

The system in Fig. 3(*c*) is similar to the one in Fig. 3(*b*), except that segment C_3 is viewed as a branch at the junction of two unequal pipes (because A_2E_2 and E_2D_2 have been converted, using Eqs. 5 and 6, into a single segment A_3D_3). The system in Fig. 3(*c*) is converted into the one in Fig. 3(*d*), which consists of two pipes in series plus a withdrawal at D_4. The head at C_3 is no longer one of the system variables, and now only the heads at A_4 and B_4 are the same as at A_1 and B_1. The final step is to convert the system in Fig. 3(*d*) into a single pipe segment as in Fig. 3(*e*), using Eq. 11.

The result of these steps is a series of equations for the properties of pipe A_5B_5 and the flow in it, expressed in terms of pipe properties and flows in the triangle [Fig. 3(*a*)], for which the heads at A_5 and B_5 are the same as at A_1 and B_1.

One could also transform Fig. 3(*c*) into Fig. 3(*d*), which would consist of $A_4C_4B_4$ (instead of $A_4D_4B_4$). This transformation is not shown here. C_4 would have the same head as C_1, while $A_4C_4 \neq A_3D_3$ and $C_4B_4 \neq D_3B_3$. If all three points (A,B,C) are significant, there is no use in further transforming into Fig. 3(*e*). Segment C_3 may represent a combination of segment C_2D_2 and the rest of the system connected to it. The transformation into C_3 is then performed using Eqs. 5 and 6 (or rather Eq. 11 if the discharge varies along the branch). One may proceed with the schematization the same way, combining into Fig. 3(*e*) all the system connected to A and B.

By the example, one may see that even for a large range of possible ratios of discharge (which relates to an even larger range of discharges), the equivalent R varies only between 38 and 54 (e.g., 46). If the anticipated Q_A could be narrowed to not larger than half of Q_C (and in the same direction), the equivalent R would be in the range of 38 to 49 (e.g., 44). This means that narrowing the range of ratios of discharge by a factor of 2 changes the approximate equivalent R by only 5%, i.e., the transformation is not very sensitive to flow assumptions.

Schematization of Time-Varying Discharge

When the discharge in a pipe varies over time, an equivalent constant discharge that will produce the same loss of energy over the entire time span of interest is sought. Consider, e.g., discharge that varies linearly from Q_1 at time t_1 to Q_2 at t_2, in a uniform pipe segment. The discharge may be increasing or decreasing, and the time span $(t_2 - t_1)$ can be of any length. Using the Hazen-Williams formula for the energy gradient in the pipe, the equivalent discharge is

$$Q_e = 0.623\left(\frac{Q_1^{3.852} - Q_2^{3.852}}{Q_1 - Q_2}\right)^{0.35} \quad \text{(17)}$$

This expression is derived by integrating $Q_1 + [(Q_2 - Q_1)/(t_2 - t_1)]^{2.952} \cdot t$ over t. More complex cases are analyzed in a similar way by Hamberg (1984).

Conclusions

Feasibility of Step-Wise Combination Approach

It is clear that the step-wise combination approach is feasible only for reducing somewhat the size and detail of a network model but cannot be applied in practice to all parts of an entire system.

This is due to the considerable amount of work that is needed to reach a substantially reduced model size by this approach and because some of the equivalence transformation equations depend on the relative magnitude of flows in network elements. Therefore, this approach is more suitable for existing networks than for planned ones. A model of an existing network is used to plan additions and modifications, such as the introduction of a new source or addition of major storage. A model of an existing system is also needed when studying its operation or implementing on-line control. The latter cases are outside the scope of this paper, but it is useful to keep in mind this additional possible use of schematized network models.

To summarize, the step-wise combination approach should be used to reduce the size and complexity of a network model, typically by removing, say, 10–50% of the elements.

In contrast with the step-wise combination approach, which was discussed herein and which looks in detail at individual elements and assembles them into a combined model, the following paper proposes to look at the system as a whole, as a *continuum*. This can be viewed, pictorially, as standing back from the system map far enough to see only the overall flow pattern and hydraulic head map, without taking interest in the individual system elements.

Acknowledgments

The research reported herein was performed as a Ph.D. thesis of Hamberg. The work was enabled by the support of Tahal, Water Planning for Israel, and consulting engineers.

Appendix I. References

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Appendix II. Notation

The following symbols are used in this paper:

A = area under curve of Q^2 versus R;
C = smoothness coefficient in Hazen-Williams formula;
D = diameter;
d = conductivity;
E = energy cost;
F = cost coefficient;
H = head;
L, l = length;
m = number of consumptions or pipes;
n = friction coefficient in Manning formula;
P = discharge (outflow at end of pipe);
p = consumption linearly distributed along pipe;
Q = discharge;
q = consumption;
R = resistance of pipe;
t = time; and
α, β, γ = coefficient and exponent in flow formula.

Subscripts

e = equivalent; and
i, j = index of consumption or pipe.