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Optimal Annual Operation of a Coastal Aquifer

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Optimal annual operation of a coastal aquifer is determined by using a multiple objective linear programing model based on a multicell model of the aquifer and a network representation of the hydraulic distribution system. The decision variables are pumping and/or recharge quantities in each cell. Four objective functions are based on (1) a desired groundwater surface map, (2) a desired location of the sea water–fresh water interface toe in each coastal cell, (3) a desired concentration map of a selected conservative contaminant, and (4) minimization of the energy for pumping and recharge. An approximate linearized expression of the location of the interface has been developed to enable the use of linear programing as the optimization method. A trade-off procedure is employed for identifying the most desirable solution. The model is applied to a segment of the coastal aquifer in Israel (a 44-km strip along the coast with a width of 7 to 15 km) and results are discussed.

INTRODUCTION

Israel's water system has three main natural reservoirs: the Sea of Galilee (Lake Kinneret) in the north of the country, which is the only large surface reservoir and yields annually about one fifth of the total supply, and two aquifers which supply much of the rest. The remainder comes from smaller aquifers, from flood runoff, and from reclaimed sewage.

The work reported herein concentrates on the coastal aquifer, which has the shape of a strip along 120 km of the Mediterranean coast, with width ranging between 5 and 20 km and a total surface area of about 1800 km^2. It is a sandstone aquifer, of the Pleistocene period, with thickness ranging from 150–200 m at the coast down to only a few meters at the inland boundary. The aquifer is phreatic but has at some locations, primarily adjacent to the coast line, several impervious or semipervious horizontal layers which divide it into a few subaquifers. These have mostly a local influence, and in what follows we shall consider the coastal aquifer to be a single layer.

Natural replenishment of the coastal aquifer averages annually 290 × 10^6 m^3/yr. Artificial recharge, which adds another 80 × 10^6 m^3/yr consists of imported water, flood waters, and some reclaimed sewage, infiltrated through spreading basins and through wells. The water imported to the region is from the Sea of Galilee via the National Water Carrier and from another main aquifer, a Turonian limestone formation located some distance to the east. Return flow to the saturated zone of the coastal aquifer from water applied at the ground surface, primarily for irrigation, adds some 80 × 10^6 m^3/yr, while about 100 × 10^6 m^3/yr of fresh water flows to the sea. Present pumping from the aquifer is about 410 × 10^6 m^3/yr, resulting in an average annual deficit of 60 × 10^6 m^3/yr in the aquifer's water balance.

Presently, the sea water–fresh water interface toe reaches inland a distance which varies along the coast and ranges between 250 m and 1900 m. Unless equilibrium has been reached, the interface continues its advance. The location and motion of the interface are of considerable importance in the management of the aquifer because of the strong relationship between the fresh water discharge to the sea and the length of sea water intrusion.

Water quality in the aquifer is deteriorating over time due to man's activities on the land overlying the aquifer and due to artificial recharge with water of higher salinity. Chlorides, nitrates, and other contaminants show an increasing concentration, although not everywhere by the same degree. For example, the concentration of salinity (measured in terms of Cl^-) averaged over the whole aquifer has been found to increase at a rate of about 1.5 mg/l each year. While present salinity of the water in the aquifer is only around 150 mg/l of Cl^-, the rate of increase of 1.5 mg/l/yr is of great concern because of the projected salinity in a few decades. Contaminants reach the saturated zone of the aquifer primarily with water percolating through the unsaturated zone above it, taking several years to get through to the water in the aquifer. Vertical and horizontal mixing in the aquifer are relatively slow; and therefore water quality varies across the aquifer.

Long-range management policies are derived for the coastal aquifer together with the other main aquifer and the Sea of Galilee as components of the main National Water System. Within this system, modeled schematically as a "three-reservoir system," the total annual pumping and recharge for the coastal aquifer as a whole is determined. In addition, the long-range global policies, based on the simplified "three-reservoir model" of the system also yield the desired location of the interface and specifications for a desired "map" of water quality parameters throughout the aquifer.

Operating policies for the coastal aquifer for a single season or one year have to be determined within these guidelines. It is the purpose of this paper to describe a model for determining an annual or seasonal operating policy in light of several objectives and constraints imposed by the dictated long-range consideration. Multiple-objective linear programing is used as the decision making aid. A segment of the coastal aquifer is used as an example for the application, but the procedure is general. Furthermore, an aquifer in which sea water intrusion does not exist can also be analyzed by the same model, after the appropriate equations and objectives relating to this item have been deleted, and the model is thus simplified.

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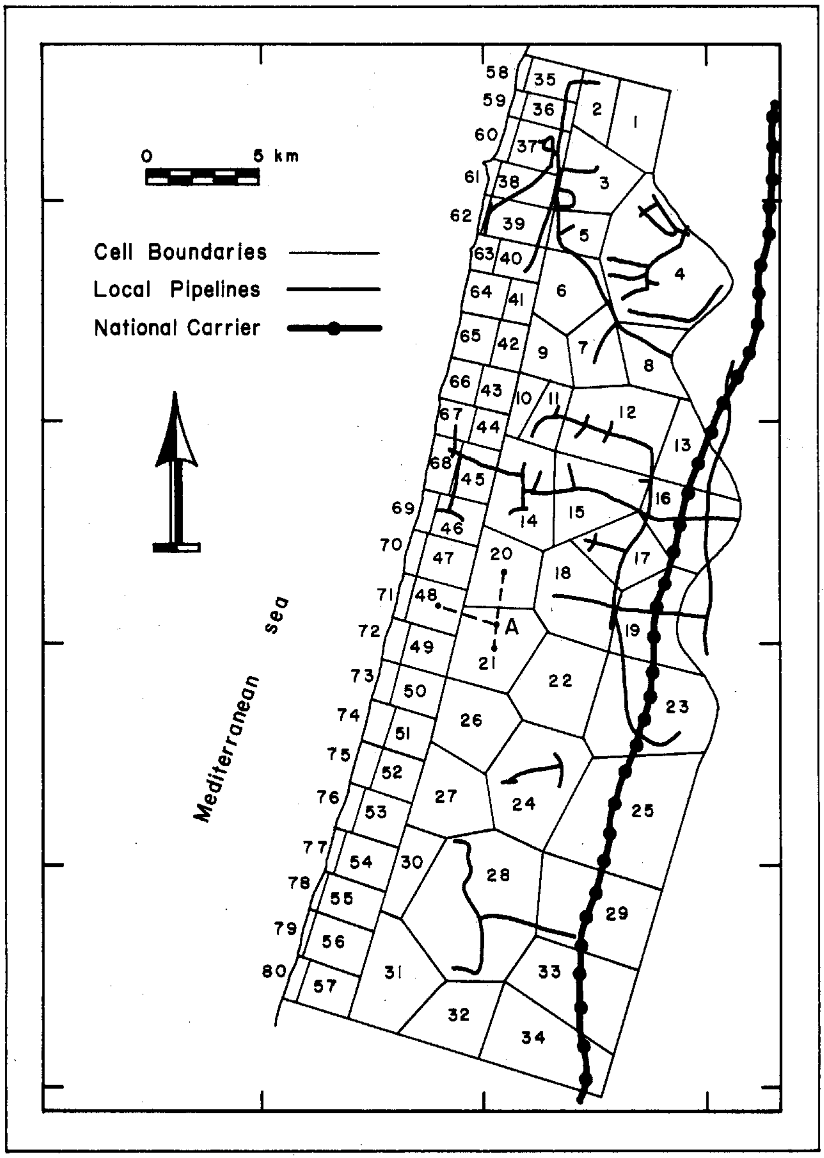

Fig. 1. The aquifer, the cells, and the hydraulic systems.

The mathematical model incorporates both hydrologic and hydraulics considerations. The aquifer is divided into cells which are selected in light of the hydrologic properties of the aquifer, of water projects (groups of wells, hydraulic network on the ground surface), and of water use regions. The constraints in the optimization model include, among others, hydrologic equations of the aquifer cells and hydraulic equations of the distribution system.

THE MANAGEMENT MODEL

Consider a phreatic coastal aquifer. Figure 1 is a map of a 44-km-long segment of the coastal aquifer of Israel. It extends between 7 and 15 km inland and has a thickness ranging between 50 and 150 meters. There are several local clay lenses in the aquifer, but these have been ignored in the present analysis and the aquifer is thus considered to be a single homogeneous layer. The boundaries of the area shown in Figure 1 are all impervious; the northern and southern boundaries are actually streamlines (the flow is towards the sea on the left) along which the selected region has been separated from its surroundings.

The area has been divided into 80 cells: 34 polygonal aquifer cells (numbers 1 to 34), 23 coastal cells (numbers 58 to 80) having a width (along the coast) of 2 km, and 23 transition cells (numbers 35 to 57) with the same width as the coastal cells. The transition cells reach to a distance of 3000 m from the coast. The boundary between the coastal and transition cells is at the location of the sea water interface toe, which changes with time. A later section will deal with this movement. The coastal cell is also called the interface cell. By moving its boundary, it always contains all of the sea water intrusion.

The aquifer cells are defined so as to coincide with water distribution projects: groups of wells and/or a specific part of the hydraulic system (which is also shown in Figure 1). Decisions made with respect to cells (pumping and recharge) can thus be related directly to a water project. Transfers of water between cells also relate to the hydraulic system. Each water project has associated with it the following: pumping, recharge, supply to its local customers, and hydraulic links to other projects and to sources outside the region.

A schematic representation of the groundwater and hy-

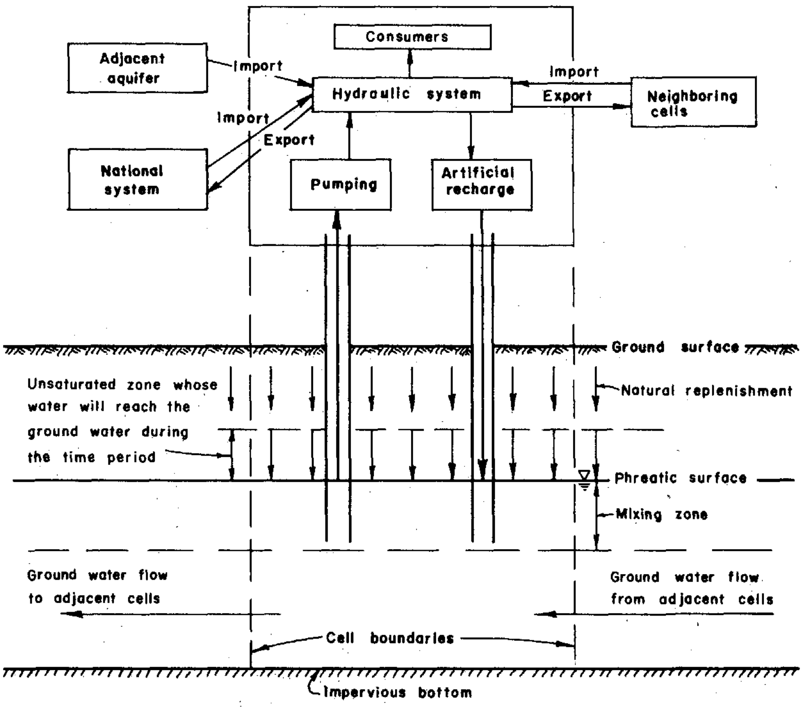

Fig. 2. Schematic representation of the groundwater and hydraulic system associated with a cell.

draulic system associated with each cell is shown in Figure 2. The various features will be explained as the structure of the management model is developed.

Time Horizon

The model is used to determine the operation during one year or one season of the year, viewed as a single time step.

Decision Variables

The independent decision variables are pumping and recharge for each aquifer cell, direct (through the hydraulic system) supply to the local consumers, and transfers between the cell (to and from) and other cells or sources outside the region. The heads throughout the aquifer, the location of the interface (for coastal cells), and the concentrations of contaminants in the cells depend on the pumping and recharge, and are state variables of the aquifer system. They appear in the objective functions and are therefore indirect, or dependent, decision variables.

Constraints

Constraint 1. In our model the water demands must be met. Hence the total quantity supplied to each consumer is not a decision variable. For each cell a constraint of the following form represents continuity at the node of the hydraulic system in the cell:

$$\text{import} + \text{pumpage} = \text{export} + \text{recharge} + \text{demand} \quad (1)$$

Constraint 2. The model accepts directives given from a model higher up in the hierarchy of models, one which views the coastal aquifer as one component in the national water system. These directives are expressed as (1) the total amount to be pumped during the given time period over the entire region of the coastal aquifer is fixed, (2) the total transfer of water from or to the national water carrier (which runs from north to south along the eastern boundary of the coastal aquifer) is given, and (3) the total transfer of water from or to an adjacent aquifer (the Turonian aquifer further inland) is also fixed.

Constraint 3. Constraints imposed by the existing hydraulic system are expressed, wherever applicable, as a maximum quantity of water which can be transferred between a cell (i.e., a water project) and an external source or destination (e.g., a point on the national carrier) or between two cells.

Constraint 4. Maximum pumpage from each cell is limited. This results from either allocations, administrated by the Water Commissioner's office, or from the physical limitation of the wells, whichever is lower. There are many private wells in the area whose operation is controlled only through allocations. This means that private well owners will extract whatever they are allowed, without considering the hydrologic condition of the aquifer. The other wells, however, are owned and operated by Mekorot, the national water supply company, whose pumping policy is guided by hydrologic consideration (such as long-term conservation of the aquifer as a source of water). Our model therefore considers the private pumping as fixed and deals with pumping by Mekorot as decision variables.

Constraint 5. Maximum groundwater level in each cell, where applicable due to hydrologic, engineering (e.g., building foundations), or agricultural (e.g., land drainage) considerations, is prescribed.

Constraint 6. Minimum groundwater level in each cell is prescribed. This may be due to well depth and pump elevations in the wells, so that no more than a certain fraction of pumping capacity is lost through lowering of the water level. Minimum water levels can also reflect hydrologic considerations, for example, so as not to allow groundwater flow into a certain area from its surroundings (e.g., because of water quality considerations). Setting minimum levels by such considerations is problematic. We would like to have these levels as decision variables, fixed by an "objective" analysis, since they have an effect on the attainment of economic, environmental or other, objectives. It is, however, necessary to carry out such an analysis at a "higher level" of the hierarchy of the national water sector, since it is the long-range effects of these minimum levels and especially their effects on water quality and on water reserves for future generations which determine how they should be fixed. For our one-season or one-year operation problem, these minimum levels are considered to be given and fixed.

Constraint 7. A continuity (or balance) equation for groundwater flow in each cell must be satisfied. For a polygonal inland cell the expression is an implicit finite difference equation:

$$\frac{1}{A_i}\sum_j\left[\frac{W_{ji}T_{ji}}{l_{ji}}(h2_j - h2_i)\right] + N_i + \frac{R_i + \beta_i(\mathrm{DP}_i + \mathrm{DM}_i) - \mathrm{PP}_i - \mathrm{PM}_i}{A_i} = S_i\frac{(h2_i - h1_i)}{\Delta t} \qquad (2)$$

where

- A_i cell area;
- W_{ji} length of the boundary between cell i and an adjacent cell j;
- T_{ji} transmissivity on this boundary;
- l_{ji} the distance between centers of the two adjacent cells;
- N_i natural recharge;
- R_i recharge;
- DP_i supply to private consumers;
- DM_i supply to Mekorot consumers;
- PP_i private pumping;
- PM_i Mekorot pumping;
- β_i the fraction of supply which reaches the groundwater as return flow;
- S_i storativity;
- Δt length of the time period;
- $h1_i$, $h2_i$ groundwater level at the beginning and end, respectively, of the time period.

The summation in (2) is performed over all cells j adjacent to the cell i.

An implicit finite difference equation has been used ($h2_i$ and $h2_j$ used in the spatial derivatives) so that this management model has the same hydrologic equations which are commonly used in our numerical groundwater model. Although the implicit formulation imposes no theoretical stability limitations, it is recommended for accuracy to maintain for all cells

$$\frac{1}{A_i}\sum\frac{W_{ji}T_{ji}}{l_{ji}} < \frac{S_i}{\Delta t} \qquad (3)$$

which is not a constraint but a condition on the values of the parameters.

Constraint 8. The groundwater continuity equation which must be satisfied for each cell has to be modified for polygonal cells located at the boundary with the transition cells (numbers 35 to 57) because there is no simple geometric configuration of the intercell boundary. The water level was assumed to vary linearly along a line connecting the centers of adjacent polygonal cells, as shown, for example, in Figure 1 for cells 20 and 21. A line perpendicular to the coastline and passing through the center of a transition cell (e.g., cell 48) then defines a water level at a point in the polygonal cell (point A in cell 21) to be used in computing the gradient for the flow between the two cells. The continuity equation for a polygonal cell may include several such terms. For example, cell 21 has a connection with cells 48 and 49.

Constraint 9. The continuity equation for a transition cell contains these same terms (with a reversed sign) and a term for the flow of fresh water between it and its adjacent coastal cell. Figure 3 is a cross section, perpendicular to the coastline, through two such cells. A linearized model of the sea water–fresh water interface is used, as shown. The location of its toe, at a distance L from the coast, varies with time. A simplified linear solution for this location has been developed and reported elsewhere [*Bear et al.*, 1980]. The procedure gives the new location of the interface in cell i due to a change in the flow of fresh water into the coastal cell from inland:

$$L2_i = L1_i\left[1 + \frac{[(\mathrm{QFL}_i + N_iL1_i - \mathrm{QF1}_i)]\,\Delta t}{f(\mathrm{QF1}_i)(N_iL1_i - \mathrm{QF1}_i)}\right] \qquad (4)$$

where

- $L2_i$ intrusion length at the end of the time period, t_2;
- $L1_i$ intrusion length at the beginning of the time period, t_1;
- QFL_i new fresh water flow per unit width, at ($x = L1$) for ($t > t_1$) as a result of which the interface moves;
- N_i natural recharge;
- $\mathrm{QF1}_i$ fresh water flow per unit width at ($x = 0$, $t = t_1$);
- $f(\mathrm{QF1}_i)$ a function given below by (6);
- Δt length of time period, equal to $t_2 - t_1$.

The flow per unit width (along the coast) from the transition cell j into the interface cell i is

$$\mathrm{QFL}_i = (h2_j - h2_i)\frac{T_{ij}}{l1_{ij}} \qquad (5)$$

The function $f(\mathrm{QF1}_i)$ is [*Bear et al.*, 1980]

$$f(Q) = \frac{n}{N}\left[B - \frac{Q}{\sqrt{\alpha}}\sin^{-1}\frac{B\sqrt{\alpha}}{Q}\right] \qquad (6)$$

where

- n effective porosity (with respect to fresh water displacement by the moving interface);
- B depth to the impervious bottom of the aquifer from sea level (see Figure 3);
- $\alpha = KN[(1 + \delta)/\delta^2]$;
- K hydraulic conductivity;
- $\delta = \gamma_f/(\gamma_s - \gamma_f)$;
- γ_f, γ_s specific weights of fresh and sea water, respectively.

The value of $f(\mathrm{QF1})$ can be computed directly as a function of the known flow to the sea, QF1, at the beginning of the time period. $L2$ in (4) is thus a function of the known conditions at the beginning of the time period and of the decision variables in the management model.

Equation (4) is a linear approximation of the interface movement due to a change in the flow of fresh water from the main body of the aquifer, via the transition cell, into the coastal cell. It has been compared to results generated by detailed numerical models [*Shamir and Dagan*, 1971; J. Bear et al., manuscript in preparation, 1983] and found to be an adequate approximation. The linearization makes it possible to formulate the management model as a linear program.

Equation (4) results from a detailed analysis of the interface motion, without assuming the interface depth to vary linearly

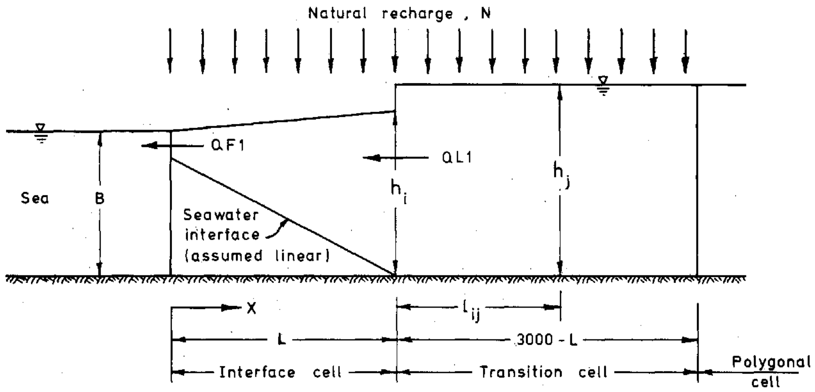

Fig. 3. Cross section through an interface and transition cells with a linearized interface.

with distance from the coast, and yields the motion of the toe over the time period under consideration. The linearly varying interface shown in Figure 3 is used to enable formulation of the balance equations for the coastal and transition cells.

With the known geometry of the interface and the transition cells and the flow to the sea for the time increment under consideration, the continuity equation for the interface cell can be formulated. Denoting with index i an interface cell (e.g., cell 72 in Figure 1) and with j its adjacent transition cell (e.g., cell 49 there), the balance equation becomes

$$\frac{S_i(h2_i - h1_i)}{\Delta t} = N_i + (h2_j - h2_i)\frac{W_{ij}T_{ij}}{l1_{ij}A1_i} - \frac{W_i\mathrm{QF1}_i}{A1_i} \qquad (7)$$

where the indices 1 and 2 denote, respectively, the beginning and end of the time period, and $\mathrm{QF1}_i$ is the fresh water flow into the cell at the coastline (negative when the flow is to the sea, as it normally is) at the beginning of the time period and is given by the known flow conditions at the beginning of the time period, which are assumed to be known. As shown in Figure 3, the representative head for the interface cell is taken at its inland boundary. Then $l1_i$ is the distance from there to the center of the intermediate cell at the beginning of the time period.

Constraint 10. A continuity equation is formulated for the hydraulic system in each cell:

$$\mathrm{PP}_i + \mathrm{PM}_i + \mathrm{IMTC}_i + \mathrm{IMNC}_i + \mathrm{IPTC}_i - \mathrm{OM}_i - \sum_j \mathrm{QM}_{ji} + \sum_j \mathrm{QP}_{ji} - R_i = \mathrm{DP}_i + \mathrm{DM}_i \qquad (8)$$

where

- PP_i private sector pumping in the cell (given);
- PM_i Mekorot sector pumping in the cell (to be determined);
- IMTC_i import from the adjacent aquifer by Mekorot;
- IPTC_i import from the adjacent aquifer by the private sector;
- IMNC_i import from the national system;
- OM_i transfer from the cell to the national system;
- QM_{ij} pipeline flow from cell i to cell j by Mekorot;
- QP_{ij} pipeline flow from cell i to cell j by the private sector;
- DP_i demand by the private sector in the cell;
- DM_i demand by Mekorot customers in the cell.

The summations Σ_j are over all cells j adjacent to cell i.

Each of the transfer and pumping quantities may have an upper bound due to the hydraulic capacity of the hydraulic components of the distribution system, as already mentioned above.

Constraint 11. The location of the interface may be constrained to be within some desired range, by requiring

$$L2_i^{\min} \leq L2_i \leq L2_i^{\max} \qquad (9)$$

Constraint 12. A balance equation for the total quantity of some selected conservative contaminant is formulated for each aquifer cell. Just as an illustration of the method, we have used Cl^- as the pertinent water quality indicator. Other quality constituents may also be introduced. The balance equation for cell i is:

$$\mathrm{CQM}_i = (\mathrm{C1}_i\mathrm{PM}_i + \mathrm{CNC}_i\mathrm{IMNC}_i + \mathrm{CTC}_i\mathrm{IMTC}_i + \sum_j \mathrm{CQM}_j\mathrm{QM}_{ij})(\mathrm{PM}_i + \mathrm{IMNC}_i + \mathrm{IMTC}_i + \sum_j \mathrm{QM}_{ij})^{-1} \qquad (10)$$

where

- CQM_i concentration in the water supplied in cell i;
- $\mathrm{C1}_i$ concentration in the groundwater at time t_1;
- CNC_i concentration in the national carrier at the point from which the quantity IMNC_i is imported;
- CTC_i concentration in the water of the adjacent aquifer from which the quantity IPTC_i is imported.

This concentration is to be kept below a given upper value:

$$\mathrm{CQM}_i \leq \mathrm{CU}_i \qquad (11)$$

Constraint 13. For the concentration of Cl^- in the groundwater of cell i at the end of the time period we have from a balance of Cl^- in the cell:

$$C2_i = \{C1_iA_i\mathrm{HC}_i - C1_i(\mathrm{PP}_i + \mathrm{PM}_i) + \mathrm{CA}_i[N_i + \beta_i(\mathrm{DP}_i + \mathrm{DM}_i)] + \mathrm{CQM}_iR_i + \sum_j Q_{ji}W_{ji}C1_{ij}\}(A_i\mathrm{HC}_i)^{-1} \qquad (12)$$

where

- $C1_i$ concentration at the beginning of the time period in cell i;
- HC_i thickness of the "mixing zone" in the cell (see explanation below);
- $C1_{ij}$ concentration at the beginning of the time period on the boundary between cells i and j;
- Q_{ji} flow from cell j into cell i due to the gradients evaluated

at their common boundary, at the beginning of the time period;

CA_i concentration in the water reaching the saturated zone from above through the unsaturated zone (this value is assumed known (see below));

β_i the fraction of the demand ($DP_i + DM_i$) which reaches the groundwater as return flow.

This equation is based on an assumed "mixing zone," the upper part of the saturated zone, which is the layer in which water quality is directly affected by pumping and by salinity added with recharge and accretion through the unsaturated zone [*Bear*, 1979, p. 446]. After a sufficiently long period of time, probably years, dispersion and vertical flows will mix the entire depth of the aquifer. For one season or one year, however, it is more reasonable to assign the mixing to that part of the thickness which is delineated according to the depth of the wells and the general flow pattern in the cell. Mixing over the cell area is assumed to be uniform. The thickness of the "mixing zone" may be viewed as another model parameter, to be identified as part of the procedure for identifying model parameters.

Transport due to convection is computed in (12) with the concentrations and the flows on the boundaries between cells. This reduces some of the "numerical dispersion" encountered in numerical groundwater quality models based on the dispersion equation.

If it is desired to deal with several conservative pollutants, then for each a similar set of constraints must be formulated. For nonconservative constituents the equations have to be modified according to the appropriate physical processes.

Objective Functions

Management of the aquifer is to be guided by several criteria. Each is developed into a linear objective function, and all functions are then used in a trade-off scheme to develop the best compromise solution.

Groundwater levels. Given a desired water level in each cell, $H2_i$, we may seek to minimize an appropriate measure of the distance between the water levels at the end of the season and the desired ones, for example,

$$\text{Min}\left\{Z = \sum_i (h2_i - H2_i)^2\right\} \tag{13}$$

which leads to a quadratic programing problem, or equivalently,

$$\text{Min}\left\{Z_1 = \sum_i |h2_i - H2_i|\right\} \tag{14}$$

which can be transformed into a linear program. We have used (14) as a measure of an overall fit, over the entire aquifer or some part thereof, to a desired groundwater contour map. A rational selection of such a desired map, i.e., $H2$ for each cell, is a difficult problem because it must be based on a broad, long-term view of the aquifer's role and performance within the entire water resource system. These considerations are beyond the scope of the present paper, and we simply assume that the desired groundwater map is given.

Transformation of the absolute value formulation (14) into a linear program is achieved by defining two new variables:

$$a_i = h2_i - H2_i \tag{15}$$

$$b_i = H2_i - h2_i \tag{16}$$

and performing

$$\text{Min}\left\{Z_1 = \sum_i a_i + b_i\right\} \tag{17}$$

subject to

$$a_i \geq 0 \qquad b_i \geq 0 \quad \forall i \tag{18}$$

Location of the interface. Given a desired location for the interface toe in each interface cell, $LM2_i$, we seek to minimize the distance between the location of the interface toe at the end of the season and the desired distance, i.e.,

$$\text{Min}\left\{Z_2 = \sum_i |L2_i - LM2_i|\right\} \tag{19}$$

where the summation is over all the interface cells for which a distance LM2 is given. Transformation into a linear program is as before.

$L2_i$ is given by (4), with QFL_i and $f(QF1_i)$ inserted from (5) and (6).

As with water levels, deciding on the desired interface location in each cell, $LM2_i$, depends on aquiferwide water resources management considerations, which are not easy to formalize. Our management model assumes that the desired location has been determined and is available.

Water quality. Dealing with water quality in the context of a single time period model is problematic because of the time lag and signal modifications that take place during the passage of contaminants through the unsaturated zone. Actions taken during any time period have a long-range effect, since salinity of the water applied for irrigation at the surface may end up in the groundwater after several years, and in Israel's coastal aquifer, sometimes as many as 10 and 20 years. Thus the link in our model between decisions and groundwater quality consequences is incomplete.

This has led us to test two alternative objective functions for water quality. The first is similar to the above stated objectives for water levels and the interface location. Given a desired concentration of Cl^- in each cell, $CM2_i$, the objective function is

$$\text{Min}\left\{Z_3 = \sum_i |C2_i - CM2|\right\} \tag{20}$$

where $C2_i$ is the concentration in the cell at the end of the time period.

The second objective is to minimize the total quantity of Cl^- applied at the ground surface over the entire aquifer:

$$\text{Min}\left\{Z_4 = \sum_i [CQM_i DM_i + \sum_j CQP_{ij} DP_{ij}]\right\} \tag{21}$$

where

CQM_i concentration in the water supplied to Mekorot customers;

DM_i quantity supplied to these customers;

CQP_{ij} concentration in the water supplied to private consumers from source j;

DP_{ij} quantity from source j to private consumers in cell i.

An additional possibility is to attempt to control water quality (e.g., by trying to achieve desired target salinities) only in selected subzones rather then in the entire aquifer.

There is presently no well-defined basis for establishing a map of desired Cl^- concentration, nor is there sufficient infor-

TABLE 1. Results of Single Objective Optimizations: Values of the Objectives

			Value at Optimal Solution of Objective			
Objective	Symbol	Units	z_1 (Levels)	z_2 (Interface)	z_5 (Quality)	z_6 (Energy)
Levels	z_1	m	263.42	265.33	266.91	263.42
Interface	z_2	m	1995.08	1995.00	1996.21	1995.08
Quality	z_5	ppm	947.80	947.50	944.92	947.80
Energy	z_6	10^6 kwh	5.594	5.744	6.046	5.594

mation of water use at the surface which would allow formulation of the objective according to (21). We therefore use as quality objective minimization of the sum of all concentrations, i.e.,

$$\text{Min}\left\{Z_5 = \sum_i C2_i\right\} \tag{22}$$

Energy. With the rise in cost of energy this is becoming an increasingly important criterion. Energy is used in pumping, recharge and transfer in the hydraulic system. The objective function is:

$$\text{Min}\left\{Z = \sum_i \left[\frac{\text{PM}_i(\text{HQM}_i - h2_i)}{\text{EEP}_i} + \frac{R_i(\text{HR}_i - \text{HQM}_i)}{\text{EER}_i} + \sum_j \frac{\text{QM}_{ij}(\text{HQM}_j - \text{HQM}_i)}{\text{EEQ}_{ij}}\right]\right\} \tag{23}$$

where

HQM$_i$ head required in the hydraulic system in cell i;
HR$_i$ head needed for recharge;
EEP$_i$, EER$_i$, EEQ$_{ij}$ efficiencies for pumping, recharge and transfers, respectively.

Equation (23) is a nonlinear objective function, since it contains products of a decision variable (e.g., PM$_i$) and an unknown dependent variable (e.g., $h2_i$). A linearized form of this equation is

$$\text{Min}\left\{Z_6 = \sum_i \left[\text{PM}_i\text{EP}_i + R_i\text{ER}_i + \sum_j \text{QM}_{ij}\text{EQ}_{ij}\right]\right\} \tag{24}$$

where EP$_i$, ER$_i$, EQ$_{ij}$ are the average specific energies (per unit of water volume) used last year in the system for pumping, recharge and transfers, respectively. The values are obtained from data collected regularly by Mekorot Water Company. Each value already includes the effects of head differences and efficiencies. This is admittedly an approximation which ignores the fact that the heads (and possibly the efficiencies as well but to a much lesser extent) are functions of the decisions on pumping, recharge, and transfers. Examination of the results shows that typical changes in groundwater levels make only a small difference; for example, (HQM$_i$ − $h2_i$) does not change much (in percentage) when $h2_i$ is varied over some reasonable range. Equation (24) is therefore taken as a reasonable approximation for the energy objective.

DATA REQUIREMENTS

The following data are needed to construct and run the multiple objective decision model: (1) geometry of the cells in the aquifer model, (2) storativity for each cell, (3) transmissivity on the boundaries between adjacent cells, (4) type of boundary (impervious, fixed head, given flux) on each aquifer boundary, (5) natural recharge for each cell, for the period of analysis (season, year), (6) initial water level in each cell, (7) initial position of the interface in each coastal cell, (8) initial concentration of the selected pollutant(s) in each cell, (9) concentrations of the selected pollutant(s) in the water imported to the region (from the national carrier and the adjacent aquifer), (10) demands to be met in each cell, (11) fixed pumping by private well owners, who have been given a licence for the current year and are no longer under consideration, (12) upper and lower bounds on various variables: pumping and recharge in each cell, transfers in the hydraulic system between cells and from the external sources, groundwater levels, concentrations, interface location, (13) desired water level in each cell, (14) desired concentration in each cell, (15) desired interface location in each coastal cell, (16) thickness of the "mixing zone" in each cell (see equation (12)), (17) return flow coefficient for each cell (β in equation (12)), and (18) the efficiencies for computing the energy in pumping, recharge and transfer (EE's in (23)).

MULTIPLE-OBJECTIVE DECISION MAKING

Trade-off functions between pairs of objective are constructed by the constraint method [*Cohon and Marks*, 1975]. Given the multiple objective problem

$$\text{Min}\ \{\mathbf{Z}(\mathbf{x}) = [z_1(\mathbf{x}), z_2(\mathbf{x}), \cdots, z_p(\mathbf{x})]\} \tag{25}$$

subject to

$$\mathbf{X} \in X \tag{26}$$

Where X is the feasible set, we first solve p independent problems:

$$\text{Min}\ z_k(\mathbf{x}) \qquad \text{subject to } \mathbf{x} \in X \qquad k = 1, \cdots, p \tag{27}$$

Each gives an extreme point, $z_k{}^* = z_k(\mathbf{x}_k{}^*)$. We then select a pair of objectives, say, $z_k(\mathbf{x})$ and $z_l(\mathbf{x})$, and produce points on their trade-off curve through the parametric solution of

$$\text{Min}\ z_k(\mathbf{x}) \qquad \text{subject to } \mathbf{x} \in X \qquad z_j(\mathbf{x}) < L_j \tag{28}$$

The bound L_j is a parameter which is varied systematically in the range

$$m_j < L_j < z_j{}^* \tag{29}$$

where m_j is the lowest value of $z_j(\mathbf{x})$ experienced during the p individual optimizations, according to (27). When constructing the trade-off function for a pair of objective functions, all other objectives are left unconstrained. The values generated for these other objectives, at solutions lying on the trade-off function, are recorded and used in the next phase of the solu-

TABLE 2. Results of Single Objective Optimizations: Pumping

		Value at Optimal Solution of Objective			
Variable	Actual Value	z_1 (Levels)	z_2 (Interface)	z_5 (Quality)	z_6 (Energy)
PM$_3$	4.274	5.000	5.000	0.681	5.000
PM$_5$	11.414	11.181	8.681	13.000	11.181
PM$_6$	1.509	0.000	2.500	2.500	0.000

Values are in 10^6 m^3/yr.

TABLE 3. Results of Single Objective Optimizations: Heads

Variable	Desired Value $H2_i$	Initial Value $h1_i$	Value at Optimal Solution of Objective z_1 (Levels)	z_2 (Interface)	z_5 (Quality)	z_6 (Energy)
$h2_3$	2.60	17.50	13.79	13.83	14.87	13.80
$h2_5$	2.50	14.10	5.11	5.55	4.84	5.11
$h2_6$	5.37	3.67	4.53	3.53	3.50	4.53

Values in meters.

tion process. The pairwise trade-off functions may, in theory at least, contain some inferior solutions [*Cohon*, 1978, pp. 121–123]. No inferior solutions remain after the full evaluation procedure, as discussed below.

The pairwise trade-off functions are used as guides towards a compromise solution. There are several ways to proceed; selection of an appropriate method is a subjective matter and depends on the organizational, institutional and decision making environment. One way is to observe the trade-off functions and impose constraints on values of the various objective functions. These constraints eliminate portions of the solution space. This can be done through examination of the two-way trade-off functions and the attendant values of the other objectives, using graphical displays and a tabular presentation of all the objectives at extreme efficient points (the p solutions of (27)).

The process is iterative. At each iteration, p single-objective optimization problems are solved, with tighter bounds imposed on the values of the objectives, bounds generated by submitting the results of the previous p optimization problems to negotiation among the "interests" represented by the objectives [*Haith and Loucks*, 1976; *Alkan and Shamir*, 1979]. As will be seen in the following section, the situation in Israel's coastal aquifer, at least in the part we analyzed, is rather constrained and required little in the way of negotiation to reach a compromise solution for the objectives we selected.

EXAMPLE

To test the methodology, an analysis was carried out for the aquifer shown in Figure 1, for the hydrologic year October 1976 to September 1977. A report by the hydrologists [*Kapuler et al.*, 1978] gives the desired interface location. This is the location deemed to be optimal for the long-range steady state condition, as determined by hydrologic and economic considerations (external to the present study). The interface location in each coastal cell was used to formulate the interface objective function and also to determine the desired groundwater levels, as follows.

TABLE 4. Results of Single Objective Optimizations: Water Quality

Variable	Initial Value $C1_i$	Value at Optimal Solution of Objective z_1 (Levels)	z_2 (Interface)	z_5 (Quality)	z_6 (Energy)
$C2_3$	80	75.75	75.75	78.56	75.75
$C2_5$	75	61.91	65.01	59.62	61.91
$C2_6$	167	170.00	166.60	166.60	170.00

Results are concentrations in ppm.

TABLE 5. Results of Single Objective Optimizations: Position of the Interface Toe

Variable	Desired Value $LM2_i$	Initial Value $L1_i$	Value at Optimal Solution of Objective z_1 (Levels)	z_2 (Interface)	z_5 (Quality)	z_6 (Energy)
$L2_{62}$	1800	305	296.2	296.2	296.3	296.2
$L2_{63}$	1800	250	245.2	245.2	245.2	245.2
$L2_{64}$	1800	1035	1035.8	1035.8	1035.8	1035.8

Values in meters.

At the desired interface toe location in each cell the steady state water level is given by

$$\mathrm{HB} = B/\delta \tag{30}$$

The fresh water flow there is [*Bear*, 1979, p. 396]

$$Q_f(L) = \frac{KB^2(1+\delta)}{2L\delta^2} - \frac{NL}{2} \tag{31}$$

where N is the natural recharge in the interface cell (in millions of m^3/yr per unit surface area of the aquifer) and L is the intrusion length. Taking the distance from the interface toe to the inland boundary of the aquifer and assuming that the regional flow pattern is essentially towards the coast, the net contribution of natural recharge to the groundwater over this entire length LS is

$$N_e = \frac{Q_f(L)}{\mathrm{LS}} \tag{32}$$

This must hold so that a flow $Q_f(L)$ toward the sea is indeed realized over the interface toe. Next the analytic equation for a phreatic aquifer flow to the sea, with recharge, is used to compute water levels along a line from the interface toe to the inland boundary of the aquifer. The values given by this continuous solution are converted to values at cell centers by an interpolation scheme.

Other data needed have been listed in a previous section. These are used to construct the management model as a multiple-objective linear program. MPSX/370 was the optimization program used, on an IBM 370/168.

Tables 1 to 5 summarize the results of the first iteration single objective optimizations. Table 1 gives the values of the four objectives, Table 2 gives the pumping in three selected cells, Table 3 gives heads in the same cells, Table 4 shows concentrations in the same cells, and Table 5 presents intrusion lengths in three selected coastal cells.

The results in Tables 1 to 5 show small differences between values generated at the four single-objective optimizations.

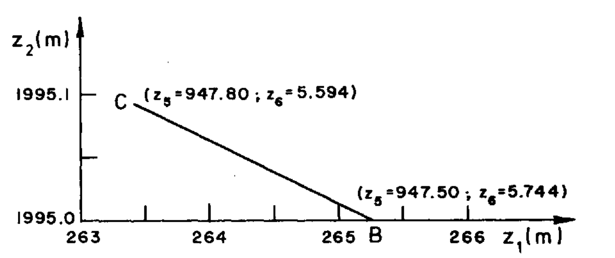

Fig. 4. Trade-off function of levels z_1 and interface location z_2.

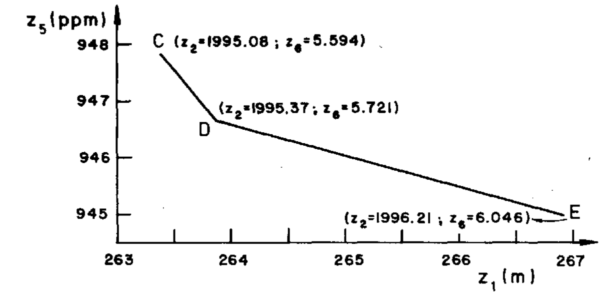

Fig. 5. Trade-off function of levels z_1 and quality z_5.

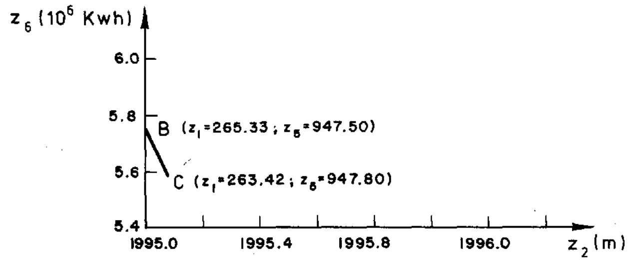

Fig. 7. Trade-off function of interface location z_5 and energy z_6.

Observe, for example, that the interface objective z_2 changes only from 1995.00 to 1996.21. The difference of 1.21 m amounts to an average difference of only 5 cm in each of the 23 interface cells. Still, an average change of 5 cm away from the desired location, caused by an operation which is optimal for quality may be a considerable price to be paid. The 1.21 m should not be compared with the overall value of 1995 m, since the magnitude of the latter is a function of how distant the present interface is from the ultimate desired location.

Figures 4 to 7 give the trade-offs between several pairs of objectives obtained by the procedure discussed in a previous section: the constraint method used for the pair while other objectives are left free to vary without constraints. As can be seen from these figures, the ranges of variation of all objectives is rather restricted. This is due to the particular circumstances in this segment of the coastal aquifer, where in 1976–1977 there was little freedom of action under the hydrologic and hydraulic constraints. There is no trade-off between levels (z_1) and energy (z_6): they turn out to be complementary objectives and reach their optimum at the same solution point. Figure 7 shows the trade-off surface for three objectives: levels, interface location, and quality.

Selection of the final (compromise) solution now proceeds as follows. Observing Figure 4 we ask the decision maker(s) whether they prefer to improve the level objective z_1 by $265.33 - 263.42 = 1.91$ m against a loss of $1995.08 - 1995.0 = 0.08$ m in the interface location objective z_2 or not. Assuming they do, the better point is *B*. Similarly, if a preference is established for an improvement of $1996.2 - 1995.0 = 1.2$ m in the interface objective z_2 over a loss of $947.80 - 944.92 = 2.88$ ppm in the quality objective z_5, from Figure 6, then point *B* is preferred over *E*. Figure 5 can be used in a similar fashion to investigate preferred points from trade-offs between levels and quality.

For the problem analyzed here, *B* seems a preferred compromise solution. In reality, the selection process is to be carried out in consultation with the decision makers who are asked to respond to specific questions posed to them regarding preferences between alternative solutions.

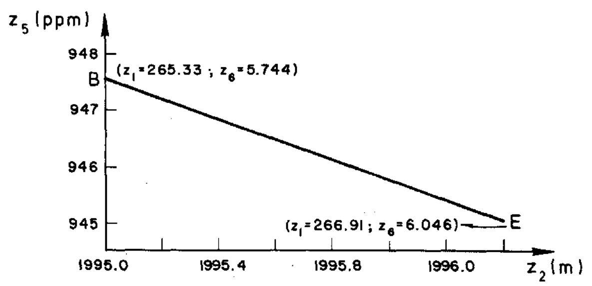

Fig. 6. Trade-off function of interface location z_2 and quality z_5.

DISCUSSIONS AND CONCLUSIONS

The management model described and demonstrated here provides a means for determining an operating policy for one season or one year in a coastal aquifer, under several objectives. It contains in the constraints a model of the physical system: (1) water balance in aquifer cells, (2) mass balance of a conservative pollutant in aquifer cells, (3) sea water–fresh water interface location in coastal cells, and (4) continuity equations for the hydraulic system. The objectives are (1) approaching a given groundwater map, (2) approaching given locations of the interface, (3) approaching a given map of pollutant concentration, and (4) minimizing energy for pumping and recharge. The "given" values in the first three are assumed to be the results of analysis carried out at a "higher level" of the water resources management hierarchy, a level responsible for long-range planning and operations. These results are then imposed upon the seasonal/yearly operation: in our formulation as objectives, but they could also have been set as constraints.

An approximation is developed for the interface location and motion, which maintains linearity of the interface constraints and objective function. A mild linearization is also necessary in the energy objective function. Other relations are all linear, and thus a multiple objective linear programing method can be used.

For the example studied, it turns out that the operation is very tightly constrained for the presently existing wells and hydraulic system. Therefore the multiple objective decision-making process is rather limited in scope; after one round of single objective optimizations a final (compromise) solution can be selected.

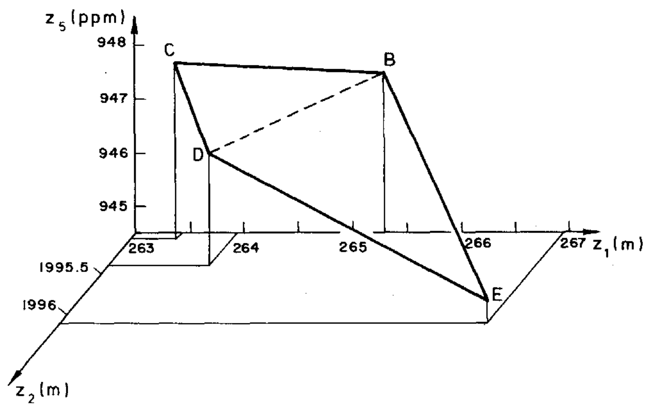

Fig. 8. Trade-off surface for levels z_1, interface location z_2, and quality z_5.

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