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MOTION OF THE SEAWATER INTERFACE IN A COASTAL AQUIFER BY THE METHOD OF SUCCESSIVE STEADY STATES

J. BEAR[1], U. SHAMIR[2], A. GAMLIEL[2] and A.M. SHAPIRO[1]

[1] *Faculty of Civil Engineering, Technion, Haifa 32000 (Israel)*
[2] *Department of Geology, Kent State University, Kent, OH 44242 (U.S.A.)*

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ABSTRACT

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The paper describes a method for determining the motion of the seawater interface in a phreatic coastal aquifer during a specified time period in response to changing hydrologic conditions, provided a seaward flow of fresh water is maintained everywhere above the interface. The method of successive steady states is used, leading to an approximate analytical expression which gives the motion of the interface toe during the time period as a function of the initial conditions and the change in freshwater flow to the sea above the toe during the time period. Sensitivity of the results to values of parameters and to hydrologic conditions is investigated, and the results are compared with those obtained by other methods of analysis.

The motivation for developing the approximate analytical expression for movement of the interface was to facilitate the introduction of seawater intrusion as a criterion in the multiobjective management model for coastal aquifers.

1. INTRODUCTION

In order to incorporate seawater intrusion as a criterion, and/or as a constraint, in the optimal management of a coastal aquifer, an explicit expression is required for the movement (advance or retreat) of the interface during a specified time interval, in response to variations in water balance components. These may include planned activities, e.g., pumping and/or artificial recharge, and stochastic inputs, such as natural replenishment. The objective of the present paper is to develop such an expression.

Under certain conditions, an equilibrium may exist, with a stationary interface and a certain seaward freshwater flow above the interface. Then, as this flow varies (e.g., by increasing pumping landward of the interface toe), changes will be produced in the shape and position of the interface. This is a very simplified description of what happens in reality. Actually, in an

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exploited aquifer, and especially during a development period when pumping increases continuously, the interface is never stationary. It is always in motion, depending on changes introduced in components of the (fresh) water balance and hence in the seaward freshwater flow above the interface.

We consider here a simplified model of a sharp interface, separating fresh water from seawater, although in reality, a transition zone always exists between the two zones due to hydrodynamic dispersion.

Theoretically, the flow in each of the two zones — the one occupied by fresh water and the other by seawater — is three dimensional in nature. Close to the coast, the freshwater zone overlies the salt-water one, with the moving interface serving as a boundary between the two zones. Further inland we have a zone of fresh water only.

Often the upper boundary of the freshwater zone is a phreatic surface with accretion. Under such conditions, the mathematical statement of the flow problem leading to the determination of the shape and position of the moving interface is *nonlinear*. The solution is difficult even when numerical methods are employed (Shamir and Dagan, 1971; Pinder and Page, 1976; Sa da Costa and Wilson, 1979; Mercer et al., 1980).

In view of the fact that the geometry of both zones is such that the thickness of each zone is much smaller than the horizontal lengths of interest, the flow in the two zones may be considered as essentially horizontal. This enables the application of the hydraulic approach to the flow in both zones (Bear, 1979). A great simplification is then achieved in the statement (and in the solution) of the problem. Obviously, the error introduced is larger in regions where the basic assumption of "essentially horizontal flow" fails. However, in spite of the simplification introduced by the hydraulic approach (which actually eliminates the phreatic surface and the interface as boundaries of the flow domains), we are still faced with a rather difficult problem due to the movement of the interface. Bear (1972, 1979) reviews the problems briefly described above.

Although numerical solutions of interface movement based on the hydraulic approach are available, they cannot yield an explicit relationship between movement of the interface and known flow parameters, e.g., freshwater seaward flow above the toe of the interface. Hence a further simplification is introduced.

It is assumed that in the vicinity of the coast, the flows in both the freshwater zone and the salt-water one are essentially along a line perpendicular to the coast, and that above the interface itself the flow is always seaward.

2. MOTION OF THE INTERFACE

Fig. 1 is a schematic section through a phreatic coastal aquifer, bounded from below by a horizontal impervious layer, and recharged from above by natural replenishment, N. Seaward flows, Q_f and Q_s of fresh water and

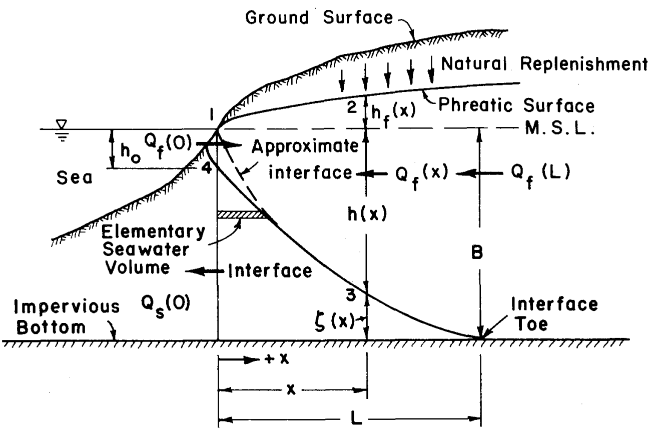


Fig. 1. Schematic cross-section of a phreatic coastal aquifer.

seawater, respectively, per unit width, are considered positive. The elevation of the phreatic surface above sea level is denoted by h_f; h is the depth from the same datum to the interface (where it exists); B is the depth to the aquifer bottom; and L is the intrusion length — the distance from the coast line to the toe of the interface.

The following analysis is restricted to cases where the phreatic surface rises monotonously over the intrusion length. This means that the freshwater flow is everywhere seaward in this region.

Assuming a steady-state freshwater balance for a control volume (*1—2—3—4* in Fig. 1) extending from the coast to a distance x:

$$Q_f(x) + Nx - Q_f(0) = 0 \tag{1}$$

where $Q_f(0)$ is the flow to the sea at $x = 0$. We adopt the Dupuit approximation (of essentially horizontal flow) and the Ghyben—Herzberg relation associated with it:

$$h_f = [(\gamma_s - \gamma_f)/\gamma_f]h = h/\delta \tag{2}$$

Eq. 1 then becomes:

$$\frac{K(1+\delta)}{\delta^2} h \frac{\partial h}{\partial x} + Nx - Q_f(0) = 0 \tag{3}$$

where K is the hydraulic conductivity.

Integrating eq. 3, and using the boundary condition $h(0) = h_0$:

$$-2Q_f(0)x + Nx^2 + K(1+\delta)\delta^{-2}[h^2(x) - h_0^2] = 0 \tag{4}$$

Hence:

$$x = N^{-1}Q_f(0) - N^{-1}[Q_f^2(0) - NK(1+\delta)\delta^{-2}\{h^2(x) - h_0^2\}]^{1/2} \tag{5}$$

This equation describes the parabolic shape of the interface in steady state and the dependence of this shape on $Q_f(0)$. The other solution of eq. 4, namely the one with a plus sign in front of the term in square brackets (the square root) in eq. 5, is impossible physically; it corresponds to the interface rising and the phreatic surface falling landward.

In the following, we shall assume $h_0 = 0$, which is justified in most practical circumstances. With

$$A = KN(1+\delta)/\delta^2$$

we get:

$$x = N^{-1}[Q_f(0) - \{Q_f^2(0) - Ah^2(x)\}^{1/2}] \tag{6}$$

From $h = B$ at $x = L$ (= the toe of the interface), we obtain for the intrusion length the expression:

$$L = N^{-1}[Q_f(0) - \{Q_f^2(0) - AB^2\}^{1/2}] \tag{7}$$

which relates the intrusion length — in steady state — to the freshwater flow to the sea. When $N = 0$, $Q_f(L) = Q_f(0)$. By integrating eq. 4 with $N = 0$, we obtain the known result (e.g., Bear, 1979, p. 396):

$$L = K\frac{1+\delta}{2\delta^2}\,\frac{B^2}{Q_f(L)} \tag{8}$$

For the limit case $Q_f(L) = 0$ and $N \neq 0$, then $Q_f(0) = NL$, and we obtain from eq. 7:

$$L = [K(1+\delta)N^{-1}]^{1/2}(B/\delta) \tag{8a}$$

To obtain the motion of the interface, we start with the volume of seawater below the interface, given by:

$$V_{sw} = n\int_{h_0}^{B} x\,\mathrm{d}h \tag{9}$$

where n is the porosity. The integration is performed with horizontal elementary volumes of height dh each, extending from $x = 0$ to $x(\zeta)$ as shown in Fig. 1. Assuming $h_0 = 0$ and using the expression for x from eq. 6, we obtain:

$$V_{sw} = \frac{n}{N}\left[Q_f(0)B - \frac{1}{2}B\{Q_f^2(0) - AB^2\}^{1/2} - \frac{Q_f^2(0)}{2A^{1/2}}\sin^{-1}\frac{BA^{1/2}}{Q_f(0)}\right] \tag{10}$$

A check of this result is obtained by observing that for $N = 0$, eq. 9 integrates to the expression given by Vappicha and Nagaraja (1976):

$$V_{sw} = \frac{nK(1+\delta)}{6Q_f(0)\delta^2}(B^3 - 3h_0^2 B + 2h_0^3) \tag{11}$$

Following Henry (1959), if we use:

$$Q_f(0) = Kh_0/\beta\delta$$

and hence

$$h_0 = \beta\delta Q_f(0)/K \tag{12}$$

where β is a numerical coefficient (equal to 0.741 according to Henry, 1959), we obtain:

$$V_{sw} = \frac{n\beta(1+\delta)}{6\delta h_0}(B^3 - 3h_0^2 B + 2h_0^3) \tag{13}$$

Bear et al. (1980) also develop an expression for V_{sw} for the case $h_0 \neq 0$.

The above discussion is related to steady state. Next, consider the motion of the interface due to changes in freshwater flow.

We express the seawater flow into the aquifer as a function of the freshwater flow to the sea:

$$Q_s(0) = -\frac{dV_{sw}}{dt} = -\frac{dV_{sw}}{dQ_f(0)}\frac{dQ_f(0)}{dt} \tag{14}$$

Substituting eq. 10 into eq. 14 leads to:

$$Q_s(0) = -\frac{n}{N}\left[B - \frac{Q_f(0)}{A^{1/2}}\sin^{-1}\frac{BA^{1/2}}{Q_f(0)}\right]\frac{dQ_f(0)}{dt} = F[Q_f(0)]\frac{dQ_f(0)}{dt} \tag{15}$$

where F is defined by eq. 15.

If we allow $h_0 \neq 0$ and follow the whole development, we get:

$$Q_s(0) = -\frac{n}{N}\left[(B-h_0) - \frac{Q_f(0)}{A^{1/2}}\sin^{-1}\frac{(B-h_0)A^{1/2}}{Q_f(0)}\right]\frac{dQ_f(0)}{dt}$$

$$= -\frac{n}{N}F[Q_f(0)|_{h_0 \neq 0}]\frac{dQ_f(0)}{dt} \tag{15a}$$

The difference between eq. 15 and eq. 15a in $Q_s(0)$ is less than ~5% for the example where $B = 102$ m, $h_0 = \beta\delta Q_f(0)/K = 334$ m, $K = 8395$ m yr.$^{-1}$, $A = 8413$ m^2 yr.$^{-2}$ and $N = 0.536$ m yr.$^{-1}$.

Consider now the motion of the interface during a time period Δt, as shown in Fig. 2. At time t the interface is PA, and at time $(t + \Delta t)$ it has moved to its new position, PCB. At the same time, the phreatic surface has moved from ODE to OFG. We now formulate the continuity equation for the control volume OPACFD, under the following simplifying assumptions:

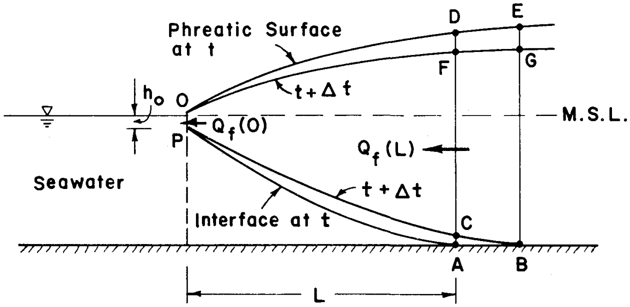


Fig. 2. Motion of the interface and the phreatic surface during t.

(1) The volume of the wedge of seawater, ACB, is negligible, compared to the total volume of fresh water in the control volume, due to the small distance of interface advance, AB, during Δt.

(2) The volume EDOFG, due to the change in the phreatic surface, is negligible compared to the total volume of fresh water in the control volume [recalling that (FD) = (AC)/δ].

The continuity equation is:

$$Q_s = Q_f(L) + NL - Q_f(0) \tag{16}$$

Using eq. 15, this becomes:

$$F(Q_f(0))\frac{dQ_f(0)}{dt} = Q_f(L) + NL - Q_f(0) \tag{17}$$

At time t, L and $Q_f(0)$ are known. Then, as a result of a new value $Q_f(L)$ starting at time t, and for a replenishment N during Δt, the value of the freshwater discharge at the coast line at the end of the time period, $t + \Delta t$, is:

$$\begin{aligned} Q_f^{t+\Delta t}(0) &= Q_f^t(0) + \frac{[Q_f^{t+\Delta t/2}(L) + NL^t - Q_f^t(0)]}{F[Q_f^t(0)]}\Delta t \\ &= Q_f^t(0) + \Delta Q_f^t(0) \end{aligned} \tag{18}$$

$Q_f^{t+\Delta t/2}(L)$ is the representative value of $Q_f(L)$ for the time step (e.g., the average over Δt). Introducing the new value of $Q_f(0)$ into eq. 7 yields the new location of the toe:

$$L^{t+\Delta t} = N^{-1}\ [Q_f^{t+\Delta t}(0) - \{(Q_f^{t+\Delta t}(0))^2 - AB^2\}^{1/2}] \tag{19}$$

Thus, the new location of the toe can be computed for given initial conditions (at time t) and the new flow of fresh water above the toe.

One should recall that the freshwater flow above the toe is the outcome of inputs and outputs in the coastal aquifer landward of this point. Thus, this new flow incorporates in it the planned pumping and artificial recharge (which are the decision variables in the management problem), and the (assumed) natural replenishment.

This result for the new location of the interface can be used quite easily in computer programs for forecasting and simulating aquifer behaviour. Since it is nonlinear, it can be used in models for optimal management only if the model uses a nonlinear optimization technique. To be used in a linear programming management model eq. 19 may be linearized, as follows.

With $h_0 = 0$ and $h(L) = B$, eq. 4 becomes:

$$NL^2 - 2Q_f(0)L + K(1+\delta)\delta^{-2}B^2 = 0 \tag{20}$$

Considering $Q_f(0)$ and L to be functions of time and differentiating with respect to t, leads to:

$$2NL\frac{dL}{dt} - 2L\frac{dQ_f(0)}{dt} - 2Q_f(0)\frac{dL}{dt} = 0 \tag{21}$$

from which:

$$\frac{dL}{dt} = \frac{L}{NL - Q_f(0)}\,\frac{dQ_f(0)}{dt} \cong \frac{L^{t+\Delta t} - L^t}{\Delta t} \tag{22}$$

Using this result in eq. 17 yields:

$$L^{t+\Delta t} = L^t[1 + \Delta Q_f^t(0)/\{NL^t - Q_f^t(0)\}] \tag{23}$$

which is a *linear* relation between the new location of the interface and the known quantities L^t, $Q_f^t(0)$ and $Q_f^{t+\Delta t/2}(L)$, with $\Delta Q_f^t(0)$ given in eq. 18.

To summarize, the following assumptions have been made;

(a) Flow is everywhere along a line perpendicular to the coast.

(b) The Dupuit approximation of essentially horizontal flow is valid.

(c) The Ghyben—Herzberg law is used. The error due to this approximation is largest when there is a water divide of fresh water above the interface, and freshwater flows both sea- and landward. Such situations cannot exist as a steady state. The error is due to the fact that in the region landward of the water divide, the depth to the interface, h in Fig. 1, increases, while the height of the phreatic surface, h_f, decreases — contrary to the Ghyben—Herzberg condition. Thus, our results hold for $Q_f(x) \leqslant 0$ for $0 \leqslant x \leqslant L$, i.e. the flow over the interface toe is always seaward.

(d) $h_0 = 0$. The error due to this approximation can be estimated from comparing $(h^2 - h_0^2)^{1/2}$ with $(h^2)^{1/2}$ for realistic values of the two variables. For $h = 100$ m and say, $h_0 = 2$ m and $= 10$ m, the ratio of the first to the second expression is 0.9998 and 0.9949, respectively.

(e) Neglecting the volume of fresh water in the control volume (Fig. 2) due to changes in the phreatic surface. To assess this approximation consider a change of 10 cm in the phreatic surface at the toe of an interface which intrudes 1500 m inland. For a porosity $n = 0.25$ the incremental volume (per 1 m coast) is approximately $\frac{1}{2} \times 1500 \times 0.1 \times 0.25 \simeq 19\,m^3$, as compared to a change of $150\,m^3$ due to the change in interface location (see Table I). This amounts to some 12.5% — not completely negligible, but acceptable.

(f) Neglecting the wedge of seawater, ABC (Fig. 2). For an initial intrusion length of 1500 m in a 100-m thick aquifer and a 20-m motion of the toe, the neglected volume is only ~ 0.01% of the total seawater volume.

3. COMPARISON WITH A NUMERICAL SOLUTION

Because of lack of field data we can only compare with a numerical solution. The numerical model by Shamir and Dagan (1971) was somewhat modified by Kapuler (1972). Recently, an improved version has been developed and tested (Shapiro et al., 1983), and it is to this model that we compare the results of the nonlinear and the linear successive-steady-states methods, eqs. 19 and 23.

Five cases have been examined. They have been chosen to represent typical conditions found in the coastal aquifer of Israel (run *5*) and variations of certain parameters to test the sensitivity of the results. The basic data are:

Aquifer thickness		$B = 102\,m$
Porosity (effective)		$n = 0.25$
Net recharge		$N = 0.336\,m\,yr.^{-1}$
Hydraulic conductivity:	Runs *1*, *3*, *4* and *5*	$K = 8395\,m\,yr.^{-1}$ ($= 23\,m\,day^{-1}$)
	Run *2*	$K = 839.5\,m\,yr.^{-1}$
For $\gamma_s = 1.025$,		$\delta = \gamma_f/(\gamma_s - \gamma_f) = 34.5$

The results are presented in Fig. 3 and summarized in Table I.

The continuous curves in Fig. 3 have been computed with $\Delta t = 0.01$ yr. = 3.65 days (see eq. 18). The nonlinear (eq. 19) and linear (eq. 23) expressions give essentially the same results, which are shown by the dashed lines in Fig. 3.

Table I contains the data on the initial condition, which is a steady-state condition with a balance over the intrusion length (L^0) between the freshwater flow above the toe ($Q_f^0(L)$), the freshwater flow to the sea ($Q_f^0(0)$), and the amount of recharge above the length of the interface (NL^0). A sudden change to $Q_f^t(L)$ is made in the freshwater flow at the toe. The percentage changes are also indicated, as well as the steady-state intrusion length, L^∞, corresponding to this new flow.

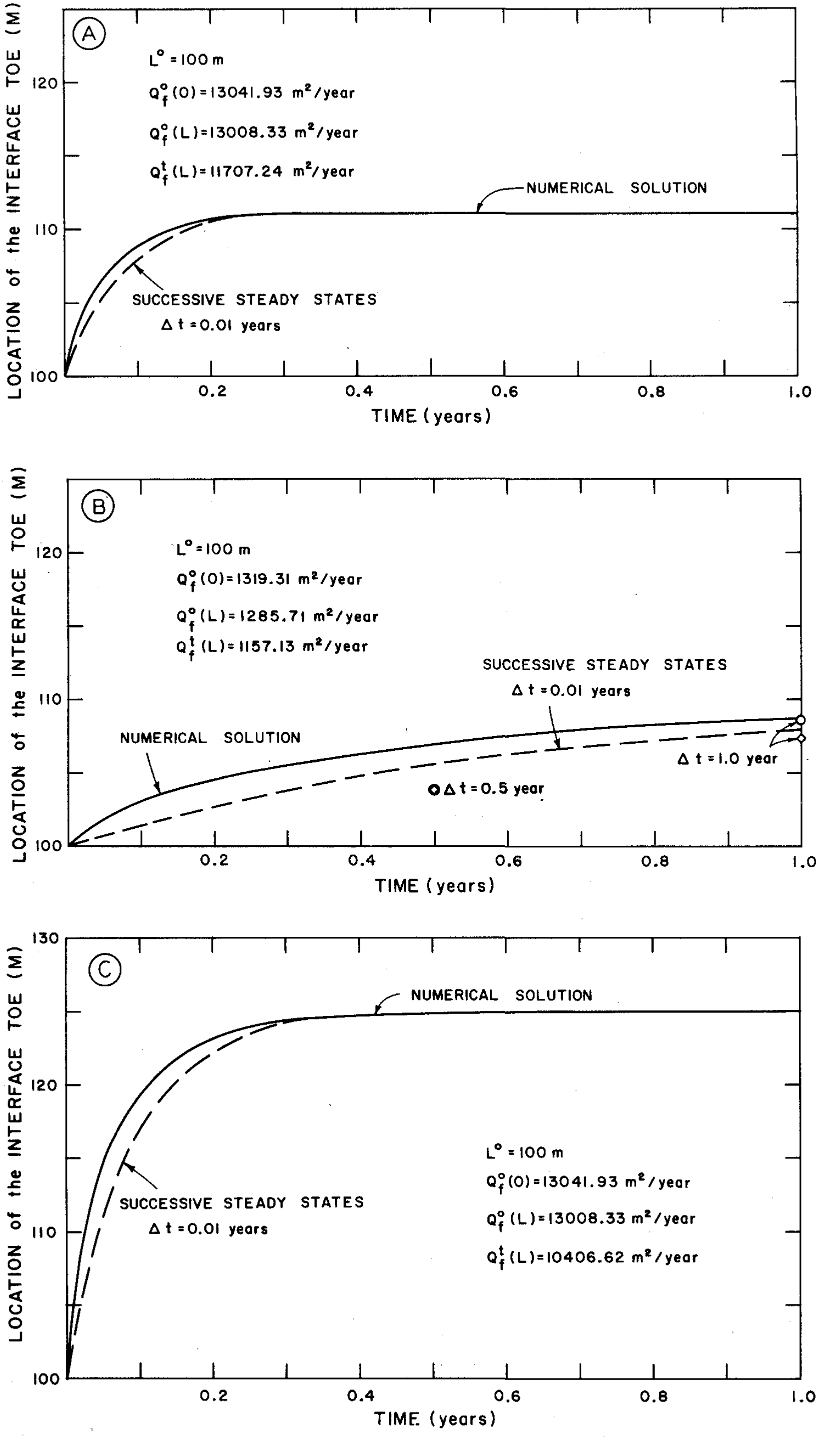
Results are compared for $t = 0.5$ and $= 1$ yr. Five values are given in each case: L_1 = numerical solution: L_2 = successive-steady-states nonlinear equation (19), computed with steps of $\Delta t = 0.01$ yr.; L_3 = successive-steady-states linear equation (23), computed with steps of $\Delta t = 0.01$ yr.; L_4 and L_6 = successive-steady-states nonlinear equation (19), computed with one step, $\Delta t = 0.5$ yr. and $\Delta t = 1$ yr., respectively; L_5 and L_7 = successive-steady-states linear equation (23), computed with one step, $\Delta t = 0.5$ yr. and $\Delta t = 1$ yr., respectively.

TABLE I

Comparison of the linear and non-linear successive steady-states results with a numerical solution

Run	Initial intrusion length, L^0 (m)	Freshwater flows before change to sea $Q_f^0(0)$ (m^2 yr.$^{-1}$)	Freshwater flows before change at toe $Q_f^0(L)$ (m^2 yr.$^{-1}$)	Freshwater flows after at toe $Q_f^t(L)$ (m^2 yr.$^{-1}$)	Change in freshwater flow at the toe (%)	Steady-state intrusion for $Q_f^t(L)$, L^∞ (m)	Location of the toe, computed by various methods*, after 0.5 yr. L_1 (m)	0.5 yr. L_2 (m)	0.5 yr. L_3 (m)	0.5 yr. L_4 (m)	0.5 yr. L_5 (m)	1 yr. L_1 (m)	1 yr. L_2 (m)	1 yr. L_3 (m)	1 yr. L_6 (m)	1 yr. L_7 (m)
1	100	13,041.93	13,008.33	11,707.24	10	111.08	110.08	110.08	110.09	162.20	138.41	110.09	110.09	110.09	434.16	176.63
2	100	1,319.31	1,285.71	1,157.13	10	110.78	107.0	105.4	105.4	103.99	103.84	109.1	108.02	108.02	108.33	107.68
3	100	13,041.93	13,008.33	10,406.62	20	124.91	124.8	124.8	124.8	434.16	176.63	124.9	124.8	124.8	−40,855	253.27
4	950	1,530.70	1,211.5	1,150.89	5	988.97	961.8	954.0	954.0	952.04	952.06	966.1	957.4	957.4	955.96	955.96
5	1,556	1,098.50	575.68	546.90	5	1,597.6	1,567.2	1,558.5	1,558.5	1,557.3	1,557.3	1,571.0	1,561.0	1,561.0	1,559.84	1,559.83

* For definition of L_1–L_7 see text.


A
L° = 100 m
Q°f(0) = 13041.93 m²/year
Q°f(L) = 13008.33 m²/year
Qtf(L) = 11707.24 m²/year
NUMERICAL SOLUTION
SUCCESSIVE STEADY STATES
Δ t = 0.01 years
LOCATION of the INTERFACE TOE (M)
TIME (years)
B
L° = 100 m
Q°f(0) = 1319.31 m²/year
Q°f(L) = 1285.71 m²/year
Qtf(L) = 1157.13 m²/year
SUCCESSIVE STEADY STATES
Δ t = 0.01 years
NUMERICAL SOLUTION
Δ t = 1.0 year
Δ t = 0.5 year
LOCATION of the INTERFACE TOE (M)
TIME (years)
C
NUMERICAL SOLUTION
L° = 100 m
Q°f(0) = 13041.93 m²/year
Q°f(L) = 13008.33 m²/year
Qtf(L) = 10406.62 m²/year
SUCCESSIVE STEADY STATES
Δ t = 0.01 years
LOCATION of the INTERFACE TOE (M)
TIME (years)

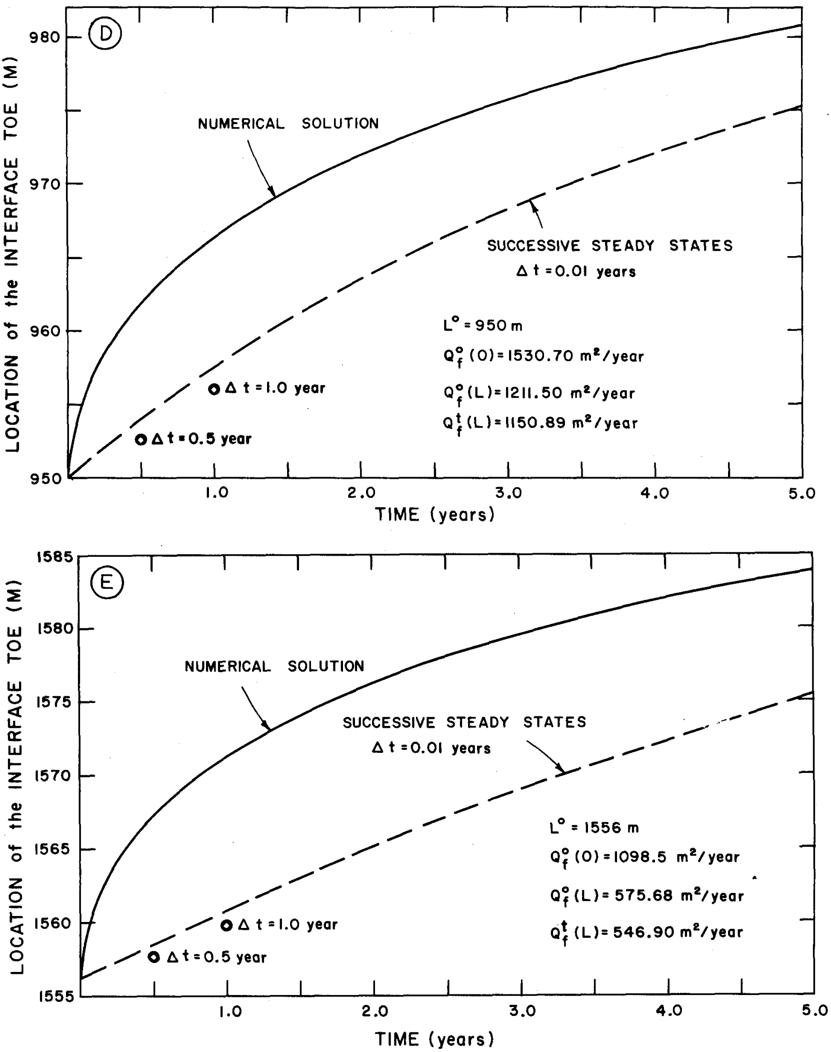


Fig. 3. Runs *1*—*5*: run *1* (A); run *2* (B); run *3* (C); run *4* (D); and run *5* (E).

The values of L_4—L_7 are shown in Fig. 3B, D and E. In the other two runs (*1* and *3*) these values are far out of the range of the figure.

The following conclusions can be drawn from the results:

(1) Computed with small Δt, the linear and nonlinear expressions give the same results.

(2) When the initial intrusion length is large — of the order of 10 to 15 times the aquifer thickness — the successive-steady-states results lag considerably behind the actual movement of the toe (assuming the numerical results to be accurate). Under these conditions, however computing the

successive-steady-states results with single time steps ($\Delta t = 0.5$ and $= 1$ yr.) does not change the results much from those for a small time step ($\Delta t = 0.01$ yr.).

(3) When the initial intrusion length is small — of the same order as the thickness of the aquifer — the successive-steady-states solution follows more closely the numerical solution. This is particularly true when resistance to flow is small (large K, cases *1* and *3*). In this case, however, the single-time-step computations (L_4—L_7) give completely erroneous results. When the resistance to flow is 10 times higher (low K, case *2*) the successive-steady-states result with $\Delta t = 0.01$ yr. deviates from the numerical solution, but a single-time-step computation gives more reasonable results.

An error analysis has been carried out on the results of the successive-steady-states method with $\Delta t = 0.01$ yr. (L_2 or L_3). The percent error has been defined as:

$$\%\ \text{error} = [(\Delta L_1 - \Delta L_2)/\Delta L_1] \times 100 \tag{24}$$

where ΔL was computed at the end of one year for a change of 1% in the freshwater flow at the toe. Fig. 4 shows the results. The % error is plotted against the nondimensional initial toe location, L^0/B, for various values of the nondimensional recharge parameter:

$$\bar{N} = N/[Kn(1 + \delta)]$$

The errors increase with L^0/B, and with $\bar{K}$ for low values of L^0/B. As L^0/B becomes large, the error for a given $\bar{N}$ begins to drop. This is due to the fact that the amount of recharge above the interface, NL, is now a more

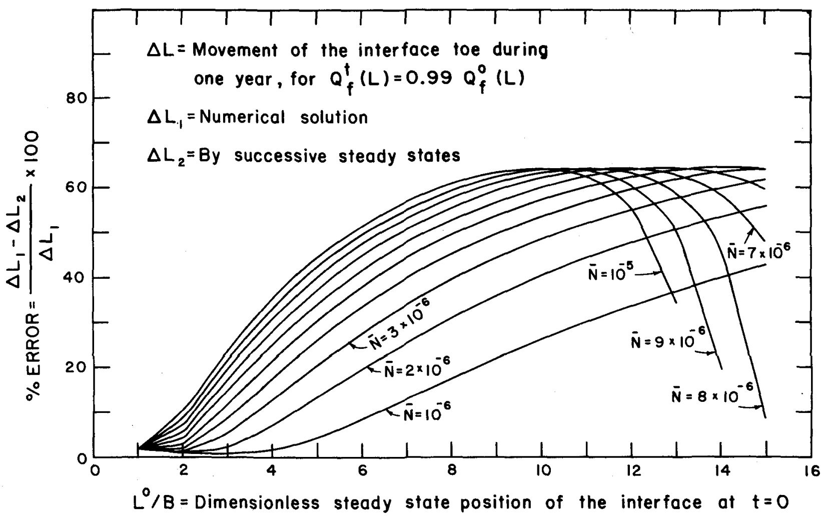


Fig. 4. Error analysis.

dominant factor as compared to the change in freshwater flow above the toe. The higher $\bar{N}$ the sooner this drop occurs.

From the results displayed in Fig. 4, it is apparent that the method of successive steady states does not accurately predict the location of the interface toe for perturbations of the boundary conditions which are "far" from the initial steady-state conditions. The cause of this arises from our assumption that at all times the interface shape maintains a geometric similarity with its steady-state profile. Thus it is implicit in the method of successive steady states that the responses induced by changing the boundary conditions are instantaneously propagated to maintain this geometric steady-state profile. Since we have neglected the elastic storage of the porous medium, pressure propagates instantaneously. However, in reality, the adjustment of the shapes of both the interface and the phreatic surface to changes at the boundaries is not instantaneous, and in general, the shape of these surfaces will not retain a geometric shape which is similar to the steady-state one.

In more sophisticated (numerical) moving interface models (see, e.g., Neuman and Witherspoon, 1971; Shapiro et al., 1983), the shape of the interface is not prescribed a priori, making it possible for changes in aquifer storage, due to the movement of both the interface and the phreatic surface, to be correctly identified. The sophistication of these numerical solution techniques, however, would negate the advantage of obtaining a simplified linear expression for the movement of the interface toe location (eq. 23), which is ultimately to be used in our management model.

4. CONCLUSIONS

We have developed the approximate expressions for the motion of the interface toe, especially the linear one, for incorporation into a management model of an aquifer in which the location of the interface is one of the criteria (Shamir et al., 1984). The management model is constructed as a linear program, and it was therefore important to have a linear expression for the motion of the interface.

The management model that we have developed focuses on the operation of the aquifer for one season or for one year, and therefore the validity of our approximation must be examined for these time periods. As discussed above there is considerable deviation between the approximate solution(s) and the results of the numerical model. The approximate results are, however, indicative of the true movement for conditions encountered in the coastal aquifer of Israel. Thus, until a better approximation can be developed, we feel that the one presented herein is adequate for incorporation into a management model of this coastal aquifer.

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