WATER RESOURCES BULLETIN
VOL. 14, NO. 1 AMERICAN WATER RESOURCES ASSOCIATION FEBRUARY 1978

A SIMPLE HYDRAULIC SIMULATOR[1]

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ABSTRACT: A simple simulator was constructed, calibrated, programmed and used in management studies of a hydraulic system containing two pumping stations in series, connected by canals and reservoirs. The features of the model, its calibration and use are described, and comparisons between measured and computed hydraulic behavior are presented.
(KEY TERMS: canal; flow simulator; surrogate model; wave front; dispersion; distortion; outflow generator.)

INTRODUCTION

This paper deals with a mathematical model of a hydraulic system, which was used as the basic component in a management model of the system. The idea was to make the simulator of the hydraulic system as simple and computationally efficient as possible, so that it could be run the very large number of times needed for the management study.

The plan in Figure 1 shows the general layout of the National Water Carrier in Israel. It conveys some 400 million cubic meters of water annually from the Sea of Galilee to the centre and south of the country. The operation of the entire carrier has been under study for some time, and the work has been carried out in a hierarchical form, where the operation of the system has been decomposed – in time and space – and dealt with by the use of several models. The work described herein deals with that part of the Carrier from the Sea of Galilee to the Eshkol Reservoir, which includes two pumping stations operating in series, connected by pipelines, canals, and reservoirs.

The management model for this section, which was used to determine optimal operating rules, has been described elsewhere (Damelin and Shamir, 1975). It is the purpose of this paper to describe in detail the mathematical model of the physical system – the hydraulic simulator – as it is incorporated into the management model.

MODELING PHYSICAL SYSTEMS FOR MANAGEMENT

A mathematical model of the physical system, whose design or operation is to be optimized, is always an integral part of the management model. When a mathematical

[1]Paper No. 77046 of the *Water Resources Bulletin.* Discussions are open until October 1, 1978.

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programming formulation is used in the optimization, then the mathematical model of the physical system is embodied in the constraints. When simulation is used, then the mathematical model of the physical system is embodied in the simulation program.

Whether optimization or simulation is used, careful consideration has to be given to the way in which the mathematical model of the physical system is formulated. The main reason is that in both cases the model has to be run a large number of times, and its computational efficiency is of great importance. In optimization, each iteration is essentially a solution of the model of the physical system. In simulation, each trial of a new alternative requires running the program several times. One can afford the large number of solutions that are needed in simulation studies only if the cost of each run is small.

Selecting the appropriate model requires, therefore, careful weighing between the level of detail of the model and its accuracy, on the one hand, and its efficiency, on the other. This paper is devoted to a case study. Simulation was to be used in managing the operation of a hydraulic system. A mathematical model of this system was needed as a component in the simulation program, and the model selected will be described and will show how a simplified description of the complex hydraulic system was devised and used.

When a simplified model of a real system is used, it may be called a "Surrogate Model" which means that not all details of the real system are represented explicitly in the model. Still, for the model to be useful at all, it has to replicate the system's behavior in its important aspects. One obviously has to determine what one considers to be the essential variables which describe the behavior of the system, and then make sure that the model is just sufficiently detailed to yield the values of these variables with acceptable accuracy.

The concept of "Surrogate Models" is not new. For examples, the black-box linear system used in the Unit Hydrograph method is such a model; an application to a hydraulic system is shown here.

DESCRIPTION OF THE MODEL

Characteristics of the Physical System

The part of the National Water Carrier dealt with in this work is shown in Figure 1. It consists of the Kinrot pumping station, which pumps water from Lake Kinneret (the Sea of Galilee) into the Jordan Canal. The water flows into the Zalmon Reservoir, from which it is pumped by the Zalmon pumping station, through the Netufa Canal and into the Eshkol Reservoir. The water is withdrawn from the Eshkol Reservoir via a 108" pipeline which feeds the center and south of the country.

The three pumps in the Kinrot station can, in their various configurations (combinations of pumps in parallel) supply 0, 24000, 46000 or 67000 m3/hour. The Jordan Canal is 16 km long, has a carrying capacity of 73000 m3/hr, and, when full, holds about 285000 m3 of water. The Zalmon Reservoir stores approximately 1600 m3 per cm. of elevation, and has a total capacity of 880000 m3. The Zalmon pumping station has three pumps which can supply 0, 25000, 49000 or 71000 m3/hour in the various configurations. The Netufa Canal is 18 km. long, is capable of carrying 73000 m3/hour and holds about 335000 m3 of water, when full. Finally, the Eshkol Reservoir stores some 7500 m3 per cm of elevation, and has a total capacity of 3800000 m3.

From the above data it is clear that the operation of the two pumping stations has to be coordinated, since considerable fluctuations of water levels in the Zalmon Reservoir

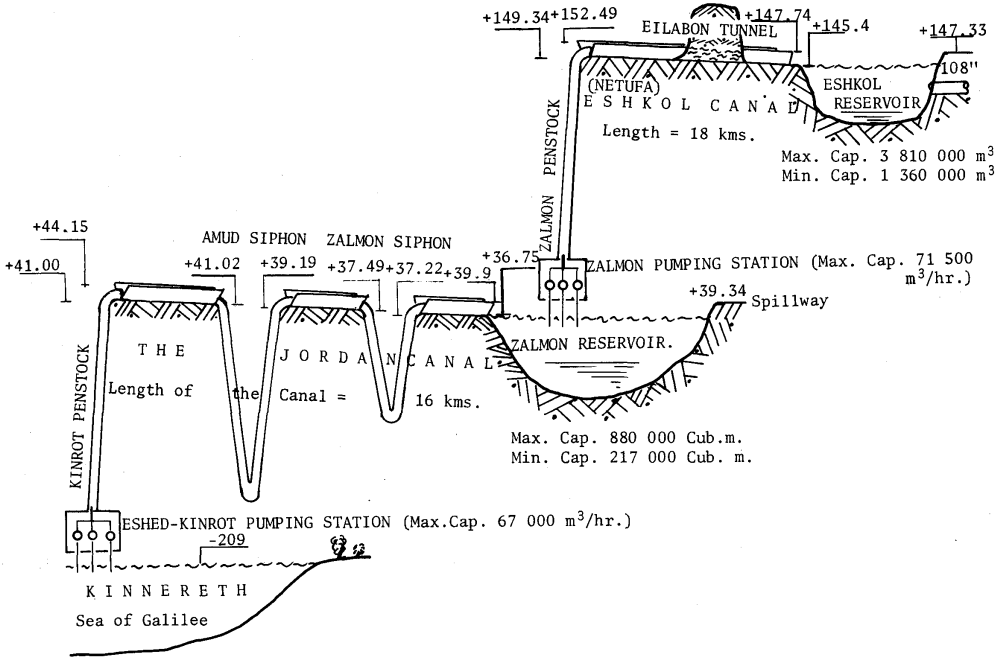

Figure 1. Schematic Layout of the National Water Carrier, Section Kinnereth – Eshkol.

result from unbalanced flows in and out. Also, any unpredicted pump failure at one station has to be offset by a matched shut-down of a pump at the other station. A total power failure results in all the water in the Jordan Canal flowing into the Zalmon Reservoir, while no water can be pumped out. Therefore, empty storage has to be provided if overflow is to be prevented. Such empty storage, on the other hand, causes higher pumping costs in the Zalmon station, due to lower water levels, and there is therefore motivation to keep the level high. A similar problem does not exist at the Eshkol because of its relatively large size.

A statistical study of the occurrence of unexpected pump and total failures showed that the time between successive failures, T, is exponentially distributed.

$$f_T(t) = \lambda e^{-\lambda t} \qquad t \geqslant 0$$

where $\lambda = 1/\bar{T}$ is the reciprocal of the mean time between failures, $\bar{T}$.

Analysis of the data yielded

Kinrot pumps: $\bar{T}$ = 400 pump hours, for each pump

Zalmon pumps: $\bar{T}$ = 730 pump hours, for each pump

Total power failures: $\bar{T}$ = 800 calendar hours

The operation of the pumps is governed by the availability of electrical energy, and is therefore subject to constraints imposed by the electric system, rather than by the demand for water or hydraulic considerations. This leads to several daily changes in the number of pumps in operation.

The travel time for the water in each of the two canals is between 4 and 5 hours, where the canal is initially dry but varies considerably when the canal is flowing full. Discrete changes in inflow to a canal result in a dispersed wave front at the exit.

Specifications for the Model

For use in analyzing operating policies of the pumping stations, a mathematical model of the physical system was needed. For this purpose, only the following time-dependent results are required from the model:

(a) Discharge from each canal into the reservoir.
(b) Volumes in the reservoir.
(c) Volumes in the canals.

If we were interested in a full description of the time-varying pattern of flows in the system, we would have to resort to a numberical solution of transient flows in open channels. Such a solution would be complex, (computer) time-consuming and expensive. Since we were interested only in the above three physical quantities a much simpler ad-hoc computational scheme was devised and will be described below.

An hourly log of all pump operations in the two pumping stations and of water levels in the two reservoirs is available. Using these data, together with the recorded hourly levels in the two reservoirs, charts of flows and volumes in the two canals were derived. These were used in evaluating and calibrating the computational procedure described below. The reservoir levels available were correct to between 1 and 2 cms.

The Hydraulic Simulator

The approach taken was strictly empirical. What was required was the ability to generate out-flows from the two separate canal sections, and the water volumes in these sections, given the inflows. Some indication of the nature of the outflows was provided by a flow-meter located in the outflow section ofthe Netufa Canal. These data were not altogether accurate, but nevertheless served to indicate the way in which outflows respond to changes in the inflow. Segments of the curve, shown in Figure 2, indicate an exponential rise or fall of the outflow resulting from the step change of inflow as pumps are switched on or off.

To simulate this behavior of the outflow, each section of the canal was simulated by the container shown in Figure 3.

The inflow $Q(T)$ is given. It is the constant discharge delivered by the specific pump configuration which is in operation during the period leading up to the time T. The outflow, varying constantly with time, has the value $q(t)$ at the instant T, and is some function, yet to be determined, of the (time varying) water level iin the container, $H(t)$. Several attempts were made for selecting an appropriate function $q = q(H)$, and it was found that a simple linear function served adequately, viz.,

$$q(t) = K.H(t) , \tag{1}$$

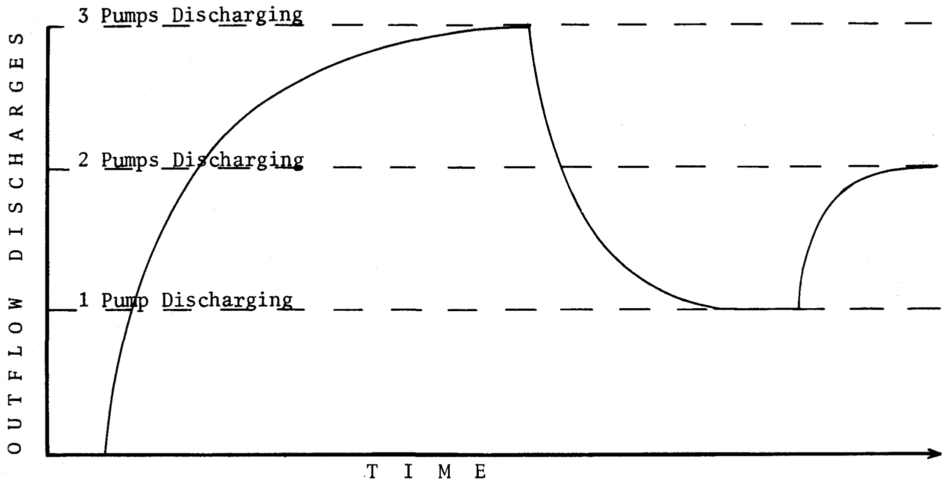

Figure 2. Schematic Representation of Changes in Outflow Resulting from Discrete Changes in the Inflow.

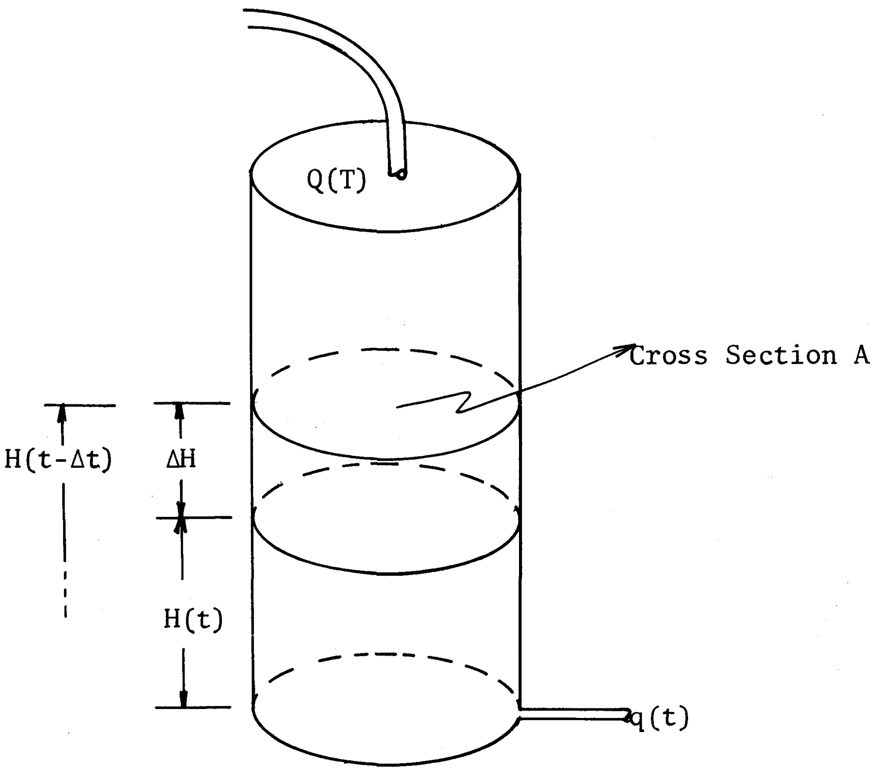

Figure 3. The Container as the Physical Model.

where K is a constant. The rate of change in the water level is given by:

$$\frac{dH}{dt} = \frac{Q(T) - q(t)}{A} \quad , \tag{2}$$

where A is the cross sectional area of the container.

From (1) and (2)

$$\frac{dq(t)}{dt} = \frac{K}{A}\{ Q(T) - q(t)\}, \tag{3}$$

Defining $P(t) = Q(T) - q(t)$, the net water addition to the container, then, since $Q(T)$ remains constant during the time interval, $dP = -dq$. Denoting $K/A = \alpha$, (3) becomes

$$-\frac{dP(t)}{dt} = \alpha P(t), \tag{4}$$

or

$$\frac{dP(t)}{P(t)} = -\alpha dt \quad , \tag{5}$$

The solution of this differential equation is

$$P(t) = e^{C} e^{-\alpha t} \quad , \tag{6}$$

Substituting back for $P(t)$

$$q(t) = Q(T) - e^{C} e^{-\alpha t} \quad , \tag{7}$$

Consider now the system at some state where the inflow is Q_o and the outflow q_o, and the inflow is changed at $t = 0$ to $Q(T)$ and kept at this value thereafter. The value of C in (7) is computed from these initial values and conditions, and is

$$e^{C} = Q_o - q_o \, , \tag{8}$$

and thus the outflow at time t is:

$$q(t) = Q(T) - \{ Q_o - q_o \} e^{-\alpha t} \quad , \tag{9}$$

We now select the basic time increment as 1 hour. It is assumed that changes in pumping configurations can take place only at specified (hourly) intervals, and therefore the state of the hydraulic system is examined at hourly intervals. If we set $t = 1$ in (9), we obtain:

$$q(t) = Q(T) - \{ Q_o - q_o \} e^{-a} , \qquad (10)$$

The volume of the water which flows out of the container in the time period from $t = 0$ to $t = T$, during which the flow into the container is kept constant at $Q(T)$, is:

$$V(T) = \int_0^T [Q(T) - \{ Q_o - q_o \} e^{-at}] dt , \qquad (11)$$

and for $T = 1$ the integration yields:

$$V = Q(T) + \frac{1}{a} \{ Q_o - q_o \}\{ e^{-a} - 1 \} , \qquad (12)$$

This is the volume which leaves the container during one hour, at the start of which the flow was q_o and the inflow was abruptly changed from Q_o to $Q(T)$. The volume of water in the container (the canal) at any time is:

$$V_d(t) = A.H(t) , \qquad (13)$$

Substituting $H(t) = q(t)/K$ from (1), and $a = K/A$, (13) may be written as

$$V_d(t) = q(t)/a , \qquad (14)$$

Changes in water volumes in the reservoir are simply computed, using the difference between inflow and outflow during the time interval.

Lag and Dispersion of Wave Fronts

The real canal differs from its idealized model – the container – in that there is a time lag between the introduction of a change in the inflow and the response of the outflow. This lag is due to the travel time of a wave front in a canal, which is different when the canal is initially empty and some flow is introduced into it and when the change in inflow occurs when there is already flow in the canal. The response at the outlet is therefore considered to be affected by two separate phenomena: a lag, which accounts for the translation time of a sharp wave front; and dispersion, which spreads out the advancing wave. Both the lag and the dispersion may be measured directly. Lag is the time taken for a wave front to traverse an empty canal, and dispersion time is the time by which lag is reduced by a wave traversing a canal containing water.

Lag times for both canal segments were found to be between 4 and 5 hours. Dispersion Time (D.T.) was found to be three hours and the dispersion of wave fronts was performed by an approximation method which spread the front over a total of six hours, three hours (= DT.) on each side of the undistorted, sharp wave front, as shown schematically in Figure 4 for the case of a change from $q = 1$ to $q = 2$. The method for dispersing the front is as follows.

If $\Delta W3$, $\Delta W2$, $\Delta W1$ are the differences in flow between any hour's flow, and the flows which are 3, 2 and 1 hours ahead of it, then the flow during any time period will have a unique set of ΔW (ΔW_i) attached to it.

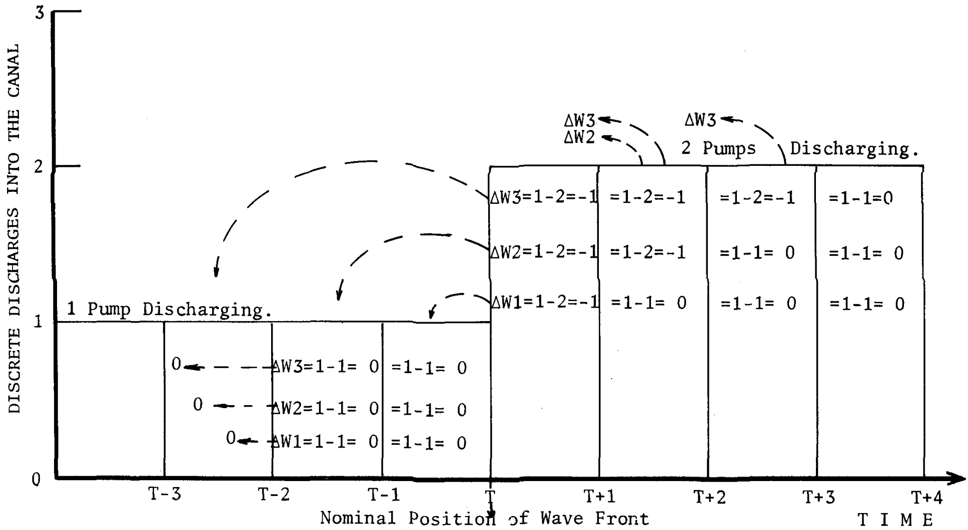

Figure 4. Flow History Preparatory to Dispersion.

The differences in flow are smoothed out (or dispersed) by deducting, algebraically, $\frac{1.\Delta W3}{15}$, $\frac{2.\Delta W2}{15}$, and $\frac{3.\Delta W1}{15}$ from each hourly flow (say at time T) and adding the amounts successively to the flows 3 hours, 2 hours and 1 hour prior to T (i.e., T-3, T-2, and T-1), (the amounts dispersed are inversely proportional to the difference in time). Where no flow changes exist, then the $\Delta W_i = 0$ and no dispersion is carried out. In the example in Figure 5 all ΔW_i at the time period T are equal to 1 (denoting proportional dispersons to the flows at periods T-3, T-2 and T-1), while at time period T+1, $\Delta W1 = 0$ (denoting proportional dispersions to the flows at periods T-2 and T-1, but no dispersion to the flow at time T which is nominally equal to the flow at time T+1).

The final dispersed front of the history presented in Figure 4 will appear as set out in Figure 5.

The dispersion procedure is accomplished with the aid of a dispersion factor $\frac{1}{\beta}$. (In the case above $\frac{1}{\beta} = 15$.) This factor is determined by the following algorithm.

$$\frac{1}{\beta} = (1 + \text{D.T.})^2 - 1 ,$$

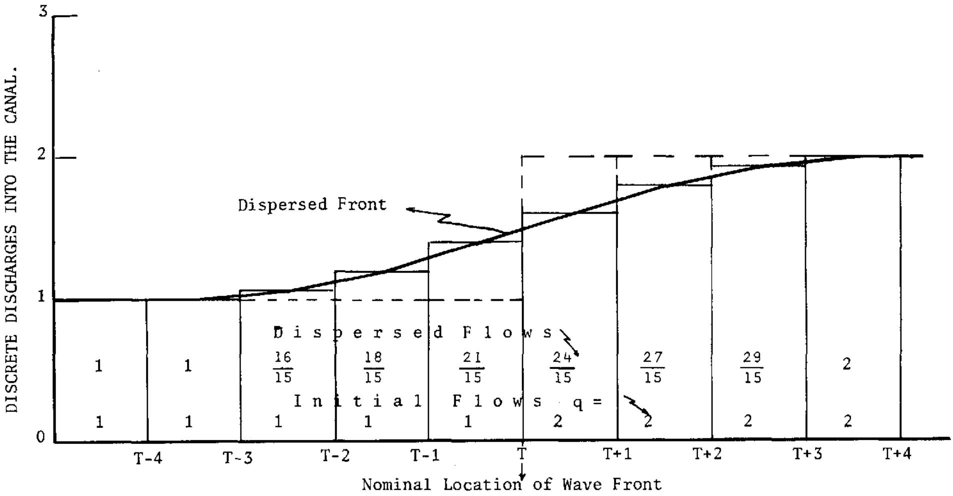

Figure 5. A Dispcrsed Wave Front for Flows Changing from q=1 to q=2.

where D.T. ≡ Dispersion Time. For D.T. = 3 hours:

$$\frac{1}{\beta} = (1+3)^2 - 1 = 15.$$

This factor is so designed that where the dispersion time is other than 3 hours, the factor β will so be adjusted as to result always in a smoothly dispersed front.

It should be stressed that this procedure is strictly empirical in nature, but has proved its worth in yielding good results.

CALIBRATION OF THE MODEL

As mentioned above, data of canal outflows resulting from known, time-varying inflows were available and were plotted at one hour intervals for several periods of one week each.

The parameter a and the method for carrying out the dispersion should represent the physical properties of the canal: length, slope, cross-section and roughness. With the method of dispersion determined, only a remains to be calibrated. Without adjusting for dispersion, a preliminary value of a was assumed, and the computed flows were compared with the historical data. Utilizing the occasions where a flow was introduced into an empty canal, a satisfactory value for a was easily determined. The fit was tested by inspection, and not by statistical analysis.

The dispersed wave front, when operated upon by the distortion effect, now completed the process and allowed for the perfection of the fit. In the Equation (10), the dispersed hourly wave front would be represented by Q(T) and the final distorted flows out of the canal by q(t). Figure 6 shows the effect of changes in the value of a, and Figure 7 shows the effect of changes in dispersion time, D.T.

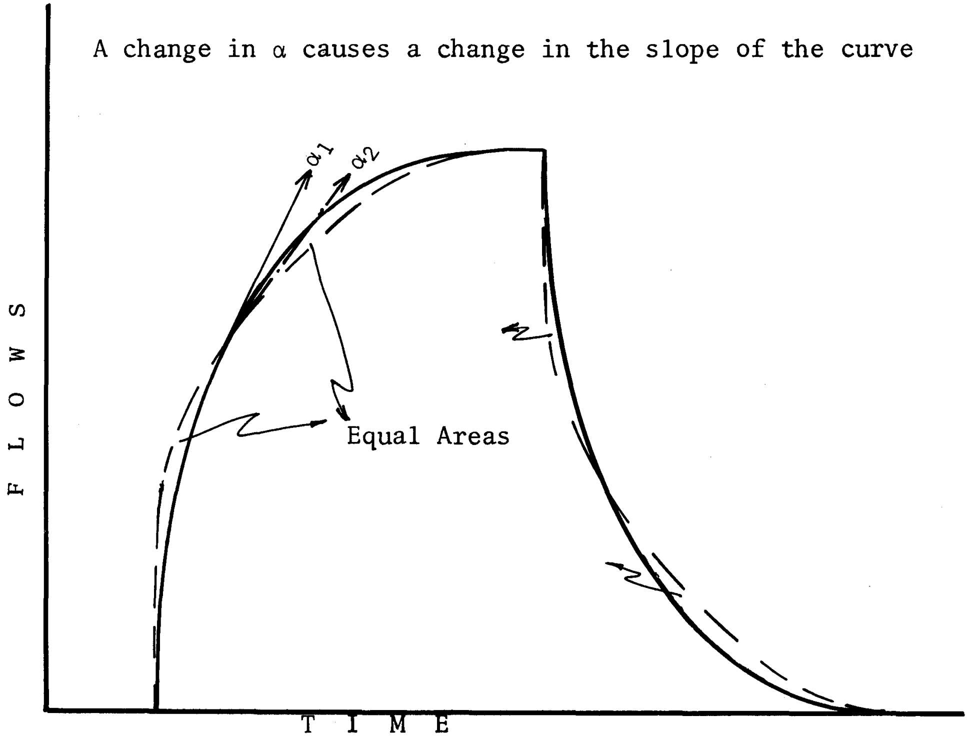

Figure 6. Effect of Changes in a on the Flows.

A value of $a = 0.88$ was found to yield the best fit for the Jordan Canal. Only a slight change $a = 0.89$, was needed to obtain the best fit for the Netufa Canal (Eshkol Canal). The fit between computed and measured data is seen in Figure 8.

DISCUSSION

The method developed, and the values of the parameters provided a satisfactory hydraulic simulator, whose greatest advantages are its simplicity and computational efficiency. The fit obtained between computed and measured data was as good as could be expected, with regard to inaccuracies in the field data. The fit can, in fact, be made to the accuracy of the field data. The accuracy could have improved by using time intervals of 1/2 hour, instead of 1 hour, but the fit with 1 hour intervals is so good that this was not warranted.

It should be mentioned that the method worked well even though the Jordan Canal is interrupted by two large inverted siphons, 720 and 300 meters long (Figure 1). The Netufa Canal is interrupted by a tunnel 800 m long. The Jordan Canal has a trapezoidal section, whereas the Netufa Canal has a curved base. The method developed, lumps all these effects into a very simple simulator, and the results show it works well.

The simulator is easily programmed for a computer, and computation time for a typical simulation run, one week for example, was negligible. The simulator's applicability

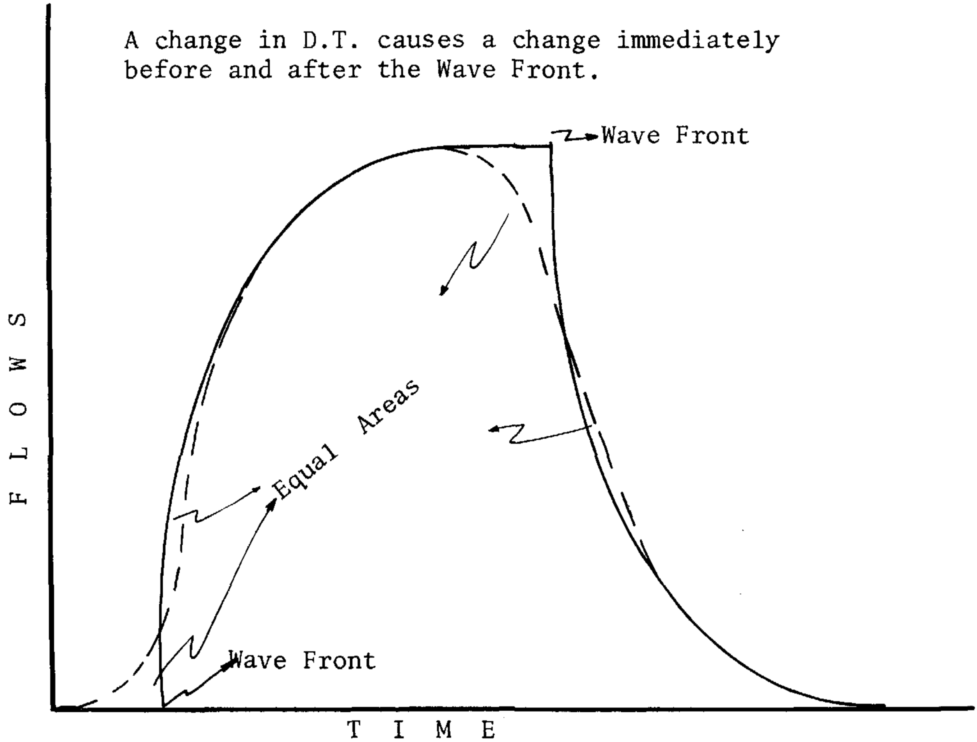

Figure 7. Effect of Changes in Dispersion Time (D.T.) on Flows.

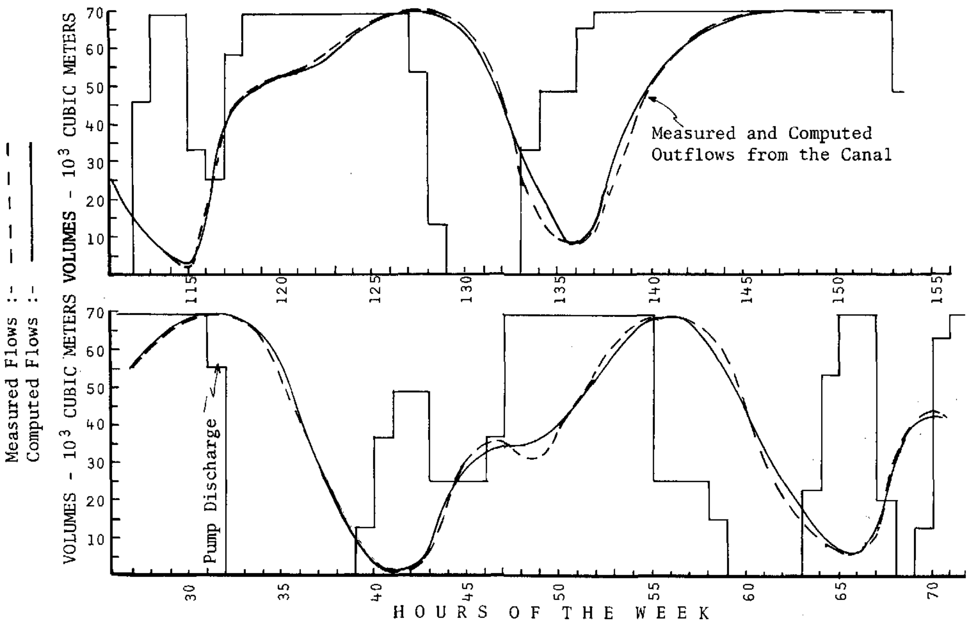

Figure 8. Comparison of Measured and Computed Outflows from the Jordan Canal.

and accuracy has been proved in the hundreds of runs which were made for various purposes.

APPLICATIONS OF THE SIMULATOR

The hydraulic simulator was used as the basic element in a mathematical model whose purpose it was to determine optimal pumping policies for the system, as discussed elsewhere (Damelin and Shamir, 1975).

Other applications were:

(a) To determine operating policies when low levels in the Zalmon Reservoir were to be held for some period, during which maintenance work was carried out. The operators asked that several policies be studied with the simulator before they were implemented.

(b) Discrepancies between the total volume of water pumped and the total supplied to customers where pin-pointed to a specific Venturimeter, by the use of the simulator. Actual dates of cases of tampering with this meter were fixed by such studies.

(c) The simulator can be used as an early warning system for any changes in the pumping capacity of pumps at the two stations, and can help in the diagnosis of incipient mechanical failure.

(d) The simulator is currently used in a study whose aim it is to compare several alternatives for increasing the pumping from the Kinneret to the National Water Carrier. Some modifications had to be made in the program, such as the addition of an option for a fourth pump at the Kinrot station. With these modifications, the model can be used to study the various alternatives for increasing the pumping: more pumping capacity, used during hours of low electric loads, vs. more pumping at peak load hours, using more expensive power.

CONCLUSION

It has been demonstrated that a simple simulator can duplicate with good accuracy the unsteady outflow from a canal, when the inflow is changed by steps. The advantage of this simulator, over numerical schemes for solving the unsteady flow in open channels, is its simplicity and computational efficiency.

LITERATURE CITED

Damelin, E. and U. Shamir, 1975. Optimal Operation of the Pumping Stations in the Kinnereth-Eshkol Section of the National Water Carrier. Journal of Hydrology.