OPERATIONS RESEARCH IN AGRICULTURE AND WATER RESOURCES
D. Yaron, C. Tapiero (editors)
North-Holland Publishing Company

MULTIPLE OBJECTIVE PLANNING OF A REGIONAL WATER RESOURCE SYSTEM

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A multiple objective methodology is used to plan the development and seasonal operation of a regional water resources system. The methodology -- based on the principles of the STEM method -- allows progressive articulation of preferences by the decision makers and interest groups. In an iterative process which presents them with results of one iteration and solicits directives for the next. Each iteration narrows the region on the "Efficiency Frontier" in which the final compromise solution is to be found. The procedure is not guaranteed to converge -- in the mathematical sense -- but does provide a practical framework for reaching a compromise.

The methodology is applied to planning of the Har Ha'Negev and Northern Arava region in the South of Israel. A linear programming model describes the region at some target date, and contains the following elements: various types of sources, divided into potable and sub-potable; urban, industrial and agricultural consumers; two separate networks -- one for potable water and the other for sub-potable -- to deliver water from sources to consumers. Decision variables are the capacities of all the sources and the network links to be developed -- above whatever capacity already exists -- subject to hydrologic and engineering constraints. The year is divided into two seasons, with different water demands in each, and the operational decisions are how much to draw from each source and how much to transfer in each link of the network in each season. Six objectives are considered. Plans are generated for two target dates.

1. INTRODUCTION

The water resources system of a region approximately 75 km by 75 km in the South of Israel is to be planned. The region is called Har Ha'Negev - Northern Arava, and lies in an arid zone with annual rainfall between 50 and 200 mm/year. The region is relatively poor in local water sources of good quality, and therefore depends to a large extent on import from the National Water System. Still, there are considerable local sources which can be developed and used, some of them at a high cost. Agriculture and industry exist in the area, and could be further developed. The region is expected to develop considerably and provide the basis for locating a sizeable population in this relatively underpopulated area. The work reported herein was carried out prior to signing of the peace treaty in 1979, which will result in an even greater pressure to develop the region.

The purpose of our work has been to formulate a plan for development of the water resources system of the region to meet future needs. As in most such projects, there is more than one objective according to which a plan is to be selected.

Also as is often the case, there is no single identifiable decision maker, and the process of arriving at the preferred plan must allow for interaction and competition between the various interests. It is clear that under such circumstances the selected plan cannot be "optimal" in the sense that it maximizes one objective; rather, it is a compromise solution, hopefully the "best" (in some sense) compromise solution.

To allow for interaction with the decision makers, a multiple objective decision making process was developed which emphasizes the two-way communication between analysts and decision makers. Graphical and tabular presentation of the results of each step in the analysis were designed to facilitate the presentation of intermediate and final results to the decision makers. Their responses to the results of each step in the analysis were used to gradually reduce the range of "acceptable solutions", leading finally to the compromise solution.

The following sections detail the methodology used, describe the region and its water resources, and demonstrate the application of the methodology to planning the water resources system of the Har Ha'Negev - Northern Arava region.

2. THE MULTIOBJECTIVE DECISION - MAKING PROCESS

2.1 Introduction

Multiobjective optimization methods can be classified in many ways (Cohon and Marks, 1975; Keeney and Raiffa, 1976; Zionts, 1979). One classification is as follows:

(1) Methods which rely on prior articulation of preferences: a single objective optimization is constructed on the basis of these preferences and solved; no further iterations are carried out.

(2) Generating methods: through a sequence of properly constructed single objective optimizations one generates points on the non-inferior set, then one of these points is selected as the best-compromise solution.

(3) Methods which rely on progressive articulation of preferences: an iterative procedure of stating preferences based on updated information and generation of more solution points to provide additional information.

The method used in this study combines elements of (2) and (3). It is essentially a modification of the Step Method (STEM) due to Benayoun et al. (1971; see also Haith and Loucks, 1976).

2.2 The method

Consider the following multiobjective linear programming problem

$$\text{Max } \underline{f}(x) = [f_1(\underline{x}) = \underline{c}_1\underline{x};\ f_2(\underline{x}) = \underline{c}_2\underline{x};\ldots\ldots\ldots;\ f_p(\underline{x}) = \underline{c}_p\underline{x}] \tag{1}$$

subject to

$$\underline{\underline{A}}\ \underline{x} \le \underline{b} \tag{2}$$

$$\underline{x} \ge \underline{0} \tag{3}$$

where x is an n-dimensional vector of decision variables, $\underline{\underline{A}}$ is an (mxn) technological matrix, $\underline{b}$ is an m-dimensional vector, and $\underline{f} = f_1, \ldots, f_p$ is a p-dimensional vector of objectives. We shall denote by X the feasible region in decision space, as defined by (2) and (3).

Define

$$M_k = \underset{\underline{x}\in X}{\text{Max}}\ \{f_k(\underline{x})\} \tag{4}$$

This is the maximum value which f_k can attain within the feasible region, and is obtained by solving a single-objective LP, ignoring all objectives except f_k. The solution point of this problem is denoted

$$\underline{x}_k^* = \{\underline{x} \mid f_k(\underline{x}) = M_k\} \tag{5}$$

At this point the other objectives take on values denoted by

$$f_\ell^k = f_\ell(\underline{x}_k^*) \tag{6}$$

The first step in the proposed procedure is to solve the p single-objective LP's. Each solution gives a solution point, $\underline{x}_k^*$, in the n-dimensional decision space and a corresponding point, $f(\underline{x}_k^*)$, in the p-dimensional objective space (Cohon and Marks, 1975). The results of these p solutions are summarized in Table 1.

Table 1: Results of Initial Single Objective Optimizations

Objective Being Maximized	Solution Point in Decision Space	Values of all Objectives							
		1	2	3	.	.	.	.	p
1	$\underline{x}_1^*$	M_1	f_2^1	f_3^1	.	.	.	.	f_p^1
2	$\underline{x}_2^*$	f_1^2	M_2	f_3^2	.	.	.	.	f_p^2
3	$\underline{x}_3^*$	f_1^3	f_2^3	M_3	.	.	.	.	f_p^3
.	.	.	.	.	.	.	.	.	.
.	.	.	.	.	.	.	.	.	.
.	.	.	.	.	.	.	.	.	.
.	.	.	.	.	.	.	.	.	.
p	$\underline{x}_p^*$	f_1^p	f_2^p	f_3^p	.	.	.	.	M_p

Now define the minimum value of each f_k in the table:

$$m_k = \underset{\ell}{\text{Min}} \{f_k^\ell\} \tag{7}$$

which is the minimum value in the k-th column of the matrix.

M_k is the absolute maximum that the k-th objective can attain, subject to the given constraints, and it can usually be achieved only if the other objectives are ignored. m_k is the lowest value that the k-th objective would take on, if it were ignored as an objective. It results from maximizing that one of the other objectives which is in most direct competition with it.

Notice that the points $\underline{x}_k^*$ are efficient points, because any change in $\underline{x}$ from $\underline{x}_k^*$ will cause $f_k(\underline{x}_k^*) = M_k$ to decrease . The noninferior (nondominated, efficient) set (Cohon and Marks, 1975) is supported on these points. However, these p noninferior points do not provide any information about the shape of the noninferior set between them. What is clear is that $[m_k, M_k]$ gives for each objective the range in which the final value of f_k is to be expected.

We now define a utility function over the range $[m_k, M_k]$ for each objective, as shown schematically in Figure 1.

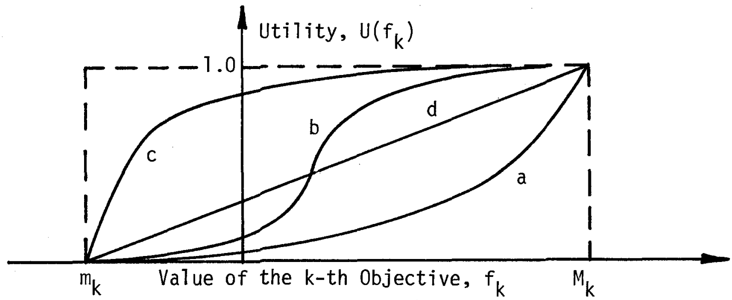

Figure 1
Utility Functions

The range of U is (arbitrarily) set to be [0,1], with

$$U(m_k) = 0 \tag{8}$$

$$U(M_k) = 1.0 \tag{9}$$

The shape of the utility function reflects the relative importance assigned to achieving different percentages of the total range $[m_k, M_k]$.

Curve (a) represents an objective which must reach close to M_k before its utility rises significantly above $U(m_k) = 0$. (b) represents an objective for which most of the utility is achieved upon crossing some threshold value f_k in the range $[m_k, M_k]$. Curve (c) represents an objective for which much of the utility is obtained by a relatively small rise above m_k. Finally, (d) shows a utility which is linear with f_k.

Constructions of such utility functions in multiple decision-makers situations is the crux of the whole multiobjective decision making problem. We do not claim to have any new suggestions on how to arrive at these curves, nor do we claim that once constructed for a particular problem they must be adhered to. Some multi-objective optimization methods require that preferences be articulated in advance, before anything is known about possible solutions to the optimization problem. Other methods allow the preferences to be articulated after some analysis was performed, but once these preferences have been stated they have to be adhered to. We believe that one must allow the decision makers to articulate their preferences progressively. The analyst's role is to provide information to the decision makers, aid them in articulating their new preferences in a procedure which generates more information for the decision makers to respond to. Thus , at every iteration of the procedure to be described now the decision makers may modify the utility functions. If this is done, there is no assurance that the procedure will lead to an acceptable compromise solution in a finite number of iterations. It will, however, narrow the range of acceptable alternatives and point out explicitely the conflicts, allowing further negotiation to be directed to these areas.

For the analysis presented herein we shall adopt the linear utility function over the range $[m_k, M_k]$ (curved in Figure 1). Thus, for each entry in the matrix of Table 1 we define its utility by:

$$U_k^\ell = U(f_k^\ell) = \frac{f_k^\ell - m_k}{M_k - m_k} \tag{10}$$

If $U(f_k^\ell)$ is a true representation of the utility, and assuming the utilities are additive, then the best among the p solutions in Table 1 is the one selected by maximizing the sum of the utilies over all objectives:

$$\max_\ell \{ UT(\ell) \} = \max_\ell \{ \sum_{k=1}^{p} U[f_k^\ell(\underline{x}_\ell^*)]\} \tag{11}$$

Denote the solution which results from this maximization by s, then

$$UT^* = UT(s) \tag{12}$$

UT^* is a number in the range (0,p). The solution, s, selected according to (11) has the best total utility, UT. It may, however, have several utilities which are low. Again, if the U's are a true representation of the utility then (11) is a reasonable criterion. Since we recognize that the definition of the utilities is open to question, we allow the selection of the "best" solution to be modified by the decision-makers upon observation of the matrix in Table 1. Another row may be preferred even though its sum of utilities, UT, is larger than UT* found by (11), if the distribution of its utilities is deemed more equitable between the objectives. Also, if there is a tie between rows on the basis of (11) it may be broken by selecting that row which has

$$\max_\ell \{ \min_k [U_k^\ell]\} \tag{13}$$

In any event, (11) is an acceptable selection in the absence of any additional considerations. We shall continue to denote by s the solution selected, whether by (11) or by some negotiation based on the entire set of UT's and the matrix of U's.

An alternative formulation of the same selection procedure would be to define

$$D_k^\ell = 1 - U(f_k^\ell) = \frac{M_k - f_k^\ell}{M_k - m_k} \tag{14}$$

and to select among the p noninferior solutions of Table 1 the best solution, s, by

$$\min_\ell (DT(\ell)) = \min_\ell (\sum_{k=1}^{p} D_k^\ell (\underline{x}_\ell^*)) \tag{15}$$

The minimum value is

$$DT^* = DT(s) \tag{16}$$

which relates to UT^* of (12) through

$$DT^* = p - UT^* \tag{17}$$

Selection of the solution s terminates the initial phase of the procedure. At this solution point the objective number s reaches its maximum possible value, M_s, while the other objectives have values f_k^s. These values will usually be lower than their corresponding maxima, M_k, except for objectives which are complementary to f_s, and may therefore also reach their maximum at the same point.

$\underline{x}_s^*$ is a feasible point and could therefore be taken as the final compromise solution. It is not reasonable, however, to expect that those representing the various objectives will allow f_s to reach its maximum while they have not. Some compromise is called for. Objective s must give up some of its value, so that the other objectives can be improved above their values at $\underline{x}_s^*$.

An iterative procedure is begun. At each iteration p single - objective problems are solved, as before, except that additional constraints are imposed. For all objectives besides s - the selected solution at the present iteration - the values of $f_k^s(\underline{x}_s^*)$ are set as the lower bounds on the values of the objectives for the next iteration. Thus, the following constraints are added to (2) and (3).

$$[f_k(\underline{x})]_{j+1} \geq [f_k^s(\underline{x}_s^*)]_j \ \forall\ k \neq s \tag{18}$$

while for objective s the constraint is

$$[f_s(\underline{x})]_{j+1} \geq [f_s(\underline{x}_s^*)]_j - [\Delta f_s]_j \tag{19}$$

j is an iteration index. The initial phase of the procedure, leading to Table 1, is denoted by j = 0. $[f_s(\underline{x}_s^*)]_j$ is the maximum value which f_s took on in the j-th iteration. $[\Delta f_s]_j$ is selected by consideration of all the p single-objective solutions, summarized by their $[f_k^\ell]_j$ values, as in Table 1. The larger $[\Delta f_s]_j$, the more the other objectives can be improved in the next iteration. Selection of $[\Delta f_s]_j$ is aided by the experience gained during previous iterations, and is performed as part of the negotiations between decision makers representing the various objectives.

Once $[\Delta f_s]_j$ has been selected, p single-objective LP's are solved, each with (4) as the objective function, and (2), (3), (18) and (19) as constraints. The values of the objectives in the p solutions are $[f_k^\ell]_{j+1}$, as in Table 1. For the assumed linear utilities, they are computed from (10), i.e.,

$$[U_k^\ell]_{j+1} = U_k^\ell[f_k^\ell]_{j+1} = \frac{[f_k^\ell]_{j+1} - m_k}{M_k - m_k} \tag{20}$$

Note that the <u>original</u> range $[m_k, M_k]$ is retained for definition of the utility. Again, one of the p solutions is selected, using the criterion (11) as a guide, i.e.,

$$\underset{\ell}{\text{Max}}\ ([UT(\ell)]_{j+1}) = \underset{\ell}{\text{Max}}\ (U[f_k^\ell(\underline{x}_\ell^*)]_{j+1}) \tag{21}$$

The selected solution, s_j, is used to determine the lower bounds on the objectives - equations (18) and (19) - and a new iteration is begun.

At each iteration we require that all objectives but one reach at least the value they had in the previous iteration, and the remaining one is allowed to drop somewhat. Therefore, it is reasonable to expect to find

$$[UT^*]_{j+1} \geq [UT^*]_j \tag{22}$$

If this not the case in any iteration we go back and revise the last $[\Delta f_s]_j$ (reduce it).

The procedure is terminated when changes in UT^* and/or in the solution point $\underline{x}_s^*$

from one iteration to the next are considered insignificant. The last $\underline{x}_s^*$ selected is then the compromise solution.

2.3 Presentation of Results in the Iterative Process

A graphical representation of the progress of the procedure is used to aid the decision makers and interest groups in assessing the progress towards a compromise and in formulating their directives for the next iteration of the analysis. Figure 2 shows this for p=2 objectives.

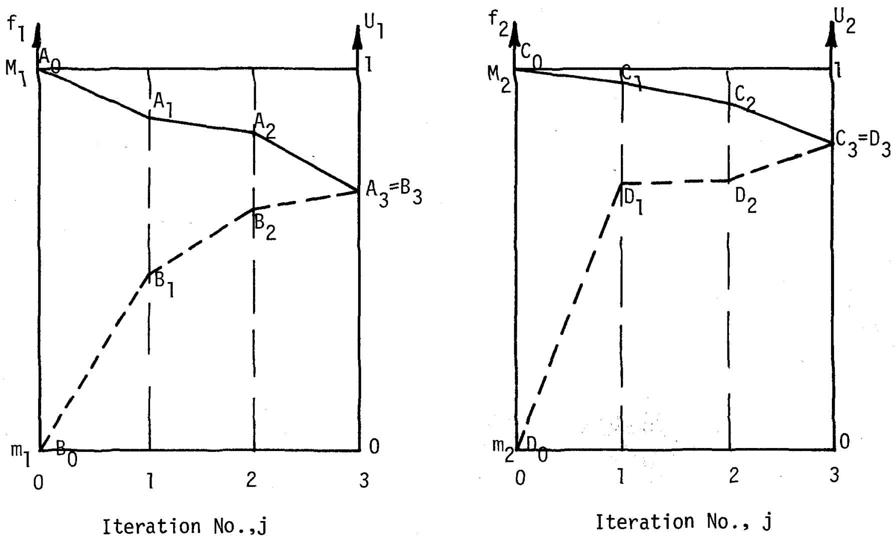

Figure 2
Graphical Representation of the Progress of the Solution

At the o-th iteration the range of possible values for each objective is found. A_0 corresponds to the solution $\underline{x}_1^*$ which maximizes f_1, and B_0 to the solution which has the lowest value of f_1, m_1, while maximizing some other objective (for two objectives it is when f_2 is maximized). The analogous points for objectives 2 are C_0, corresponding to $\underline{x}_2^*$ at which f_2 is maximized, and D_0, respectively. (Note that for the special case of only two objectives the points A, and D, correspond to the same solution, and similarly B, and C, because when one objective is maximized the other takes on its minimum value. Still, different letters are used for these corresponding points to stress the fact that for p>2 one cannot tell in advance which will be the corresponding points). The solution corresponding to either point A_0 or C_0 is selected according to equation (11) and denoted by s. By observing the ranges $[m_1, M_1]$ and $[m_2, M_2]$ the decision makers select $[\Delta f_s]_0$ for equation (19). The constraints (18) and (19) are added, and points A_1, B_1, C_1 and D_1 are generated by optimizing once f_1 and then f_2. The acceptable range of values of the objective functions has now been narrowed, from above and below.

The decision makers are now presented with the tabular and/or graphical results, and have the option of changing their minds on the Δf_s they decided upon at the previous iteration and repeating it; this may happen if the value selected is too large, causing a narrowing of the ranges which is quite different among the objectives. Otherwise, a new reference solution, s, is selected ---either the one corresponding to A_1 or to C_1 --- and $[\Delta f_s]_1$ is selected. The procedure continues, reducing the range of acceptable values for f_1 and f_2 at each iteration (except if it is decided to go back and reduce Δf_s of the previous iteration), ending after several iterations with a narrow enough range so as to define the final compromise solution within it.

The progress of the solution for p=2 objective can also be seen in Figure 3, which shows how the region of acceptable compromise --- in objective space --- is narrowed in successive iterations.

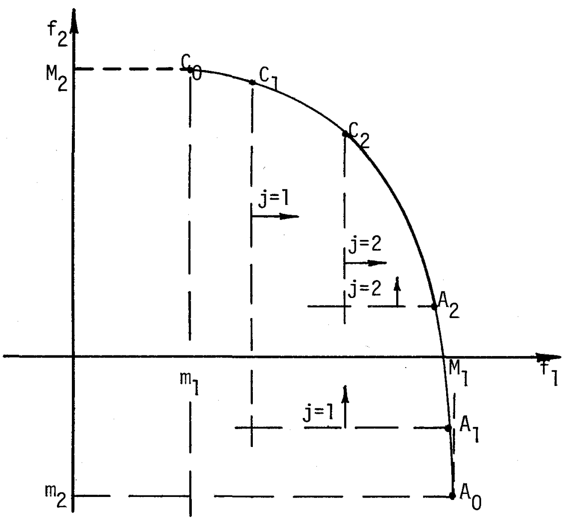

Figure 3
Narrowing the Acceptable Range

Figure 2 was found to be the most effective means of communicating the progress of the solution. In addition to it the decision makers should be shown the values of the decision variables at the solution points, since their response may depend not only on the values of f_k^ℓ and U_k^ℓ but also on the related solution points $\underline{x}^*$.

This concludes the general presentation of the decision making methodology. Next we describe the region under study and develop the water resources management model which is used in the multiobjective decision making process.

3. THE REGION

3.1 Geography, Climate, Population

The Har Ha'Negev - Northern Arava Valley region is in the arid South of Israel. Its area is approximately 4000 km^2 -- almost 20% of Israel's area (excluding occupied territories) -- and its present population is a mere 150,000 -- less than 5% of the country's population. The region has always been slated for substantial development -- when our study was undertaken, and even more so more recently, after the peace treaty with Egypt came into effect.

The region, shown schematically in Figure 4, is characterized by a desert climate, relatively abundant arable land, and scarcity of water, especially water of good quality.

There are two geoclimatic sub-regions -- Har Ha'Negev (H.H.) and Arava Valley (A.V.) -- with water resource systems which are presently separate. Future development may lead to integration of the entire region's water resources system. Some data on the two sub-regions appears in Table 2, compared to data for Tel-Aviv, on the coastal plain, and for Jerusalem.

The H.H. region consists mainly of heavily eroded calcerous rock mountain ranges, between which gravel and loam arable flat lands are formed by the eroded soils. The A.V. region was formed by a tectonic movement which created the famous "Graben" of the Great Rift Valley, which runs from North to South and contains the Jordan Valley a little further North and the Red Sea further South. The Arava Valley soil is a fill composed of coarse grained materials. The drop from the H.H. to the A.V. region is abrupt, from 500-600 meters above sea level to 200-300 meters below sea level over a few kilometers.

Table 2: Geo-Climatic Data

	Location			
	Har Ha'Negev	Arava Valley	Tel-Aviv	Jerusalem
Total Area (sq. km.)	3000	1200	-	-
Arable Land (sq.km.)	31	23	-	-
Ground Elevations (m)	280 - 640	-330 - 30	30	800
Rainfall (mm/year)	100 - 280	40 - 50	550	600
Rain Days per year	15 - 28	10 - 13	50	45
Av. Daily Temp. Aug. (^{o}C)	26 - 27	32 - 35	26	23
Av. Daily Temp. Jan. (^{o}C)	10 - 11	14 - 16	13	9
Rel. Humidity (%)	53 - 58	43 - 45	70	60
Pot. Evapo. Trans. (mm/year)	170 - 180	200 -220	120	150

3.2 Water Resources

The following sources exist in the region: (a) Groundwater, mostly brackish, (b) Surface runoff from random flash floods, (c) Reclaimed sewage, (d) Import from the National Water System, and (e) Desalinated brackish groundwater.

The sources vary in their location, elevation, capacity, temporal availability,

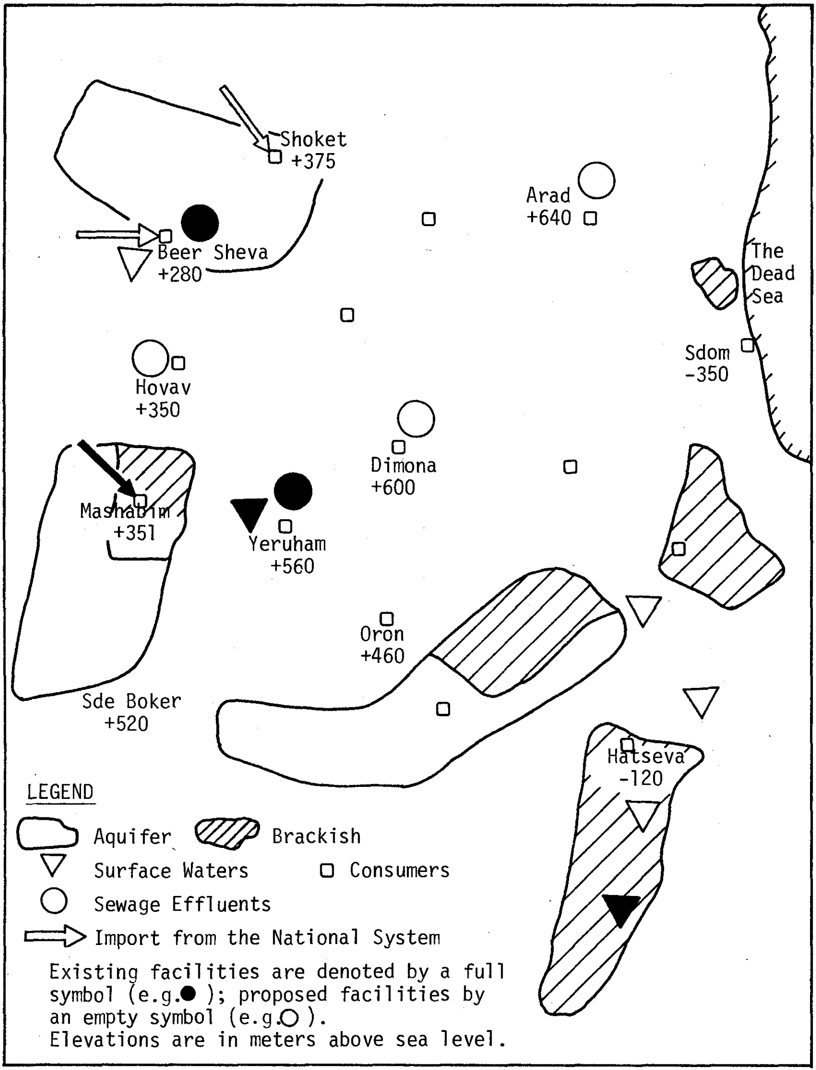

Figure 4
The Region, Consumers and Water Sources

quality, reliability and costs. Based on a relative evaluation of these properties, Table 3 gives a four-rank preference classification of the sources (1 = highest and 4 = lowest preference).

Table 3: Preference Ranking of Sources by Property

Property \ Source	Ground Water	Surface Water	Reclaimed Effluents	Desalinated Water	National Water System
Water Quality	1 - 4	2	3 - 4	1	1
Quantity Potential	1 - 3	4	3 - 4	1	2 - 3
Supply Reliability	1	4	1	1	1 - 2
Investment Unit Cost	1	3 - 4	4	4	3
Operation Unit Cost	2 - 3	1 - 2	4	4	3
Temporal Availability*	1	4	2	1	2
Geographic Location	1	2 - 3	1 - 2	3 - 4	4
Geodetic Elevation	2 - 4	1	1	1	2
Natural Renovation	1 - 4	1	1	1	2
Losses Due to Non Use	4	1	1	4	1

* Note: "Temporal Availability" refers to the temporal distribution of available water at the source; its ranking is based on comparison of this availability with the temporal distribution of demands.

3.2.1 Groundwater

Five aquifers have been identified in the region, as shown in Figure 4. Their basic data is given in Table 4.

Table 4: Groundwater Sources

Sub-Region	Aquifer No.	Annual Yield (MCM) at Salinities (mg Cl^-/ℓ) of			Total Annual Yield (MCM)	Present Utilization (MCM/year)
		< 600	600 - 1000	1000 - 4000		
H.H.	1	20 - 25	-	-	20 - 25	17 - 19
	2	-	4 - 8	-	4 - 8	-
	3	-	3 - 5	-	3 - 5	-
	Total	20 - 25	7 - 13	-	27 - 38	17 - 19
A.V.	4	1 - 2	12 - 20	19 - 27	32 - 49	17 - 20
	5	9 - 11	5 - 10	1 - 3	15 - 24	3 - 7
	Total	10 - 13	17 - 30	20 - 30	47 - 73	20 - 27
Region Totals		30 - 38	24 - 43	20 - 30	76 -111	37 - 46

Each of the aquifers identified above may be made of a number of cells, each made of a different formation. Some of these cells are replenished from rainfall, while others are not. For the latter type, withdrawals are essentially "water mining", and the potentials given in Table 4 represent estimates of how much could be extracted through a reasonable set of wells, and suffice for several decades. Groundwater presently constitutes some 85% of the supplies, but their relative contribution is expected to drop to some 60-70% in the future, as other sources are developed.

3.2.2 Surface Waters

The total surface water potential in the region is estimated at 16 MCM/year, with half in five small basins in the A.V. region and the other half in a major drainage basin which covers much of the H.H. region and drains West. Surface waters in arid zones present substantial difficulties in exploitation. They appear as flash floods, and have therefore to be captured under difficult conditions and then stored and/or recharged into the ground. Direct use and recharge of flood waters have to contend with the high concentration of suspended material normally present in these waters. Only about 3% of the supplies are presently from surface waters; this is expected to increase to some 14% in the future.

3.2.3 Reclaimed Sewage

Domestic and even industrial sewage effluents in arid areas are a valuable water resource, which increases in volume with the growth in population and the general rise in the standard of living. Only reclaimed sewage from the three main cities in the region are considered as sources of regional significance. Their present potential is 6-8 MCM/year, which is used for agriculture after secondary treatment. The potential is expected to reach 16-23 MCM/year.

Utilization of sewage in this arid land-locked region is of importance for environmental quality, because disposal is problematic. This creates an incentive for utilization of the sewage, beyond the general scarcity of water.

3.2.4 Import from the National Water System

The National Water System originates at Lake Kinneret (the Sea of Galillee), in the North of the country. It draws water from the Kinneret and from aquifers along the way, and supplies to regional water systems along its route. The study region is at the farthest point of the system -- some 250 km from its source. There is competition for the National System's water, and little can be allocated to the study area. This creates the motivation for maximum utilization of the region's own sources. Still, the region can probably not manage without some import. One of the objectives in the management model is minimization of import from the National System.

3.2.5 Desalination

At present no sources of this type are active in the region. There are substantial reserves of brackish groundwater in the area, which may be desalted by membrane processes already in practical use elsewhere in Israel. As demands rise this may be the solution for supplying the domestic demands and those of highly productive industrial and possibly agricultural activities.

3.2.6 Summary of Supply Potential

Table 5 summarizes the total water potential in the area and its distribution among the source types -- both present and future.

Table 5: Present and Ultimate Supply Potential by Area and Source Type

Time	Region	Total Water Potential (MCM/year)	Percent of Total from			
			Fresh G.W.	Saline G.W.	Surface W.	Sewage
Present	H.H.	33 - 40	51	27	3	19
	A.V.	22 - 31	11	84	5	-
	Total	55 - 71	34	51	3	11
Ultimate	H.H.	51 - 67	33	24	10	33
	A.V.	34 - 60	14	67	19	-
	Total	85 - 117	25	42	14	19

3.2.7 Water Qualities

The sources have to be classified into the potable and sub-potable categories, since in our model we did not allow for the possibilty of converting sub-potable into potable water at the source through treatment. This was done because the present thiking is that technologically and/or economically acceptable treatment methods for accomplishing this conversion are still missing, and may never become a realistic alternative. So, even though the model could be made to allow for such alternatives this was not done, and the model is therefore somewhat simpler to formulate and solve.

Classification of the sources in the model was as follows:

(a) Potable: fresh groundwater, import from the National Water System, desalination.

(b) Sub-Potable: Brackish groundwater, reclaimed sewage, flood waters.

3.3 Consumers

At present, the total yearly consumption amounts to about 50 MCM, divided 50% to industry, 32% to agriculture and 18% to municipal. The present demand is expected to undergo substantial changes as the region develops. Three dates have been considered in the study, denoted T1, T2, and T3, respectively. T1 is the present; T2 is an intermediate target date, in the late 1980's; T3 is the ultimate development date, expected in the late 1990's. It is difficult to assign calendar dates to these targets, so we simply refer to them as T1, T2 and T3. We shall report herein primarly results for T1 and T3, and later refer to the intermediate date T2, and also to the question of capacity expansion over time.

As will be explained later, the"productive" water demands, i.e. those by agriculture and industry, are decision variables of the development plans. Thus, they are not fixed a-priori as targets which have to be met, but rather determined as part of the model's solution. Still, it is instructive to examine the expected development of consumption, as put forth by the planning agencies. Table 6 contains this data.

The ranges of values for T2 and T3 represent low and high forecasts by different agencies. The following sections provide some details on the demand sectors.

Table 6: Present and Forecast Water Demands (MCM)

DATE	Area	Municipal		Industrial		Agricultural		Total	
		S	Y	S	Y	S	Y	S	Y
T1	H.H.	3.0	9.6	2.0	8.0	3.9	9.9	8.9	27.5
	A.V.	0.1	0.2	4.7	18.7	1.4	7.1	6.2	26.0
	Total	3.1	9.8	6.7	26.7	5.3	17.0	15.0	53.3
T2	H.H.	4.3-5.5	13.3-17.4	5.5-6.8	21.3-26.0	6.6-10.1	16.4-25.3	16.4-22.4	51.0-68.7
	A.V.	0.2	0.3-0.4	6.2-7.8	24.3-30.3	3.8-5.0	19.1 25.0	10.2-13.0	43.7-55.7
	Total	4.5-5.7	13.6-17.8	11.7-14.6	45.6-56.3	10.4-15.1	35.5-50.3	26.6-35.4	94.7-124.4
T3	H.H.	6.8-8.3	19.4-26.0	8.3-10.9	33.0-43.2	10.8-15.6	27.0-39.0	25.9-34.8	79.4-108.2
	A.V.	0.2	0.4-0.5	9.1-10.5	36.0-41.8	3.8-5.0	19.1-25.0	13.1-15.7	55.5-67.3
	Total	7.0-8.5	19.8-26.5	17.4-21.4	69.0-85.0	14.6-20.6	46.1-64.0	39.0-50.5	134.9-175.5

S ≡ Summer season (June - August) demand and Y ≡ Yearly demand. The seasonal distribution will be discussed later.

3.3.1 Seasonal Demand Pattern

Seasonal variations in the deman pattern are very pronounced, and vary between the two sub-regions and among the different consumers. Examination of present and expected future seasonal demand patterns has led to division of the year into two seasons: a three-month summer (June, July and August), and the nine remaining months (referred to as "winter"). Figure 5 shows the schematic demand pattern used.

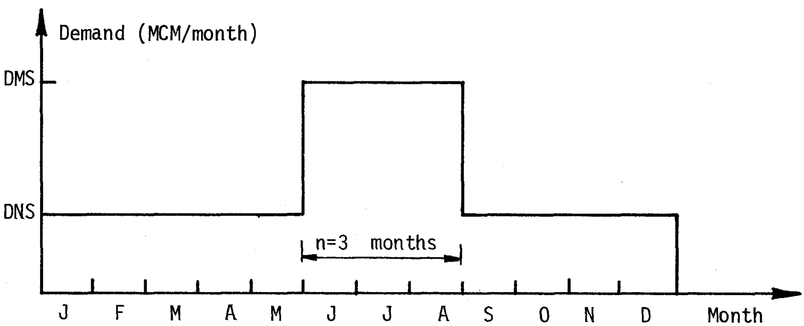

Figure 5
Seasonal Demand Pattern

Denoting:

DY = Total annual demand
DS = seasonal demand during the n = 3 month summer season
DMS = monthly demand during the summer
DNS = monthly demand during the "winter"
DDS = daily demand during the summer

We define the ratio of summer to annual demand

$$\gamma = \frac{DS}{DY} \tag{23}$$

Then

$$DMS = \frac{DS}{n} = \frac{\gamma}{n} DY \tag{24}$$

and

$$DDS = \frac{DMS}{30} = \left(\frac{\gamma}{30n}\right) DY \tag{25}$$

Table 7 summarizes the coefficients γ, (γ/n) and $(\gamma/30n)$ for the region.

Table 7: Seasonal Demand Coefficients

Economic Spector	Peak Season Coefficient	Peak Month Coefficient	Peak Day Coefficient
	γ	γ/n	$(\gamma/30n) \times 10^3$
Municipal	0.32	0.107	3.57
Industrial	0.25	0.083	2.77
Agricultural - H.H.	0.40	0.133	4.43
Agricultural - A.V.	0.05	0.017	0.57

The low agricultural demand during summer in the Arava Valley is due to the very dry and hot weather, because of which winter cultivation is preferred. Industrial demands are evenly distributed over the year.

The mathematical model is based on annual and seasonal water quantities. The peak demand coefficients from Table 7 are used in constructing the hydraulic equations for the model, as will be explained in a later section.

3.3.2 Municipal Consumption

Some 80% of the region's population are located in two main cities -- Beer-Sheva and Dimona. Three other urban centers and a few agricultural settlements account for the remainder. No additional towns are planned in the region, but the existing ones are expected to grow. The forecasts of municipal water demands (as they were prior to the peace treaty with Egypt) given in Table 6 are based on population projections. They also take into consideration the expected increase in the per - capita consumption as the standard of living rises.

These forecasted municipal demands appear in the model as fixed values which have to be met.

Since maximization of economic output and maximization of employment in the region are among the objectives (which will be detailed in a later section) the population is actually not fixed in advance and depends on the development plan. We should therefore have allowed the municipal consumption also to vary with the plan. This, however, would have only a relatively small overall effect, and therefore this additional complexity in the model was considered unwarranted.

3.3.3 Industrial Demands

Industries in the region presently employ some 12,000 people. Some of the existing and planned industries are located in and around the main urban centers; others are at sites of ore deposits or other natural resources. The existing industries are adapting themselves to the use of sub-potable waters, and the planned industries are expected tp be designed for maximum utilization of local sources of inferior quality. The main industries are: mining and minerals, chemicals and plastics, and textiles.

Industrial development is usually more limited by other factors than water - capital, manpower, etc. Still, industrial demands have an impact on the water resources planning, and the latter in turn will affect the industrial development in certain instances. The forecasted industrial demands are summarized in Table 6.

3.3.4 Agricultural Demands

Water is limiting factor for agricultural development in this arid region, in which land is in abundance. Complex problems of crop selection, agro-technical and irrigation methods have to be faced in continued development of agriculture in the region. Use of various water types of inferior quality is one of the crucial issues -- due to the overall scarcity the need to use reclaimed sewage instead of disposing it into the land-locked fragile environment, and the high cost of water in general. These considerations are represented in the model through two stated objectives -- maximum utilization of sewage effluents and minimization of water imports into the region. The use of reclaimed sewage in agriculture (as in industry) is limited, however, by the specification of a maximum admissible ratio of sub-potable to potable quality waters supplied to each consumer.

The ten existing agricultural settlements in the region presently cultivate a total of 17,000 dunams (1,700 hectares). This entire area is irrigated, using some 18,6 MCM/year. The average water duty is thus 1.09 m. The main crops are orchards and winter vegetables.

Even though industrial and agricultural supplies are to be decided upon with the model, it is instructive to examine the forecasts presently held by the planning agencies, as given in Table 6. Results obtained with the model may be quite different from these forecasts.

4. THE MATHEMATICAL MODEL

MONEG (Multiple Objective Negev) is a multiple objective linear programming model for capacity design and seasonal operation of the regional two-quality water resources system. It describes the system at a given development stage, and is used as an aid in reaching a solution which is an acceptable compromise between a number of objectives.

The model, shown in Figure 6, consists of 17 nodes at which sources and/or consumers are located. A network of pipelines connect the nodes: 28 for potable and 21 for non-potable water. Some of these pipes already exist ; others are introduced as possible additions. Where flow of a specific type of water could reasonably take place between two nodes in either direction, two possible links are introduced in the model, each allowing flow in one direction only. (This is required because of the non-negativity of decision variables automatically imposed by the LP algorithm).

4.1 Decision Variables

There are two basic types of decision variables in the model:
(a) Design variables - capacity added to each source and to each transfer link beyond the existing, and
(b) Operational Variables - seasonal quantities to be produced at each source, transferred in each link and supplied to each (productive) consumer.

All decision variables have units of discharge (volume per time), and have been expressed in (MCM/3 months) to facilitate the formulation of the seasonal water

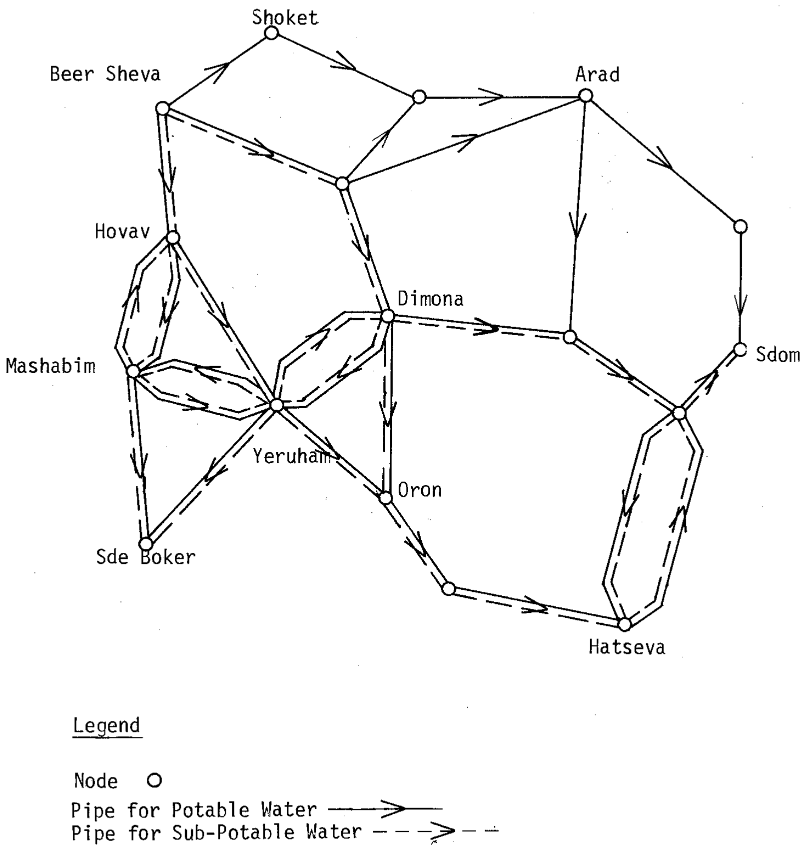

Figure 6

The Model - Nodes and Lines

balance equations. Assuming that facilities operate 700 hours per month (this is based on recorded experience, allowing about one day a month for maintenance) 1 MCM/3 months = 480 m^3/hour. There is a total of some 300 decision variables in the model.

4.2 Constraints

4.2.1 Continuity Equations

At each node a continuity equation is formulated for each of the two seasons, one equation for the potable waters only and another for the two water types combined. (This turned out to be more convenient than writing the second equation for the non-potable waters alone. Both methods amount to the same thing).

The continuity equation for each node is:

$$\sum_i \begin{bmatrix} \text{transfer} \\ \text{from node} \\ (i) \end{bmatrix} + \sum_n \begin{bmatrix} \text{production of} \\ \text{source}(n) \text{ at} \\ \text{the node} \end{bmatrix} - \sum_m \begin{bmatrix} \text{supply to} \\ \text{productive} \\ \text{consumer } (m) \end{bmatrix} = \sum_m \begin{bmatrix} \text{supply to} \\ \text{non-productive} \\ \text{consumer } (m) \end{bmatrix} \tag{26}$$

4.2.2 Annual Potential of the Sources

Due to hydrologic considerations, the annual extraction from each source is limited. This is written simply

$$\sum_\ell \begin{bmatrix} \text{extraction} \\ \text{in season} \\ (\ell) \end{bmatrix} \leq \begin{bmatrix} \text{annual potential} \\ \text{of the} \\ \text{source} \end{bmatrix} \tag{27}$$

4.2.3 Production Capacity of the Sources

These constraints define the seasonal production capacity of the sources (which are decision variables) and limit the seasonal production to the capacity. For each source and each season:

$$\begin{bmatrix} \text{seasonal} \\ \text{production} \end{bmatrix} - \begin{bmatrix} \text{Expansion of the} \\ \text{seasonal production} \\ \text{capacity} \end{bmatrix} \leq \begin{bmatrix} \text{existing produc-} \\ \text{tion capacity of} \\ \text{the source} \end{bmatrix} \tag{28}$$

4.2.4 Bounds on Seasonal Extractions

Total seasonal extraction from each source -- by the existing plus planned facilities -- is subject to an upper bound due to hydrologic and/or technological considerations. These are introduced as bounds on the decision variables of source capacity expansion, rather than as constraints, to make the LP solution somewhat more efficient.

4.2.5 Transfer Capacity of the Pipelines

These constraints define the additional seasonal transfer capacity of the pipelines (which are decision variables) and limits the quantities to be transferred in each season to the installed capacity of the line. For each pipeline and each season:

$$\begin{bmatrix} \text{seasonal quantity} \\ \text{transferred} \\ \text{in the line} \end{bmatrix} - \begin{bmatrix} \text{increase in seasonal} \\ \text{transfer capacity} \\ \text{of the line} \end{bmatrix} \leq \begin{bmatrix} \text{existing transfer} \\ \text{capacity of} \\ \text{the line} \end{bmatrix} \tag{29}$$

4.2.6 Bounds on the Supplies

For several reasons, a minimum supply to a customer may be specified in advance. This may represent political, legal or other considerations, and will force this minimum supply to be allocated even if it is not "optimal" according to some objective function. At the same time, upper limits may also be imposed on supplies. Such a limit will prevent an allocation from being made too large, even if it is "optimal" according to the stated objective function. We use this upper bound to allow consideration of scarce resources other than water -- e.g. labor, land, capital -- which place an upper limit on the possible development. Since these resources do not appear explicitely in the model, their limits are translated into upper bounds on water allocation. The bounds are on the total annual quantities of potable plus sub-potable water:

$$\begin{bmatrix}\text{minimum}\\ \text{annual}\\ \text{allocation}\end{bmatrix} \le \sum_{(k,\ell)} \begin{bmatrix}\text{allocation of}\\ \text{quality (k)}\\ \text{in season } (\ell)\end{bmatrix} \le \begin{bmatrix}\text{maximum}\\ \text{annual}\\ \text{allocation}\end{bmatrix} \qquad (30)$$

Such bounds are imposed only where necessary.

4.2.7 Ratio of Potable to Sub-Potable Supply

Each consumer has a given tolerance to substitution of sub-potable water for some of its potable water allocation. Denoting by η the maximum ratio of sub-potable to potable water in the allocation to a customer, the following constraint is formulated

$$\eta \times \begin{bmatrix}\text{allocation}\\ \text{of potable}\\ \text{water}\end{bmatrix} - \begin{bmatrix}\text{allocation of}\\ \text{sub-potable}\\ \text{water}\end{bmatrix} \ge 0 \qquad (31)$$

The ratio may be given to an individual customer or to an entire group of customers at a node. Values of η were assigned after a detailed evaluation of customers tolerance to substitution with lower quality water. Expenses to the customer and possible changes in revenue due to the change in quality were not considered in the model due to lack of sufficient data; given the data, it would be easy to do. Some customers have $\eta = 0$, i.e. no substitution can be made; others allow $\eta = \infty$. Typical values for agriculture are $\eta = 1.0 - 5.0$ (50% to 83% substitution). Industries vary widely, from a low of $\eta = 0.2$ to as high as $\eta = 50$.

4.3 Objective Functions

Six objective functions were used in the decision making analysis, representing four categories of objectives. They are summarized in Table 8.

Table 8: Objectives

No.	Objective	Symbol	Max. or Min.	Category
1	Total Cost	TC	Min.	Economic
2	Operating Cost	OC	Min.	Economic
3	Net Benefit	NB	Min.	Economic
4	Employment	EM	Max.	Socio-Political
5	Water Import	IM	Min.	Hydrologic
6	Effluent Utilization	EF	Max.	Environmental

A major part of the time and effort in this project were spent on formulation of the objectives, linearization and computation of all the coefficients. The following sections describe only briefly -- due to space limitations -- each of the objective finctions.

4.3.1 Minimum Cost

Capital and operating costs of reservoirs, sewage treatment plants, pipelines and pumps are non-linear with capacity. To allow the use of LP they all have to be linearized. A detailed study of each facility type was undertaken, using current prices, and several approximations were made. For example, it was assumed that new pipelines are designed to flow with a hydraulic gradient of $5^{o}/oo$. Similarly, actual cost data of sewage treatment and of pumps were linearized over the expected range of their operation. Operation and maintenance costs were recorded and linearized separately, as they relate to the operational decision variables. 0 - 1 variables were used, to accommodate the fixed-charge component of capital costs.

Thus, for each source -- e.g. flood water, groundwater, reclaimed sewage, import from the National System -- at each node where such a source is potentially available, and for each line, there are capital and operating cost coefficients. The first two objective functions are sums of costs over all facilities of the model.

4.3.2 Maximum Net Benefit

An extensive review of available data was aimed at obtaining the net benefit from water in agriculture and in industry. These values are difficult to come by since they must reflect only benefits attribuable directly to the water as an input in the production system. Thus, one must deduct from the gross revenues all costs associated with other inputs. We used results reported by official agencies and by other researchers, after close examination and some up-dating.

Benefit from water in irrigation in the study region was available by area, crop, and type of agricultural settlement. We combined these into a single coefficient for the agricultural consumers at each of the model nodes. Similarly , data on productivity of water in industry was given by industry type, and we combined it into a representative value at model nodes. This was done separately for potable and sub-potable waters -- wherever applicable.

The net benefit objective function is formulated as benefits minus capital and operating costs.

4.3.3 Maximization of Employment

The region is relatively sparsely populated, and it is a national goal to populate it. This will lead to a more desirable distribution of the population, and also has security implications (especially now, after signing of the peace treaty with Egypt, which will result in a transfer of troops and installations to the Negev region).

Our model deals primarily with the water resources of the region. Thus, by "employment" we mean that part of the working force whose jobs depend on the availability of water. This is obviously the case in agriculture, but also in many industries, which cannot be built or expanded without water being made available for process, cooling, etc.

The employment objective is expressed as the sum, over all activities which are allowed to expand beyond their present size, of "employees per unit of water" times the amount of water to be allocated (a decision variable). The coefficients (employees/water) were computed by a two-step procedure. First data was assembled on the quantity of water per unit area in agriculture and per unit output in industry. Then data was obtained on labor per unit output. The two were then combined to yield annual quantity of water per worker, in agriculture and in industry, for each of the model nodes. The reciprocal of these values was used as the coefficients (employee/unit quantity of water) in the model.

4.3.4 Minimum Water Imports

The region competes for its water with the remainder of the National Water System, all of it lying to the North of the study region, closer to the sources. The study region is at the far end of the system, and the water reaching it is therefore quite expensive due to conveyance costs. Quantification of the relative benefits attributable to the water being supplied to the study region as compared to other regions would be impossible within the framework of our study. Still, it is clearly desirable to minimize the quantity of water imported to the region from outside, and thus to maximize the utilization of the local sources of all types.

This objective function is easy to formulate, since it is simply the sum of those decision variables which are water imports.

4.3.5 Maximum Utilization of Reclaimed Sewage

The study region is land-locked and arid. Disposal of effluents may therefore present major environmental problems, some well understood and others potential, but probably no less hazardous. Thus, maximum utilization of the local sewage effluents, which will be done with proper controls, may contribute to the solution of the environmental problem. At the same time, this objective goes hand in hand with that of minimizing import. The two are not quite the same, because minimizing import results in utilization of the local sources according to some economic priority, while maximizing the use od sewage effluents disregards the economic costs and favours reclaimed sewage which are used.

4.4 Size of the LP Model

The resulting LP model has:

(a) 6 objectives
(b) 300 decision variables
(c) 340 constraints
(d) 40 bounds on variables.

5. RESULTS

At each iteration of the multiobjective decision making process six LP runs are made, each maximizes one objective (all objectives were converted to Max.). Each such solution contains the values of all decision variables, which are too numerous to examine in detail for each solution. The analysts and decision makers thus work primarily in the objective function space, using tables like Table 1 and the graphical representation of the solution process -- Figure 2.

Upon review of the results in iteration j the decision makers state the new constraints on values of the objective functions -- Equations (18) and (19) -- to be used in the next iteration.

Table 7 shows the results of the initial iteration for T3 (late 1990's).

Each row in the top section lists f_k^ℓ -- the value of objective k at the solution which maximizes objective ℓ (ℓ = 1 for Max(-TC) in the first row, etc.). The second section of the Table 7 lists the best and worst value over each column, M_k and m_k, respectively, and the range between them $D_k = |M_k - m_k|$. The last section of Table

7 lists the utility of each point, defined by Eq. (10) as $U_k^\ell = (f_k^\ell - m_k)/D_k$. The right hand column of this section gives $\sum_k U_k^\ell$, which is the criterion according to which one of the rows (solutions) is to be selected, according to Eq. (11).

Table 7: Unconstrained One-At-A-Time Optimization for T3 (T3RO)

Objective		Values, f_k^ℓ, of Objective No. k						
		1	2	3	4	5	6	
ℓ	Symbol	(MIL/Y)	(MIL/Y)	(MIL/Y)	(10^3EMP)	(MCM/Y)	(MCM/Y)	
1	TC	38.31	35.55	30.67	4.12	7.25	11.32	
2	OC	52.48	33.67	18.37	11.48	6.05	12.42	
3	NB	85.94	76.04	94.70	12.95	30.03	16.69	
4	EM	310.41	255.81	-145.21	61.33	67.58	8.80	
5	IM	121.09	103.09	-45.57	16.58	2.07	16.08	
6	EF	215.66	186.41	-75.65	21.65	45.73	20.40	
		Maximum, Minimum and Range of Each Objective						
	M_k	38.31	33.67	94.70	61.33	2.07	20.40	
	m_k	310.41	255.81	-145.21	4.12	67.58	8.80	
	D_k	272.10	221.51	239.91	57.21	65.51	11.60	
		Utility $U_k^\ell = (f_k^\ell - m_k)/D_k$						$\sum_k U_k^\ell$
1	TC	1.00	0.99	0.73	0	0.92	0.22	3.86
2	OC	0.95	1.00	0.68	0.13	0.94	0.31	4.01
3	NB	0.82	0.81	1.00	0.15	0.57	0.68	4.03
4	EM	0	0	0	1.00	0	0	1.00
5	IM	0.70	0.69	0.42	0.22	1.00	0.63	3.66
6	EF	0.35	0.31	0.29	0.31	0.33	1.00	2.59

The row selected is $\ell = 3$ -- the one which maximizes net benefit -- which has the best overall "closeness" to the values M_k. It also has a good distribution of the U_k^ℓ values; only the value for employment is low (0.15), but this is an objective which is apparently in almost direct competition with all others (as seen from the values of U_4^ℓ), and there is no other row (except, of course $\ell = 4$) which has an acceptable value of U_4^ℓ.

Having selected the solution corresponding to row $\ell = 3$ as the basis for the next iteration , we have to solicit from the decision makers the constraints on the values of the objective functions, according to Equations (18) and (19). First, the net benefit objective f_3 which has reached its optimal value of 94.70 MIL/Y at the point selected must now "sacrifice" some of its achievement to allow the other objective some room for improvement. The value selected by the decision makers is Δf_3 = 54.70 MIL/Y, i.e. 23% of the total range of 239.91 MIL/Y. This yields a constraint on f_3: $f_3 \geq 94.70 - 54.70 = 40$ MIL/Y

In the first iteration it is decided not to impose constraints on the other objectives and to compute a new set of single-objective optima with $f_3 \geq 40.0$ for the next iteration. The results are shown in Table 8.

Table 8: First Iteration for T3 (T3R1)

Objective		Values, f_k^ℓ, of Objective No. k						
		1	2	3	4	5	6	
ℓ	Symbol	(MIL/Y)	(MIL/Y)	(MIL/Y)	(10^3EMP)	(MCM/Y)	(MCM/Y)	
1	TC	43.77	40.08	40.00	12.95	4.95	16.69	
2	OC	47.54	39.14	40.00	12.95	3.78	16.69	
3	NB	86.00	75.73	85.53	12.95	26.59	17.30	
4	EM	86.00	75.88	40.00	50.16	26.59	16.69	
5	IM	63.31	46.31	40.00	12.95	3.07	16.69	
6	EF	85.55	76.04	40.00	20.73	8.98	20.16	
		Utility $U_k^\ell = (f_k^\ell - m_k)/D_k$						$\sum_k U_k^\ell$
1	TC	0.98	0.97	0.77	0.15	0.96	0.68	4.51
2	OC	0.97	0.98	0.77	0.15	0.97	0.68	4.52
3	NB	0.82	0.81	0.96	0.15	0.63	0.77	4.10
4	EM	0.82	0.81	0.77	0.80	0.63	0.68	4.51
5	IM	0.91	0.94	0.77	0.15	0.98	0.68	4.43
6	EF	0.83	0.81	0.77	0.29	0.89	0.98	4.57

The best row in Table 8 is $\ell = 6$, which has $\Sigma U_k^6 = 4.57$. It corresponds to maximization of the use of sewage effluent in the region under the constraint that the net benefit be no lower than 40.00 MIL/Y. Notice that the values of U_k^ℓ are computed with the original value of the range D_k, the value given in Table 7. Thus, the values of U_k^ℓ in Table 8 are the relative attainment of the optimum of each objective over the original range of its possible values.

At the solution corresponding to $\ell = 6$ in Table 8 the objective attain 83%, 81%, 77%, 29%, 89%, and 98% of their range, respectively. All but employment (k = 4) may be considered to have acceptable values. Notice, however, that net benefit (k = 3) has dropped below the others, even though in the previous iteration it was the "leading" objective. This is due to the relatively large sacrifice it allowed (23%) in that iteration. Looking at Table 8, the "representative of net benefit" may object to using this iteration as a basis for the following ones and will insist on going back and re-running this iteration with a Δf_3 smaller than 54.70 MIL/Y. In view of Table 8 he may suggest Δf_3 = 36 MIL/Y, i.e. a loss of only 15% instead of 23%. If this is accepted by the decision makers the computations are repeated with $f_3 \geq 58.70$ MIL/Y and Table 8 is filled with the new results. Instead, it may also be possible to continue the iterations from the given results, imposing now a new constraint on f_3.

In our study Table 8 was accepted as the basis for the next iteration. f_6 now allowed Δf_6 = 2.16 MCM/Y, or 18.5%, thus requiring $f_6 \geq 18$ MCM/Y. The other objectives were constrained as follows:

$f_1 \leq 85$ MIL/Y; $f_2 \leq 76$ MIL/Y; $f_3 \geq 40$ MIL/Y ; $f_4 \geq 21{,}000$ Emp; $f_5 \leq 9.0$ MCM/Y

In the next iteration line $\ell = 2$ achieved the maximum combined utility of $\sum_k U_k^2 = 4.76$, an improvement of 0.19 over that of $\ell = 6$ in Table 8. Two more iterations were carried out. In the first the total utility of the best solution, which corresponds to line $\ell = 4$, rose to 4.98. In the next the value rose to 5.07, at line $\ell = 6$. The solution of this iteration is shown in Table 9.

At this point it is decided to adopt the solution corresponding to row $\ell = 6$ in Table 9 as the compromise solution. The progress of the iterations is shown in Figure 7.

The final compromise solution adopted is the one corresponding to the last line in Table 9, the one which maximizes the use of effluent, subject to constraints on the values of the other objectives developed during the previous iterations. At this point the net benefit reaches only 77% of its possible range, and employment reaches only 45% of its possible range.

The same process was followed for T1 and for T2. In each such solution the starting point was the presently existing system. In other words, the plans examined are all for going from the present system to the development level of each time horizon in a single step. The question of capacity expansion over time will be taken up in the next section.

Table 9: Fourth Iteration for T3 (T3R4)

Objective		Values, f_k^ℓ, of Objective No. k						
ℓ	Symbol	1	2	3	4	5	6	
		(MIL/Y)	(MIL/Y)	(MIL/Y)	(10^3EMP)	(MCM/Y)	(MCM/Y)	
1	TC	47.06	42.33	40.00	30.00	6.00	18.00	
2	OC	50.00	40.46	40.00	30.00	4.96	18.00	
3	NB	50.00	44.00	48.44	30.00	5.52	18.00	
4	EM	50.00	44.00	40.00	36.02	6.00	18.00	
5	IM	50.00	42.24	40.00	30.00	3.73	18.00	
6	EF	50.00	44.00	40.00	30.00	6.00	19.91	
		Utility $U_k^\ell = (f_k^\ell - m_k)/D_k$						$\sum_k U_k^\ell$
1	TC	0.97	0.96	0.77	0.45	0.94	0.79	4.88
2	OC	0.96	0.97	0.77	0.45	0.96	0.79	4.90
3	NB	0.96	0.96	0.81	0.45	0.95	0.79	4.92
4	EM	0.96	0.96	0.77	0.56	0.94	0.79	4.98
5	IM	0.96	0.96	0.77	0.45	0.97	0.79	4.90
6	EF	0.96	0.96	0.77	0.45	0.94	0.99	5.07

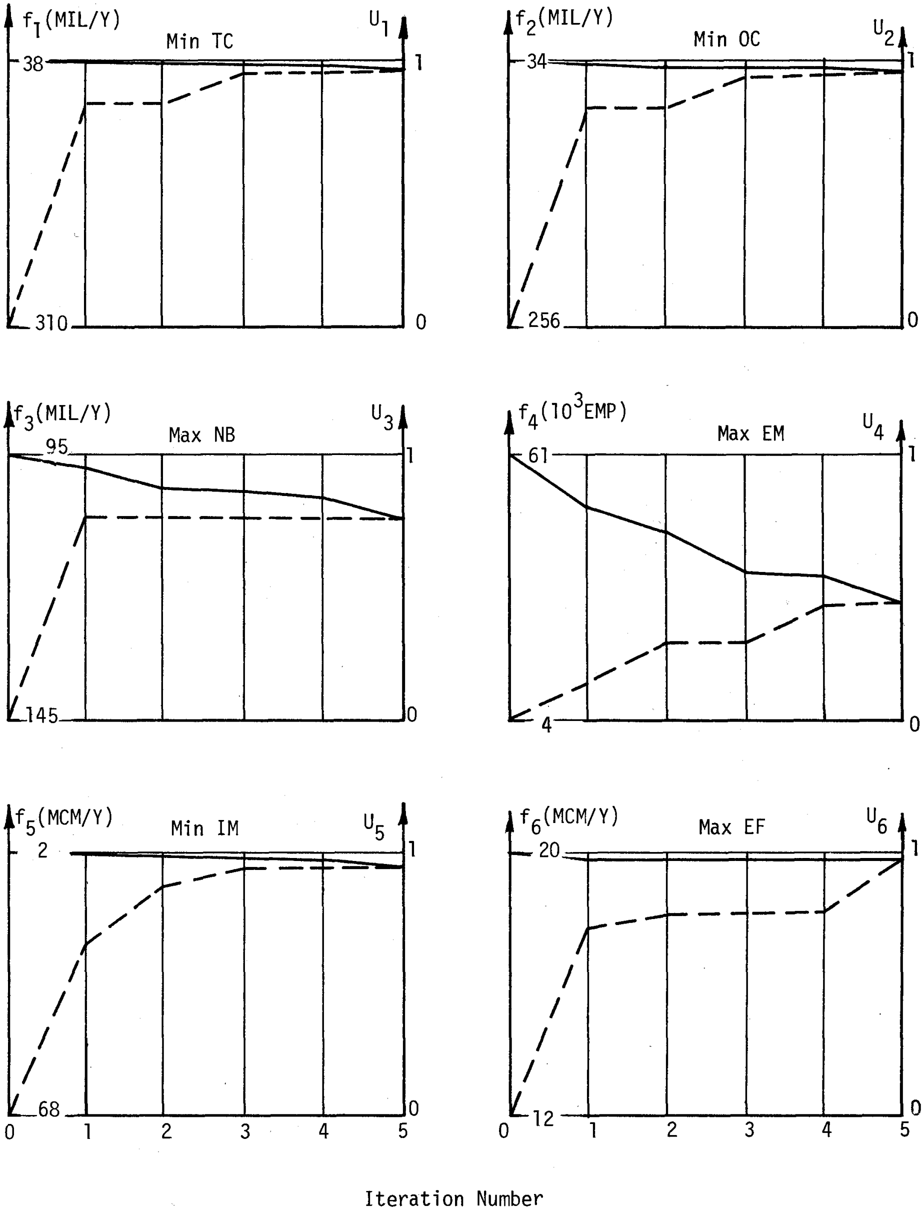

Figure 7
Progress of the Objectives Towards a Compromise Solution

Space limitation do not allow a detailed discussion of the plans corresponding to the compromise solutions. It is stressed, however, that in an actual planning study careful attention must be given to the plan itself, and not only to the values of the objectives. As the iteration process progresses and narrows down the range of acceptable solutions one should begin to look at the alternative solution points corresponding the lines appearing in each table. We used for this purpose a schematic of the system with blank spaces for each design and operationable variable. The solutions were copied from the computer output into such schematics, to facilitate the presentation. It is easier to look at a schematic with numerical data on it than at computer output in pure numerical form. This data supplement the tables of objective values, aiding the decision makers in reacting to the results of one iteration and specifying their directives for the next.

6. DISCUSSION

The multiobjective decision making process is intentionally subjective. Throughout the iterations the decision makers are presented with intermediate results and asked for their responses, which must be given in a specific numerical form and are then incorporated into the next iteration. The information presented to the decision makers is in the objective space -- tables and graphs showing the progress of all objectives through the iterative process. The work is done in objective space, rather than in decision space, because (a) directives on the progress of the iterations should be based primarily on the values of the objectives, and (b) the dimension of the objective space is very much lower than the dimension of the decision space (here $p = 6$ vs. $n = 300$). Still, at any point in the iterative process a decision maker may want to examine the decisions (i.e., the values of the decision variables) corresponding to any solution appearing in the tables, and then base his responses not only on the values of the objective functions at each iterations but also on how they translate into a physical system and an operating policy. Such a response may result from a subjective preference of a certain solution over others, from political considerations, etc. We consider this input to the decision making process to be quite legitimate, provided it is done -- as it is in our procedure -- in full view of the other decision makers, and expressed in specific numerical values imposed as constraints on objectives. Moreover, the duals of these constraints for each of the single objective optimizations indicate the effect that the conditions imposed by the decision maker have on the achievement of all objectives.

The iterative decision making process is not guaranteed to converge. It does, however, provide a structured framework for progressing towards a compromise, with explicit statements of relative preferences, made progressively as more information is generated.

The model which we used does not deal explicitely with the problem of capacity expansion over time. A separate solution is obtained for each of the three target dates. The three solutions are examined to see whether a consistent development pattern emerges. In our study it turned out that it was possible to reconcile the recommendations of the three plans and combine them into a logical capacity expansion program. Had this not been the case, there may not have been any choice but to construct one large model. The model would have to contain the proper relations between the decisions at the different times. For example, the facilities installed at one time become part of the already existing system at the following times. Also, continuity equations for the sources over each time increment have to be added. Such a model would be rather costly to run the many times needed in the decision making process. Thus, it is advantageous to try an approach which does not require the large model, even if the result cannot be shown to be optimal. A possible approach is as follows. Observe in the single time solutions which facilities are optimal at all times. Take the result of T1 and "fix" these facilities. Now solve for T2, with these facilities already existing for the incremental development. Again, fix those facilities constructed at T2 which are consistent with the optimal solution at T3, and solve at T3. Several iterations of this overall procedure may be necessary, before one arrives at a "good" capacity expansion program. Again, it must be stressed that it is possible to solve for all times simultaneously, but that this would be rather costly.

AKNOWLEDGEMENTS

The work was carried out as part of the first author's study for the M.Sc.degree under the supervision of the second author, at the Faculty of Civil Engineering, Technion - Israel Institute of Technology. Financial support was provided by Mekorot Water Co., Ltd.

Thanks are due to N. Arad and Y. Dreizin from Mekorot, to D. Telem, Y. Dotan, P. Dar and others from Tahal - Water Planning for Israel, and especially to Dr. A. Simon, from the Faculty of Agricultural Engineering at the Technion for invaluable help in setting up the interactive LP.

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