Engineering and Economic Evaluation of the Reliability of Water Supply

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Abstract. A simulation model is developed and used for evaluating the reliability of supplying a known demand pattern in a given water supply system in which shortfalls are caused by random failures of the pumping equipment. An economic model is presented that enables one using the reliability model to compute the cost of the marginal water obtained by improving the reliability of the system and to determine the desirability of the improvement from this cost computation. Results are presented from the initial tests of the reliability model, in which the sensitivity of the reliability to various variables was examined. The results of applying the reliability and economic models to an existing water supply system are presented and discussed.

The reliability of supply is a problem of great concern in the design and operation of water systems. Shortfalls occur when the demand exceeds the supply, at least temporarily. If we assume that the source is sufficiently large and that the installed equipment and the storage in the system are designed so that all demands can be met, then shortfalls will occur only when some of the equipment is not operative due to failure.

Failure of pumping equipment does not occur at regular, prescribed intervals. The length of time that each piece of equipment will operate until the next failure occurs varies randomly. Furthermore, the time required to restore the equipment to operation is also a random variable. Thus equipment becomes unavailable for use during random periods. If the equipment fails during peak demand, the storage available at that time plus the capacity of the remaining equipment may not be sufficient to cover the demand, and shortfalls may occur.

The impact of shortfall in supply varies with the consumer. In municipal water supply, a minor shortfall may merely inconvenience the population, whereas a major one may lead to disastrous results. In agriculture, where water is a resource used in a production system, a shortfall may result in reduction of the yield or even in total loss of the crop. Only in special cases can one evaluate the economic consequence of the shortfall in the supply of water for irrigation. A shortfall in the supply of water to an industrial process may lead to a temporary shutdown, the economic impact of which may not be easy to evaluate. Thus, in general, one cannot define the penalty for shortfalls as a function of their magnitude and time of occurrence.

If shortfalls occur as a result of equipment failure, one can usually guarantee a higher reliability (i.e., reduce the probability of shortfalls) by installing additional equipment or by improving maintenance procedures. Such actions incur costs, and the question is whether these costs are economically justified. One is thus led to examine the reliability of the water supply of a given system, to determine the additional amounts of water that can be supplied by improving the reliability of the system, and to compute the associated cost.

The fact that the pumping equipment installed in a given water system does supply the total amount of water demanded of it is not necessarily a proof of good planning. Possibly a lesser, and thus cheaper, allocation of equipment could have achieved the same result. Furthermore, perfect reliability is not necessarily the best economic alternative. One might prefer a situation in which some shortfalls do

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occur because less equipment is installed if the reduction in equipment entails savings that exceed the economic loss due to shortfalls.

Various components of the pumping equipment can be substituted for one another without affecting the reliability. For illustration consider the relative effect of pumping capacity and storage capacity as shown in Figure 1.

The three isoreliability curves shown in Figure 1 were drawn as continuous lines for simplicity, although pumping capacity normally exists only at discrete points; each curve corresponds to some number of commercially available pumps. Each is the locus of all combinations of pumping capacity and storage that have the same reliability R, all other factors remaining unchanged. The curves demonstrate the substitutability of pumping capacity and storage. On each of these curves there are one or possibly several points corresponding to the least cost combination of pumping capacity and storage that has the given reliability of that curve. Such may be points A, B, and C.

In reality, reliability is affected by many more than two factors. Thus one can imagine multidimensional isoreliability surfaces. Having such surfaces available would be of great value but would not be practical. Each point in the multidimensional space in which reliability is given as a function of all affecting factors has to be generated by a rather lengthy process

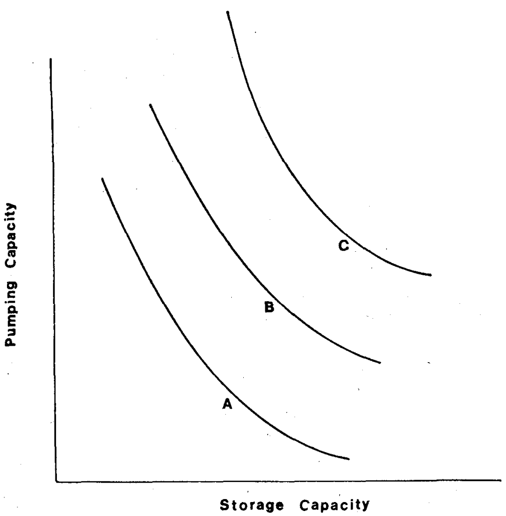

Fig. 1. Isoreliability curves for storage versus pumping capacity.

of numerical simulation, as will be explained below. Thus we retain the isoreliability surfaces as a conceptual rather than practical tool.

The first objective of this work is to devise a means of evaluating the reliability of a given set of pumping equipment serving a given demand in consideration of the random nature of equipment failure and repair duration. This tool is then used to examine the effect on reliability of variations in the design and maintenance procedures at a given or varying demand pattern. Finally, the reliability figures at different designs, maintenance procedures, and demand patterns can be used as the basic building blocks in a model aimed at evaluating any of the following: (1) location of least cost points on isoreliability surfaces, i.e., selection of the pumping equipment in such a way that a required level of reliability is achieved at minimum cost; (2) computation of the cost of the marginal water associated with any variation in design or maintenance procedure aimed at improving system reliability at a given demand pattern; and (3) computation of the cost of the marginal water associated with any variation aimed both at augmenting water supply and at maintaining the same level of system reliability.

RELIABILITY MODEL

The reliability model deals with a single pressure zone within a regional water supply project. A pressure zone is that part of the project in which the pressure is governed by the water level in a single storage reservoir. The zone has one or more sources of supply (such as supply reservoirs and wells), one or more booster stations, several consumers, and a storage reservoir into which water is pumped during periods of low demand and from which water is withdrawn during periods of high demand. For the purpose of this work, we shall lump together all consumers and assume that the combined single consumer draws his water from the storage reservoir.

The system contains fixed elements, such as the sources, the pipelines, the storage reservoir, and the operated equipment, which includes the pumps, their motors, and the related control equipment. We define this operated equipment as pumping equipment. The policy for operating the pumping equipment at normal times is usually based on two considerations: minimization of energy expenditure and reliability. Implementation of this policy is by manual operation or by some means of automatic control, such as that described below.

The amount of water that the system fails to supply during any period of time is called the shortfall for that period. Using this definition, we define the annual reliability factor for year j as R_j equal to 1 minus the ratio of the shortfall during year j to the total demand during year j.

As the shortfall in any year is a random variable, so is R_j. The average reliability factor, defined for years $j = 1, \cdots, T$ as $R = (1/T) \sum_{j=1}^{T} R_j$, is an estimate of the expected value of the probability that the equipment will supply the total demand required of it in any year.

A model was constructed and was used to examine the reliability of a given water system to supply a known demand pattern over the year. The complexity of the way in which the system evolves in time, determined by normal and emergency procedures and driven by random events, precludes the use of an analytic model from which one could explicitly compute a reliability factor. Therefore the model is based on simulation. Through simulation, the model generates synthetic data to simulate the random events of equipment failure and repair and follows the system over time.

Model components and basic assumptions. The following description of the model should be sufficient for an understanding of how the model works and what the results are. Some of the more technical details are omitted, and the interested reader is referred to *Damelin et al.* [1970].

1. The system consists of a pumping station with a number of units and/or some pumping wells, all of which discharge into a single reservoir of known dimensions from which a single consumer draws his water.

2. The consumption is assumed to be known in advance for any hour in the future. The demand pattern follows seasonal, weekly, and hourly variations. The model can also account for an increase in demand over the years.

The demand is assumed to be deterministic, although the simulation model could handle random variations in the demand pattern, because of a lack of sufficient field data to define the parameters of the random component of the demand pattern. Instead we elected to examine the sensitivity of the reliability to variations in the demand pattern, as will be demonstrated in an example below.

3. The pumping equipment comprises the pumps, the motors, and the controls. We have assumed that sufficient data are available to define the probability of times between successive failures of the pumping equipment and the probability of repair durations. The probability distribution functions of these two random variables were assumed a priori on the basis of previous work done on this subject [*Arad*, 1968]. The values of the parameters in these functions are to be estimated from field data.

4. We have assumed that all other equipment used to store and deliver water is perfectly reliable and available for use at all times. Specifically, leaks from the reservoir, pipe bursts, and depletion or contamination of the sources were not considered. Such factors could be easily incorporated into the simulation model, but because of a lack of sufficient data we could not define with any confidence the statistical estimates of parameters of such events.

5. The power supply is assumed to be perfectly reliable; i.e., complete power failures are not considered. The reason is the same as that given in point 4 above. Practice shows, however, that power failures are responsible for some major shortfalls. Power failures make any electrically driven pumping equipment unavailable for use regardless of the number or size of pumps. The total shortfall, however, still depends on the equipment, since when the power supply is restored the capabilities of the equipment will determine the time duration required to restore normal supply. Including power failures in the model is an easy thing to do in principle but can be justified only if sufficient data are available to determine the statistics of their occurrences.

6. Control over the activation and deactivation of pumps under normal conditions is achieved by set points in the reservoir (Figure 2) and/or by a clock. The setting of these points and of the clock is based on considera-

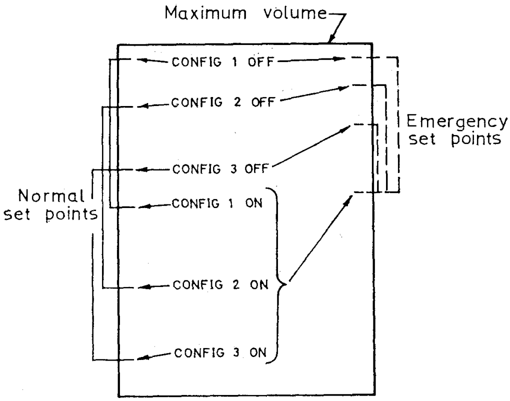

Fig. 2. Activation (when level is falling) and deactivation (when level is rising) of configurations by normal and emergency set points.

tions that are beyond the scope of the present model. We have merely assumed that the set points and clock settings are given.

7. When failure of equipment occurs, all considerations of economy are dropped, save that of supplying the demand, if at all possible. Thus we establish new set points that have been selected so that all available equipment is used to its capacity, the added energy expenditure being disregarded.

The new set points, called emergency set points, are implemented only a lag time after the occurrence of a failure. The lag is the time needed to detect that a failure has occurred and to have someone reach the location and install the new points. A remote data collection system can reduce the lag time substantially by informing the operators of failures the minute they occur. See Figure 1 for an example of normal and emergency set points for three pumping configurations.

Objectives. The reliability model has the following objectives:

1. The model must simulate the performance of pumping equipment in a pressure zone with one reservoir for a given demand pattern under normal conditions and during failures of equipment.

2. The model must compute annual shortfalls in a sequence of runs, each simulating a year of actual operation, and the average value of these annual shortfalls for the entire sequence by simulating the operation over the assumed life-span of the equipment.

3. The model must compute the numerical values of the annual and average annual reliability factors.

4. The model must test the sensitivity of the reliability and of the frequency and magnitude of the shortfalls of a given set of pumping equipment, operating within a pressure zone, to various demand patterns.

5. The model must test the sensitivity of the reliability and of the frequency and magnitudes of shortfalls to design variations (such as number and size of pumps), changes in the reservoir capacity, and the existence or absence of remote monitoring and control equipment.

6. The model must test the sensitivity of the reliability and of the frequency and magnitude of shortfalls to the statistical parameters of the distributions of interfailure times and repair durations.

Implementation of the model. We have assumed that no shortfalls occur when no equipment has failed. This assumption is usually the case in reality, because the normal practice is to install a sufficient combination of pumps and storage to enable the system to supply the maximum hourly demand for which it is designed. This condition was satisfied by all systems considered in this work. The simulation can thus proceed during such normal times in steps of 1 day and does not have to follow the system hour by hour. The model uses data tables that give working hours for each unit during any such normal day and adds up these hours until a day is reached during which some failure occurs. When such a day is reached, the model switches to an hourly simulation.

The hourly simulation is started at midnight, and the reservoir is assumed to be full. The normal set points are used to activate and deactivate standard configurations by following the changes in storage in the reservoir until such time as the failure occurs; this failure may be at any time of the day or night. The same normal set points are then used during the lag time (until the failure has been detected and the emergency set points are put into effect), but now only the units that have not failed are actually activated by the level in the reservoir reaching a set point. Emergency set points (Figure 2) are implemented at the end of the lag time; the hourly simulation continues, keeps track of the changes in reservoir storage, registers any shortfalls, and uses nonstandard configurations. The selection of nonstandard configurations is performed by adding available pumps until a pumping capacity equal to or greater than the normal configuration called for has been activated. This addition is obviously limited by the total capacity still available when one or more of the pumps is in failure. During the hourly simulation, the level changes according to the demand and the pumping rate. If the demand exceeds the total available pumping capacity for a sufficiently long time, the reservoir may be depleted and a shortfall may occur. The total amount of water demanded and not supplied and the duration of the shortfall and the demand level at which the shortfall occurred are noted and recorded. The total amount of water demanded and not supplied is also added up to determine the annual shortfall, from which the annual reliability factor is computed.

The hourly simulation continues until the (generated) repair duration is over and the unit is again available. When one pump is in failure, another may fail as well; this duplication is taken care of in the model. Only when all equipment is available does the model switch back to the daily simulation.

The simulation of a year is repeated several times. Each run results in different values of the annual shortfall and annual reliability factor. From these values one computes the average reliability factor. The number of runs required to yield an acceptable estimate of the true average value is found by experimentation with the simulation model, as will be described later.

Synthetic data for interfailure times and repair durations are generated by the Monte-Carlo method. Interfailure times of pumping equipment are assumed to be random variables with an exponential distribution function. Thus the probability density function of the time to the next failure T is

$$f_T(t) = \lambda e^{-\lambda t} \qquad t \geq 0 \qquad (1)$$

where λ is a constant equal to the reciprocal of the mean time between failures, which in turn depends on the types of equipment. This result is based on previous work [*Arad*, 1968]. The probability that the time to the next failure T will be less than or equal to t hours is given by the following cumulative probability function:

$$F_T(t) = P(T \leq t) = \int_0^t \lambda e^{-\lambda \xi}\, d\xi = 1 - e^{-\lambda t} \qquad t \geq 0 \qquad (2)$$

The specific values of λ have to be determined from actual operation and maintenance records and are supplied to the program as input data.

Repair duration is assumed to be a random variable with a log normal distribution, as shown in previous work [*Arad*, 1968]. The probability density function of repair duration M is

$$f_M(m) = \frac{1}{(2\pi)^{1/2}\sigma(\log m)} \exp\left\{-\frac{1}{2}\left[\frac{\log m - \mu(\log m)}{\sigma(\log m)}\right]^2\right\} \qquad m \geq 0 \qquad (3)$$

where $\mu(\log m)$ is the mean of the logarithms of repair durations and $\sigma(\log m)$ is their standard deviation. Both parameters have to be determined from maintenance records and are supplied to the program as input data. The probability that the duration of a repair M will be less than or equal to m hours is given by the following cumulative probability function:

$$F_M(m) = P(M \leq m) = \frac{1}{(2\pi)^{1/2}} \int_{-\infty}^{x} e^{-(z^2/2)}\, dz \qquad m \geq 0 \qquad (4)$$

where

$$x = \frac{\log m - \mu(\log m)}{\sigma(\log m)}$$

The log normal variate m is defined over the entire range ($0 \leq m < \infty$). The actual repair duration has some lower limit below which minor repairs are not even recorded. The upper limit of the repair duration is in reality the time needed for complete overhaul of the equipment, or the time needed to obtain and install a replacement. In generating synthetic values of the repair durations, we have truncated the distribution at the lower and upper ends and thus forced the values to lie between specified minimum and maximum values. Whenever a generated value was larger than the maximum, it was made equal to the maximum. Similarly, generated values below the minimum were made equal to the minimum. Thus the generated values only approximately follow the log normal distribution. The lag time is added to each generated repair duration to get the total time from the failure to the restoration of the equipment to work.

Computer program. The program is written in PL/1 for the IBM 360/50 computer. The simulation of 1 year is repeated the number of times specified by the data. Running time for simulating 20 years, including input and output, is approximately 5 min for a problem with five pumps and approximately 10 min for a problem with 11 pumps.

The following input data are printed out: pump data, pump configuration data, reservoir capacity, normal set points, lag time, length of the time step, and some other data on controlling the output from the program. The pattern of weekly demand for water over the year is printed out in a graph form.

For each simulated year, the model computes the amount of water actually supplied, the shortfall, the annual reliability factor, the number of times that a shortfall occurred during this year, and some other information on failures and shortfalls. At the same time, the corresponding cumulative quantities are also computed, and all these results are printed out, as shown in Tables 5*a* and 5*b* for a sample problem.

ECONOMIC MODEL

Reliability has an economic value. Perfect reliability is not necessarily the best economic solution, as has already been mentioned. To be able to compute the penalty due to imperfect reliability, one has to assign an economic loss function to shortfalls according to their magnitude and the time at which they occur. We consider this assignment of economic loss functions to be impossible, at least for the moment, since the actual value of water as a resource used by some production system, say agriculture, has not been defined to everyone's satisfaction. We have, therefore, taken a somewhat different approach.

Economic evaluation of alternative designs of a new project. First, during the design of a new installation, one can consider alternative sets of pumping equipment, all of which have the same reliability for the given demand pattern. Then one can choose the most economic set among them. To accomplish this decision, one runs the simulation model for each alternative and adjusts some or all of the decision variables in the model (such as the number and sizes of units installed or the size of the storage reservoir) until the desired reliability is achieved. Then one selects the least costly alternative.

Any proposed changes in the system that are expected to change its reliability can be evaluated with the aid of the economic model. Examples of such changes are: (1) addition or deletion of pumps or wells, (2) changes in normal or emergency operating procedures as defined by the corresponding set points, (3) changes in reservoir capacity, (4) changes in maintenance policies aimed either at increasing or at possibly decreasing interfailure times or at reducing repair durations, (5) installation of monitoring equipment that gives immediate indication at a control center of failures to enable a reduction in the lag time, (6) installation of automatic control equipment, with or without central control by an on-line computer, to enable instantaneous response to failures by remote activation of emergency procedures.

By testing the sensitivity of the reliability to the values of the parameters of interfailure times and repair durations, one can determine how much effort and money should be put into a program for collecting more extensive and more accurate field data. The higher the sensitivity of reliability to these parameters, the more one should be willing to spend on obtaining accurate data for defining them.

There are side benefits that have not been considered in the economic model and that in the present formulation of the model can only be termed intangibles. These side benefits (such as the desire of the water supplier to avoid any shortfalls even at high cost) should be subjectively considered by the decision maker to maintain the consumers' confidence in his supply.

Economic evaluation of design variations of *an existing system to improve its reliability.* One use of the economic model is in the evaluation of design variations such as modifications or additions to an existing system. When one is not satisfied with the reliability of a system, one may propose adding pumps, augmenting the storage, improving the maintenance policy, or installing monitoring and/or control equipment. Each such proposal has to be evaluated economically by use of the following model.

Let $S_{n,j}$ be the shortfall during year j of operating the nth alternative of equipment (hereafter referred to as state n of the system), $S_{n+1,j}$ be the corresponding shortfall for state $n + 1$, and i be the prevailing interest rate. Let M_{n+1} be the capital cost of changing the system from state n to state $n + 1$, and let $E_{n+1,j}$ be the associated change in annual operating cost in year j. The cost C_{n+1} of the marginal water gained by improving the reliability of the system from that of state n to that of state $n + 1$ is computed from

$$M_{n+1} + \sum_{j=1}^{T} \frac{E_{n+1,j}}{(1+i)^j} = C_{n+1} \sum_{j=1}^{T} \frac{(S_{n,j} - S_{n+1,j})}{(1+i)^j} \quad (5)$$

where T is the life-span of the project. The value C_{n+1} was taken to be constant over time. To compute C_{n+1}, one has to know M_{n+1}, $E_{n+1,j}$, and T; one also has to run the reliability model twice, once for state n to obtain the values of $S_{n,j}$ and once for state $n + 1$ to obtain the values of $S_{n+1,j}$.

The proposed modification from a given state n to another state $n + 1$ now has to be evaluated on the basis of the corresponding C_{n+1}.

The variable C_{n+1} is random. For each pair of shortfall sequences $S_{n,j}$ and $S_{n+1,j}$, $j - 1, \cdots, T$, one value of C_{n+1} is computed. To get an estimate of the expected value of C_{n+1}, one has to repeat the simulation of T years many times and find the average value of C_{n+1}. In our work we have performed only one T-year simulation for each design alternative and used the single value of C_{n+1} as computed from (5). This marginal cost can be compared either to the marginal cost of some other proposed modification that is designed to bring the reliability to the same improved level or to the cost of augmenting the supply from some other source. The decision maker must choose the course of action on the basis of the results of computing C_{n+1}.

Economic evaluation of design variations of an existing system to allow for increased demand at a given reliability. Another use of the economic model is in the evaluation of design variations aimed at satisfying increased water demand and maintaining the reliability level unchanged. Let $Q_{n,j}$ be the supply during year j of operating the nth alternative of the system, $Q_{n+1,j}$ be the supply during year j of operating the $(n + 1)$th alternative of the system, and i be the prevailing interest rate. As the reliability at state n is identical to that at state $n + 1$, the cost of the marginal water gained by changing from state n to state $n + 1$ is computed from

$$M_{n+1} + \sum_{j=1}^{T} \frac{E_{n+1,j}}{(1+i)^j} = C_{n+1} \sum_{j=1}^{T} \frac{Q_{n+1,j} - Q_{n,j}}{(1+i)^j} \quad (6)$$

where M_{n+1} is the capital cost of changing the system from state n to state $n + 1$, $E_{n+1,j}$ is the associated annual cost in year j, T is the life-span of the project, and C_{n+1} is the cost of the marginal water.

To compute C_{n+1}, one has to run the reliability model for state n and then several times for state $n + 1$, each time increasing the supply $Q_{n+1,j}$, until the reliability of state $n + 1$ approaches that of state n. At this point, we compute C_{n+1} from (6). To compute C_{n+1}, one has to know M_{n+1}, $E_{n+1,j}$, and T.

INITIAL TESTS OF THE RELIABILITY MODEL

A purely imaginary pressure zone was set up for testing the reliability model. Data were selected to describe a system that, although nonexistent, is representative of real systems. The pressure zone contained a single source: a pumping station with three parallel units, all of which discharged through a single pipe into a storage reservoir from which a known hourly demand is to be drawn. The demand chosen was higher than that expected in a real system with similar pumping equipment. This choice was made with the aim of artificially increasing the number and magnitude of shortfalls so that a larger sample could be generated in a relatively small number of simulated years.

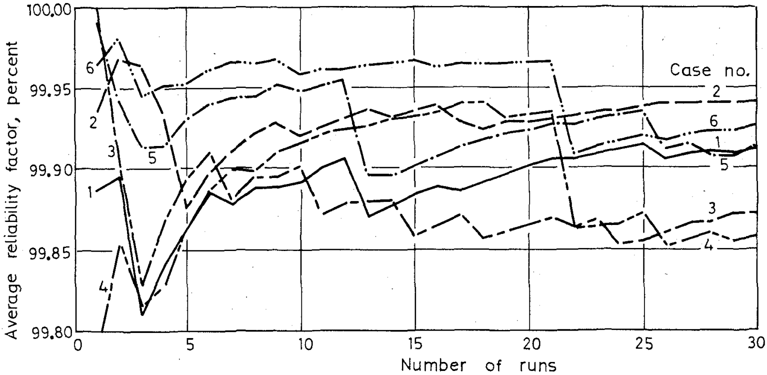

Fig. 3. Average reliability factor versus number of runs of cases 1–6 of the test problem.

Six different cases were examined. In each case some of the parameters that affect reliability were changed. These included changes in the statistical parameters of interfailure times and repair durations and changes in the positions of the set points that determine the operation of the pumps. In each case, the program was run to simulate 30 years of operation. Full details of all the cases, the results and their discussion can be found in *Damelin et al.* [1970]. Figure 3 is a summary of the results and shows, for each case, the change in the average yearly reliability factor with the number of simulated years. The average value tends to stabilize after a relatively small number of simulated years, although some large variations do occur later as well, e.g., cases 3 and 4 in the twenty-second year. These variations correspond to simulated years in which extreme although relatively rare shortfalls have occurred. These large shortfalls resulted from the artificially large demand that was imposed on the system.

From the initial tests we learned that a run of 20 simulated years is sufficient to generate a representative sample of annual reliability factors, as can be seen from Figure 3. Experience was also gained during these runs concerning the relative effect of various parameters on the reliability. This experience was then used to plan the simulation runs for real systems, as will be described below.

TESTS ON REAL SYSTEMS

Two water supply projects were selected for the tests of the reliability and economic model. One contains a large pumping station with four pumps and draws its water from a large surface reservoir. The other contains wells and a booster station. Results will be given here only for the latter, the Nahal Oren project, which supplies part of the city of Haifa.

The project, shown schematically in Figure 4, consists of eight pumping wells and a booster station with two units; this equipment pumps

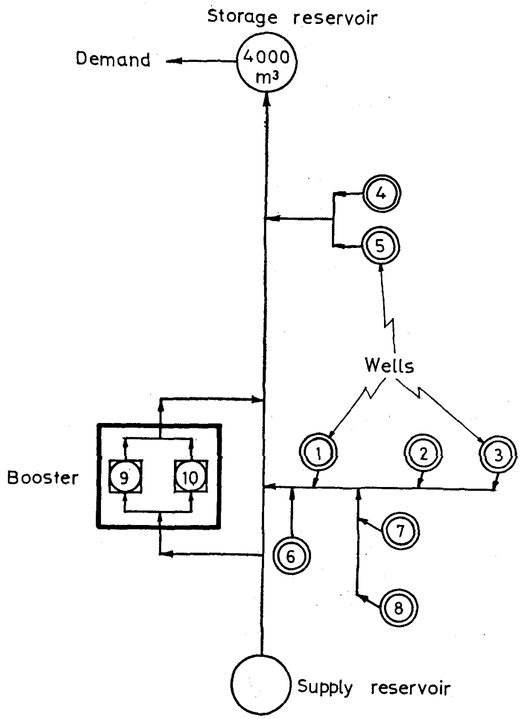

Fig. 4. Schematic layout of the Nahal Oren project.

TABLE 1. Pump Data from the Nahal Oren Project

Pump No.	Capacity, m^3/hr	Mean Time between Failures, hours	Mean Repair Duration	Logarithm of Repair Duration		Clock Limitations	
				Mean	s.d.	Hour Off	Hour On
1	115	1300	50	1.2217*	0.6977*	...	...
2	280	650	50	1.2217	0.6977	...	...
3	280	1500	52	1.2269†	0.7007†	...	...
4	115	1100	50	1.2217	0.6977	1	5
5	153	900	50	1.2217	0.6977	1	5
6	130	950	50	1.2217	0.6977	1	11
7	350	800	52	1.2269	0.7007	...	...
8	100	1200	50	1.2217	0.6977	...	...
9	450	1000	52	1.2269	0.7007	11	16
10	395	1000	52	1.2269	0.7007	6	23

Clock limitations are given for April to September only.
* 4% of the durations are below 1 hour, and 1% are over 700 hours.
† 4% of the durations are below 1 hour, and 1% are over 720 hours.

into a reservoir with a capacity of 4000 m^3 from which the demand is drawn. The distance between the wells is several hundred meters, and the length of the entire system is some 10 km. In addition to the wells, the project is supplied from a reservoir that is kept full by another project.

Table 1 contains pump data. Standard and nonstandard configuration data appear in Tables 2 and 3, respectively. The weekly demand quantities are shown in Figure 5. The daily demand patterns are shown in Figure 6. In each week Sunday had a type 1 pattern, Monday–Thursday had a type 2 pattern, Friday had a type 3 pattern, and Saturday had a type 4 pattern. In most cases lag time was taken to be zero, except in case 5 where it was taken to be 6 hours and in case 6 where it was taken to be 12 hours.

Test runs. In case 2 the statistical parameters of repair durations for all pumps were changed from those of case 1 given above to the following: the mean repair duration for all pumps was set equal to 67 hours; i.e., the mean of the logarithms of repair durations equaled 1.2882, and the standard deviation of the logarithms of repair durations equaled 0.7357. This designation corresponds to 4% of the repair durations being below 1 hour and 1% of the durations being over 1000 hours.

In case 3 another pump was added. This

TABLE 2. Standard Configuration Data from the Nahal Oren Project

Configuration No.	Pumps	Combined Capacity, m^3/hr	Configuration On (Level Falling) at Volume, m^3	Configuration Off (Level Rising) at Volume, m^3
1	none	0	...	...
2	1	115	2970	3828
3	1–2	395	2970	3828
4	1–3	675	2970	3828
5	1–4	790	2970	3828
6	1–5	943	2970	3828
7	1–6	1073	2970	3828
8	1–7	1423	2904	3630
9	1–8	1523	2904	3630
10	1–9	1973	2838	3564
11	1–11	2368	2838	3564

TABLE 3. Nonstandard Configuration Data from the Nahal Oren Project

Pump No.	Capacity, m^3/hr	Cumulative Capacity, m^3/hr	Pump On at Volume, m^3	Pump Off at Volume, m^3
1	115	115	3600	3828
2	280	395	3600	3828
3	280	675	3600	3828
4	115	790	3600	3828
5	153	943	3600	3828
6	130	1073	3600	3828
7	350	1423	3600	3700
8	100	1523	3600	3700
9	450	1973	3600	3650
10	395	2368	3600	3650

pump had all the same physical and statistical properties as pump 2 of case 1 above and was designated as pump 3; all subsequent pump numbers were increased by 1.

In case 4 the reservoir capacity was increased by 50% (from 4000 m^3 to 6000 m^3), and the settings of both normal and emergency set points were correspondingly increased; e.g., pump 1 on at 2970 m^3 was raised to 4455 m^3 and so on. All other data were the same as those in case 1.

Cases 5 and 6 had the same data as case 1, except that lags of 6 hours and 12 hours, respectively, were introduced. Case 7 had the same data as case 1 except that the demand pattern was uniformly increased by 10%. For each case the program was run to simulate 20 years of operation.

Results. A summary of the results for the seven cases is presented in Tables 4*a* and 4*b*. Typical output from the computer program appears in Tables 5*a* and 5*b*, which show the results for case 1 after 20 years.

The following cumulative data have also been calculated for the 20-year period:

1. The volume of water not supplied (shortfall) totaled 726,773 m^3.

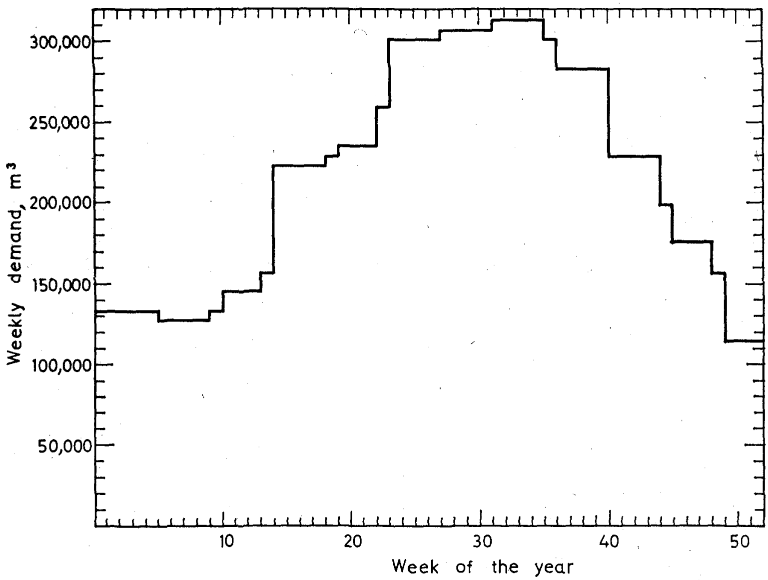

Fig. 5. Weekly demand pattern of the Nahal Oren project.

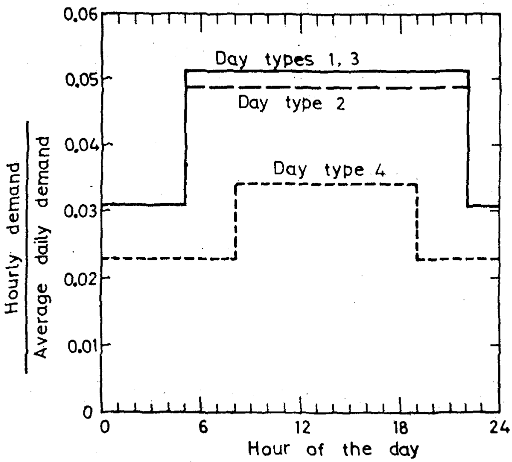

Fig. 6. Hourly demand pattern of the Nahal Oren project.

2. Shortfall occurred during 2130 hours.
3. The volume of water pumped was 221,718,286 m³.
4. The average reliability factor for the 20-year duration was 0.996722.
5. The present worth of the shortfall up to the twentieth year was 323,007 units. This information is needed for evaluation in the economic model.

The following noncumulative data have been calculated for the twentieth year:

1. The shortfalls during this year occurred seven times during 38 hours and totaled 6868 m³.
2. The reliability factor for this year was 0.999383.
3. The longest shortfall occurred for 11 hours and had a volume of 2006 m³.
4. The next longest shortfall occurred for 11 hours and had a volume of 2506 m³.

Analysis of results. The distributions of total average shortfalls that occurred at varying rates of shortfall are shown for all cases in Figures 7 and 8. The data are extracted from the first line of the shortfall distribution tables, e.g., Table 5*b* for case 1.

The distributions of the probability of occurrence of shortfalls at various weekly demand levels for cases 1 and 6 are shown in Figure 9, and those for case 7 are shown in Figure 10. For each demand level D, the curves show the probability that at any time the shortfall will be equal to or greater than any specified hourly rate q; i.e., $P[\text{shortfall} \geq q] |_D$. The probabilities were calculated by dividing the average annual number of hours during which shortfall occurred, in all weeks in which the demand level remained within certain limits, by the total number of hours in these weeks. For example, in case 7, the weekly demand remained within the limits of 343,800 ± 6600 m³/wk for a period of 2184 hours (13 weeks). Shortfalls of 450 ± 50 m³/hr occurred during an average of 41.5 hr/yr. Therefore the probability that shortfall at this rate will occur during these weeks is 41.5/2184 = 0.01901.

In case 2 the effect on the average reliability of a 40% increase in the maximum repair duration (from 700 to 1000 hours) was tested. The result was an 18% increase in the average annual shortfall. In case 1 the maximum shortfall rate was 900 m³, and the penetration into weekly demands was between 264,000 and 288,000 m³. In case 2 the maximum shortfall rate was 1100 m³ and the penetration into weekly demands was between 240,000 and 264,000 m³.

The addition of an eleventh pump in case 3 greatly reduced the average annual shortfall (compared to that of case 1) from 36,000 m³ to 3000 m³. The maximum shortfall rate and the shortfall penetration were similar to those of case 1, but the hours during which shortfall occurred were reduced from 2130 hours to 304 hours.

The addition of reservoir capacity in case 4 resulted in a reduction of average annual shortfall from 36,000 m³ to 11,000 m³. The maximum shortfall rate and the penetration did not change, but the hours during which shortfall occurred were reduced from 2130 hours to 582 hours. A further comparison of case 1 with case 4 will be dealt with in the application of the economic model.

Cases 5 and 6 test the effect of adding lag time of 6 and 12 hours, respectively. At first glance it appears that the addition of a lag actually increases the reliability of the project, a result that appears to be unreasonable. This result is largely accounted for by the extremely large shortfall of the worst year in case 1, which amounted to 224,000 m³. The comparison of case 5 (lag time of 6 hours) with case 6 (lag time of 12 hours) shows the anticipated decrease in the average reliability factor. Significant

TABLE 4*a*. Results for the Worst Year of Seven Tests from the Nahal Oren Project

Case	Average Reliability	Average Annual Shortfall, m^3	No. of Shortfalls	Duration of Shortfalls, hours	Reliability Factor	First Principal Shortfall		Second Principal Shortfall		Total Volume for Year, m^3
						Volume, m^3	Duration, hours	Volume, m^3	Duration, hours	
1	0.99672	36,000	30	385	0.97942	12,000	17	12,000	16	224,000
2	0.99613	43,000	28	396	0.98382	52,000	83	31,000	43	177,000
3	0.99977	3,000	7	50	0.99873	7,000	13	3,000	12	14,000
4	0.99900	11,000	20	203	0.99152	13,000	17	12,000	17	94,000
5	0.99797	23,000	20	193	0.99366	6,000	14	7,000	14	70,000
6	0.99689	35,000	21	237	0.99055	17,000	32	9,000	15	104,000
7	0.99252	91,000	60	698	0.97052	16,000	20	16,000	17	349,000

TABLE 4*b*. Results for a Typical Year of Seven Tests from the Nahal Oren Project

Case	Average Reliability	Average Annual Shortfall, m^3	No. of Shortfalls	Duration of Shortfalls, hours	Reliability Factor	First Principal Shortfall		Second Principal Shortfall		Total Volume for Year, m^3
						Volume, m^3	Duration, hours	Volume, m^3	Duration, hours	
1	0.99672	36,000	29	197	0.99612	4,000	12	3,000	12	43,000
2	0.99613	43,000	18	140	0.99611	8,000	16	5,000	11	43,000
3	0.99977	3,000	3	12	0.99977	2,000	10	0	1	3,000
4	0.99900	11,000	6	35	0.99913	3,000	9	3,000	9	10,000
5	0.99797	23,000	7	54	0.99799	7,000	14	4,000	12	22,000
6	0.99689	35,000	13	98	0.99682	6,000	15	6,000	13	35,000
7	0.99252	91,000	33	308	0.99171	7,000	15	5,000	15	100,000

information is, however, supplied by Table 5*b*, showing the distribution of shortfalls in the printed output. Here it was found that the penetration of shortfall for case 1 (lag time of zero) occurred only to the weekly demand level between 264,000 m^3 and 288,000 m^3, i.e., during 17 weeks of the year. Lag time of 12 hours (case 6) resulted in penetration down to the weekly demand level between 192,000 m^3 and 216,000 m^3, i.e., during 31 weeks of the year. The distribution of shortfall at varying weekly demand levels is shown for cases 1 and 6 on Figure 9. The conclusion we may draw from the above results is that the system becomes progressively prone to shortfall at lower weekly demand levels as the lag time increases, but, despite this tendency, there are no significant increases in the shortfall because of the following:

1. The system contains 10 pumping units and the loss of one pump for a few hours does not represent a large proportion of the total pumping potential. Generally speaking, a pumping system is more stable as the number of pumping units employed becomes larger. During lag time, shortfalls at deeper penetration occur, but thereafter rapid recovery takes place when emergency procedures are initiated.
2. The control set points are so arranged that six pumps are activated simultaneously. The probability is high that not all six pumps would be required to supply the required demand and that five pumps could suffice.

Lag time is directly linked to the speed at which failure information is transferred to the operator. Installation of a remote control into which information is continuously fed may decrease lag time to a minimum. Lack of such a center may result in lag time of several hours; i.e., prolonged time may elapse from the moment of failure occurrence until the operator realizes that a failure has occurred, activates emergency procedures, and commences repair.

The analysis of case 1 with a lag time of 0 hours (immediate response of the operator to failures), case 5 with a lag time of 6 hours, and case 6 with a lag time of 12 hours sheds doubt on the economic justification of investing in a remote control center. For the reasons enumerated above, the project reliability is hardly sensitive to lag time, as long as this lag does not exceed 12 hours.

Case 7 represents a 10% uniform increase in the weekly demand. Observe that the increased demand can be achieved at the expense of an increase in the average annual shortfall to 91,000 m^3. The shortfall of the worst year amounted to 349,000 m^3. The average number of hours of shortfall per annum was 257, and the penetration of shortfall was down to the weekly demand level between 238,000 m^3 and 264,000 m^3. The distribution of the accumulated volume of shortfall at various shortfall rates for this case is given in Figure 8.

Application of the economic model. The economic model was used to assess two alternatives for the improvement of the reliability of the project. The first alternative is the improvement of the reliability of the project through the addition of a well (case 3). In case 1, which is the base case, the discounted value of the accumulation of shortfall over the life-span of the project was 323,007 m^3; i.e., $\sum_{j=1}^{T}$ $[S_{1,j}/$

TABLE 5*a*. Reliability Simulator Output after Twentieth Year from the Nahal Oren Project

Pump	On/Off Switches	Whole Days Idle	Total Hours in Repair	Total No. of Failures	Total Hours Worked
1	43,053	3	5426	83	108,210
2	41,503	3	9028	158	104,793
3	43,919	3	4888	94	109,093
4	45,373	3	5400	106	126,416
5	44,698	3	5403	135	125,140
6	36,743	395	4331	90	74,618
7	41,387	1012	3919	107	81,704
8	42,404	3	4513	78	96,126
9	47,000	6	6387	120	111,716
10	41,576	7	3279	64	70,271

TABLE 5*b*. Reliability Simulator Output after Twentieth Year from the Nahal Oren Project

Weekly Demand, m^3	Average Shortfall, hr/yr															
	100 m^3/hr	200 m^3/hr	300 m^3/hr	400 m^3/hr	500 m^3/hr	600 m^3/hr	700 m^3/hr	800 m^3/hr	900 m^3/hr	1000 m^3/hr	1100 m^3/hr	1200 m^3/hr	1300 m^3/hr	1400 m^3/hr	1500 m^3/hr	1600 m^3/hr
24,040	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	.0.	0.0	0.0	0.0	0.0
48,080	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
72,120	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
96,160	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
120,200	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
144,240	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
168,280	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
192,320	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
216,360	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
240,400	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
264,440	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
288,480	0.6	0.8	2.9	2.5	1.0	0.6	0.9	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
312,520	6.9	24.0	23.1	15.2	4.8	2.6	10.1	9.6	0.5	0.0	0.0	0.0	0.0	0.0	0.0	0.0
Total	7.6	24.8	26.0	17.8	5.8	3.3	11.0	9.6	0.5	0.0	0.0	0.0	0.0	0.0	0.0	0.0

The weekly demand levels are arranged on a scale from zero to the maximum weekly demand. For example, there were on the average 24 hr/yr during which a shortfall of 100–200 m^3 occurred during weeks when the weekly demand was greater than 288,480 m^3 and as much as 312,520 m^3.

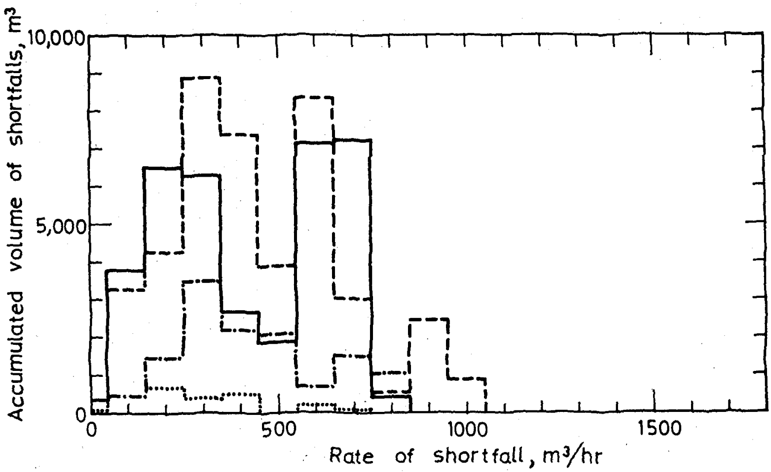

Fig. 7. Distribution of shortfalls for cases 1–4 of the Nahal Oren project. The solid line represents case 1 in which the maximum repair duration was 700 hours. The dashed line represents case 2 in which the maximum repair duration was 1000 hours. The dotted line represents case 3 in which one pump was added. The dash-dot line represents case 4 in which the reservoir was enlarged.

$(1 + i)^j] = 323{,}007$ m³, where $S_{1,j}$ is the shortfall in the year j for case 1, T is the life-span of the project (taken here to be 20 years), and i is the interest rate (taken here to be 8%). This value results from exercising the reliability model for case 1.

Repeating for case 3, we find that $\sum_{j=1}^{20} [S_{3,j}/(1 + 0.08)^j] = 23{,}767$ m³. The difference in the discounted value of the accumulation of the shortfalls over the life-span of the project is 299,240 m³.

The cost of providing and equipping a new well is estimated to be I£225,000 (Israeli pound; \$1 = I£3.5). The annual maintenance cost is estimated to be I£12,500. Using (5), we find that the cost of the marginal water gained by varying the project design from case 1 to case 3 is $C_3 =$ I£1.51/m³.

The second alternative is the improvement of the reliability of the project through the augmentation of the storage capacity of the project by 2000 m³ (case 4). Here, the discounted value of the accumulation of shortfalls over the life-span of the project is 104,084 m³. The difference in this value between case 1 and case 4 is 218,923 m³.

The cost of the additional storage is estimated at I£220,000 with negligible annual maintenance cost. Using (5), the cost of the marginal

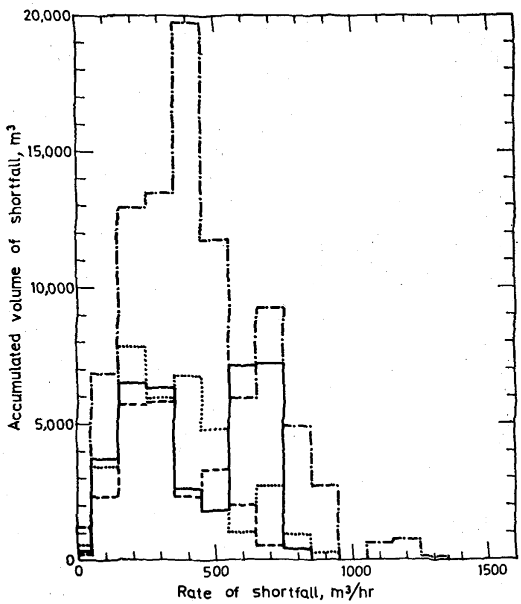

Fig. 8. Distribution of shortfalls for cases 1, 5, 6, and 7 of the Nahal Oren project. The solid line represents case 1 in which the maximum repair duration was 700 hours. The dashed line represents case 5 in which the lag time was 6 hours. The dotted line represents case 6 in which the lag time was 12 hours. The dash-dot line represents case 7 in which the demand was increased by 10%.

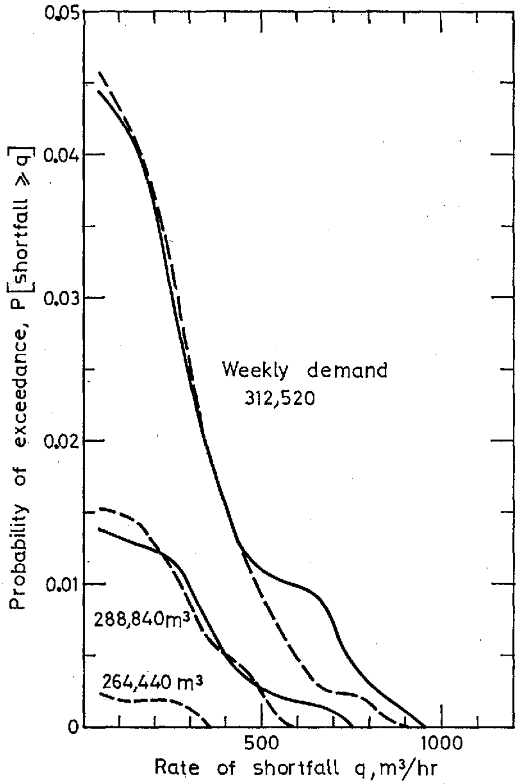

Fig. 9. Probability of occurrence of shortfalls for cases 1 and 6 of the Nahal Oren project. The solid line represents case 1, and the dashed line represents case 6.

water gained by varying the project design from case 1 to case 4 is $C_4 = \text{I£}1.00/\text{m}^3$.

Conclusion. The immediate conclusion from the above section is that the addition of storage capacity is preferable despite the higher reliability involved in the addition of a pumping unit. Such a conclusion should, however, be tested against the added benefit endowed by an extra pumping unit as a potential for increasing the project supply. One may now increase the demand to a point where the reliability of case 3 decreases to that of case 4; one may then apply the economic model. The added supply capability will lower the cost of the marginal water of case 3. The comparison between the two cases, now based on the same reliability for the two cases, is more meaningful. Our model cannot perform this comparison because we have assumed that the benefit function for added supply is not known.

SUMMARY AND CONCLUSIONS

Water projects are continuously undergoing alterations in their physical layout partly because of changes in the demand pattern and partly because of the ever existing desire by the operators to save costs. In some cases, the physical alteration is aimed at both meeting a new demand pattern and saving costs.

A given physical layout of a project is intimately associated with the reliability of water supply. Any change in the layout will immediately involve some change in the reliability. This work establishes the tie between a given physical layout of a pressure zone within a water project and the associated reliability of water supply. The water project is defined by (1) the capacity of each pumping configuration, (2) the volume of the storage reservoir, (3) the control routine (set points and clock limitations), (4) the statistical parameter of the interfailure time distribution for each pumping unit, (5) the statistical parameters of the repair duration distribution for each pumping unit, (6) the hourly demand pattern, and (7) the time lag from failure to the initiation of emergency procedures.

Exercising the reliability model yields (1) the

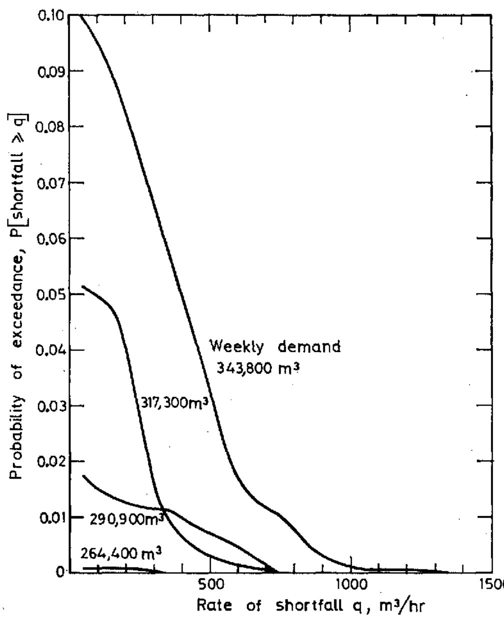

Fig. 10. Probability of occurrence of shortfalls for case 7 of the Nahal Oren project.

number and volumes of shortfall occurrences and the number of hours during which shortfalls occurred for each year in the life-span of the project, (2) the yearly and average reliability of water supply, and (3) the average number of hours in the life-span of the project during which shortfalls occurred, categorized by the volume of the shortfall and the demand level in which the shortfall occurred.

This work also allows the determination of the marginal cost of water associated with any alteration in the physical layout of the project. Determining the marginal cost of water is a step of prime importance in evaluating the economics of proposed alteration in the existing project. Also, in cases in which more than one alternative suggestion exists, this work allows the choice of the economically preferred alternative. Alterations in the physical layout may include (1) installing additional pumping units, (2) dismantling redundant pumping units if such exist, (3) enlarging the storage capacity, (4) changing maintenance procedures to affect interfailure time, and (5) changing control routines and/or installing a manned or an automatic remote control center. Exercising the economic model on any alteration of the physical layout for a given interest rate yields the marginal cost of water associated with the alteration.

REFERENCES

Arad, N., A method for the evaluation of the system and cost-effectiveness of large multistage flash desalting plants, *Rep. R-1142,* 265 pp., Plann. Res. Corp., Washington, D. C., 1968.

Damelin, E., U. Shamir, and N. Arad, Engineering and economic evaluation of the reliability of water supply in regional water projects, *Rep. 1603,* 79 pp., Mekoroth Water Co., Tel-Aviv, Israel, 1970.

Hanke, S. H., Demand for water under dynamic conditions, *Water Resour. Res., 6*(5), 1253–1261, 1970.

Turnovsky, S. J., The demand for water, Some empirical evidence on consumers' response to a commodity uncertain in supply, *Water Resour. Res., 5*(2), 350–361, 1969.

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revised February 2, 1972.)