Optimal Route for Pipelines in Two-Phase Flow

URI SHAMIR | ISRAEL INSTITUTE OF TECHNOLOGY*
HAIFA, ISRAEL

ABSTRACT

A technique is described, which makes it possible to select the optimal route for a pipeline designed to carry oil and gas in two-phase flow. The pipeline is assumed to operate under the pressure differential naturally available between the source and the point of delivery.

A discrete grid is established to describe the corridor through which the pipeline is to pass. Topographic and terrain data are given for all grid points. Cost data is given for all factors which affect the capital cost of the pipeline. The equation for the two-phase flow becomes a global constraint, to be satisfied by the selected route. Dynamic programming is then used to solve the minimization problem.

A computer program is described, with which a sample problem was solved, and the results that were obtained are also presented.

INTRODUCTION

Great sums of money are spent annually on the construction of pipelines for the oil industry. Many of these pipelines are designed to carry gas and oil in simultaneous two-phase flow from wells to various collecting and processing facilities. The procedures for selecting the route for such pipelines have followed the traditional approach of engineering judgment and selection of the cheapest among a few alternative routes laid out by hand on maps and aerial photos.

Aerial photo interpretation, to yield soil types, tree cover, existence of swamps and muskeg, and other factors affecting costs, is being used in route selection. Geologists and soil engineers are brought in to evaluate soil conditions on the basis of aerial photos, as well as by examination of the route itself and soil samples.

This data is then used to select a route and to design the pipeline. The present project was undertaken with the objective of improving the engineering practice. We sought to proceed beyond the stage of mere trial and error and to develop a rigorous method for determining the optimal route by using the techniques of systems engineering.

The over-all problem of conveying fluids in one- and two-phase flow pipelines was reviewed. It ranges from a single pipe carrying a single-phase fluid, through two-phase flow lines, to gathering systems containing networks of pipes and other equipment, such as valves and compressors, to collect the products of a large number of wells and deliver the mixed product to processing plants. All these were considered part of the over-all project, which deals with optimal design of pipeline systems.

Initially, one aspect of the over-all project had to be selected. It was decided to tackle the problem of optimizing the route for a single pipeline carrying two-phase flow. This problem presents some complications, and it was felt that if it could be solved, single-phase pipelines would present no added difficulties.

TWO-PHASE FLOW PIPELINES

It is common practice in the oil industry to use a single pipe to carry both oil and gas from producing wells to collecting facilities and plants. The alternative is to separate the two phases at the source and carry them in separate pipelines. Economics of the two alternatives should be the basis for a choice between them. The present work is therefore a useful tool for making a better choice possible by yielding the optimal solution for the two-phase line alternative. As will be shown later, the method, as well as the computer programs, can also be used to determine the optimal route for a pipeline carrying flow of a single fluid.

COMPUTING SIMULTANEOUS FLOW OF LIQUID AND GAS IN A PIPELINE

The regime of flow in a pipeline carrying both liquid and gas depends on many parameters. The regime, in turn, determines the pressure losses along the pipeline. The procedures for computing the two-phase flow are both elaborate and rather inaccurate. No attempt is made in the present work to change or to improve the existing methods, as this is beyond its scope. We do need, however, to modify the sequence of the computations to suit the requirements of the optimization problem.

As will be explained later, the two-phase flow equation will constitute a constraint in the

Original manuscript received in Society of Petroleum Engineers office Nov. 10, 1969. Revised manuscript of paper (SPE 2836) received March 25, 1971.

[1]References given at end of paper.

*This work was undertaken while the author was staff consultant at Underwood McLellan & Associates Ltd., Calgary, Alta.

optimization problem. The constraint has to be specified before the route is selected. Therefore, it has to be formulated before the length of the pipeline and the "sum of the ups"* in it are known. It will be shown that the two-phase flow constraint can be written in the form

$$(\Delta p_L) L + (\Delta p_h)\, h \leq \Delta p \;, \qquad (1)$$

where L = length of the pipeline, ft

h = sum of the ups, ft

Δp = available pressure difference between the ends of the pipeline

Δp_L = pressure loss per foot of length, due to friction

Δp_h = pressure loss per foot of "hills", due to hills

There are in use several procedures for computing Δp_L and Δp_h. In the present study, we have used the Flanigan method[1] to compute the coefficients Δp_L and Δp_h (see Appendix).

Flanigan's method is admittedly simple. Nevertheless, it is widely used, probably because of its simplicity. In the present work another method, that of Baker,[2] was also used. In some cases the two methods gave results that were significantly different. This indicated that more work has to be done to improve the state of our knowledge about pipelines carrying gas and liquid in two-phase flow under field conditions. This aspect of the problem was not, however, the one dealt with in the present work. All we need is to be able to state that the coefficients Δp_L and Δp_h of Eq. 1 can be computed *before* a route is selected, and there is *some* accepted procedure for computing them. Thus, for the present we accept Δp_L from Eq. A-5 and Δp_h from Eq. A-6 as the coefficients needed to formulate the two-phase flow constraint for the optimization problem. Note again that the length of the pipeline and the sum of the ups were not used in obtaining these values. This is a crucial point for the formulation of the optimization problem.

At this point, it should be noted that single-phase flow is but a special case of the two-phase flow. The head losses are due only to friction, and it is a relatively simple problem to compute Δp_L.

A computer program was written to compute Δp_L and Δp_h for two-phase flow by Flanigan's method. Its use made it easy to examine many design conditions, i.e., various flow rates, diameters, etc.

ECONOMIC CONSIDERATIONS

In the present work we will be comparing alternative routes on the basis of capital costs alone. We are assuming that the pipeline will operate under the pressure differential naturally available between the source (well) and the point of delivery (refinery). There is, however, no difficulty in taking into consideration operating costs as well. For example, if compressors are used to provide the pressure at the inlet, the capital cost of this equipment, plus operating (power, manpower, maintenance) costs for the period for which the pipeline is designed are to be included in the objective function. The annual operating costs have to be converted into present cash value, using the unit costs and the interest rate. The objective function will still be separable (a term to be explained later), thus making it possible to use the same optimization technique.

It is also realized that unit prices depend on the time of the year, the region in which the pipeline is located, the accessibility of the route, etc. For any specific project a cost file has to be set up reflecting these factors; and it will be used in the optimization. The method of analysis is not altered by the actual cost figures or the inclusion of operating costs, even though the optimal route may be.

CONSTRAINTS

Often the route is forced to pass through certain points, or prevented from passing through others, prior to the route selection. This is sometimes due to valid constraints. Too often, however, it is done for what seems an overriding consideration which is assumed to be unavoidable. Such dictates then become constraints in the optimization problem. An example of such a consideration might be: "The pipe has to cross the river at this point, as the cost of the crossing there is much less than it would be at any other point." If this consideration is accepted as a constraint, and the route forced to go through the specified point, we are not sure that the resulting route is indeed the economically optimal one. The correct way of taking this condition into account is to assign to each point along the river the cost of making the crossing there, and then let the algorithm pick the cheapest *over-all* route. When comparing two routes for over-all cost, the cheaper of the two may well be the one having the higher cost of crossing the river. This is because the savings along the rest of the route more than make up for the difference in the cost of the two crossings.

The same applies to other constraints. Whenever a condition has an economic value, it should be included in the objective function and not taken as a constraint.

THE OPTIMIZATION PROBLEM

The optimization problem dealt with here includes the following given facts: the discharges of gas and oil; the necessary properties of the gas and the oil; the inlet and outlet sections; the inlet and outlet pressures; the corridor through which the pipe has to pass; the terrain in the corridor; soil types, tree cover, water courses and rivers, roads and railways, and other factors in the corridor which affect the cost of the pipeline; cost data for materials and labor used in the construction; legal, technical,

*"Sum of the ups", alternately called "hills", is the sum of the vertical rises of all uphill sections along the pipe (facing downstream).

practical, and any other types of constraints to be satisfied by the route to be selected; an objective function, which is a measure of the desirability of the line (e.g., the capital cost); and the diameter of the pipe.

We are asked to find that route through the corridor, from the inlet section to the outlet section, along which the pipeline satisfies the two-phase flow constraint and all other specified constraints and for which the objective function has an extremal value (minimum cost in our case).

As a first step, a corridor is selected through which the line has to pass, connecting the inlet and outlet sections. Based on engineering judgment, the choice of the corridor, its location and width, is made by hand. The wider the corridor, the more certain we are not to have eliminated the global optimal route. But at the same time, the wider the corridor, the more computation time is required to solve the optimization problem with a fixed degree of accuracy.

For solving the optimization problem, the continuous two-dimensional area of the corridor is replaced with a finite grid, roughly at right angles to the longitudinal direction of the corridor section. These are lines cutting the corridor from one boundary to the other, as shown in Fig. 1.

Along each section grid points are selected. The pipeline will pass through one point on each section. The length of pipe connecting two grid points on adjacent sections is called a segment. The sections and associated grid points have to be selected judiciously. They should constitute an adequate representation of the terrain, the soil types and tree cover, as well as all topographic and other factors relevant to the pipeline's construction, operation and cost. The finer the grid, the more accurate the representation. But, on the other hand, the computational requirement for the optimization increases as well. It is thus a problem of balancing the desired accuracy and the associated cost of performing the analysis. Rough guidelines might be ½ to 2 miles between sections and a few hundred to a thousand feet between grid points.

DYNAMIC PROGRAMMING

After formulating the optimization problem, its structure was reviewed to determine the most suitable optimization technique. The objective function is separable, which means that it can be written as a sum of terms, each representing the cost of a single segment of the pipe. There is only one global constraint, the two-phase flow equation. A global constraint is a condition to be satisfied by the entire route. All other constraints are local, i.e., conditions to be satisfied by an individual segment.

This structure suggests that dynamic programming is the most suitable technique. The technique was developed by Bellman,[3] and is given in detail in many books. The best reference for the present work is of Hadley.[4]

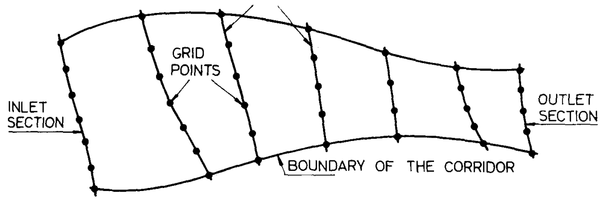

FIG. 1 — ESTABLISHING A GRID.

NOTATION

$i = 1, \ldots, n_i$ = index of the sections

$j = 1, \ldots, n_j(i)$ = index of the points on the section, $n_j(i)$ is the number of points on the ith section

$X_{i,j}$ = X-coordinate of Point (i,j)

$Y_{i,j}$ = Y-coordinate of Point (i,j)

X, Y = local Cartesian coordinates selected in any convenient manner

$Z_{i,j}$ = elevation of Point (i,j) relative to to any selected datum

$\xi_k(i,j)$ = length of pipe available from the inlet to Point (i,j), $k = 1, \ldots, K$

$\eta_\ell(i,j)$ = hills available from the inlet to Point (i,j), $\ell = 1, \ldots, L$

$\ell_{m,j}$ = length of the segment from Point $(i-1,m)$ to Point (i,j)

$h_{m,j}$ = hills in the segment from Point $(i-1,m)$ to Point (i,j)

Using this notation, the objective function is:

$$\min G = \sum_{i=2}^{n_i} f[(i-1,m)\ ;\ (i,j)]\ , \quad . \ . \ . \ . \ . \quad (2)$$

where $f[(i-1,m);\ (i,j)]$ is the cost of the segment from Point $(i-1,m)$ to Point (i,j), and where m for the next segment takes on the value that j had, to make the segments belong to a continuous line.

The recursive equation of the dynamic programming algorithm is:

$$f^*_{i,j}[(i,j)\ |\ \xi_k(i,j)\ ;\ \eta_\ell(i,j)] =$$

$$\min_{\substack{(i-1,m)\\ m\epsilon M}} \{f[(i-1,m);\ (i,j)] + f^*_{i-1,m}[(i-1,m)\ |\ \xi_k(i,j)$$

$$-\ell_{m,j};\ \eta_\ell(i,j) - h_{m,j}]\}\ , \quad . \ . \ . \ . \ . \ . \ . \ . \ . \quad (3)$$

where $f^*_{i,j}[(i,j)\ |\ \xi_k(i,j)\ ;\ \eta_\ell(i,j)]$ is the cost of the optimal route from the inlet to Point (i,j), *given* the available length $\xi_k(i,j)$ and the available hills $\eta_\ell(i,j)$.

The two-phase flow constraint is

$$(\Delta p_L) \cdot \xi_k(i,j) + (\Delta p_h) \cdot \eta_\ell(i,j) \leq \Delta p \ . \quad . \ . \ . \quad (4)$$

Other constraints may be imposed on the segment from $(i-1,m)$ to (i,j). The length of the segment is

$$\ell_{m,j} = [(X_{i,j} - X_{i-1,m})^2 + (Y_{i,j} - Y_{i-1,m})^2]^{1/2}. \quad (5)$$

as the vertical component $(Z_{i,j} - Z_{i-1,m})$ has a negligible effect. The hills in the segment are

$$h_{m,j} = (Z_{i,j} - Z_{i-1,m})^+ , \quad \ldots \ldots \quad (6)$$

where $(a)^+ = a$ when a is positive and $(a)^+ = 0$ otherwise.

The minimization is performed on all $m \epsilon M$ (m is a member of the group M), where M denotes the group of points on Section $(i-1)$, which may be connected to Point (i,j) by a pipe segment. The group M may include all points on Section $(i-1)$. Constraints may eliminate some of the points. The smaller the Group M, the less computations are required. Thus, one should try to reduce the size of M. One may achieve this by eliminating those points on Section $(i-1)$ from which a segment to Point (i,j) is highly unlikely to be part of an optimal route. This arbitrary elimination of segments should be done carefully, as by a wrong selection one may eliminate a perfectly good route. The elimination is done for the sake of efficiency. Thus, once again, one should weigh the added efficiency against the possibility of eliminating a good route. This point will be illustrated in the sample problem.

Let us now elaborate on the notion of the *available resources*: ξ, the available length of pipe, and η, the available hills from the inlet to the point in question, (i,j).

These are values of total length and hills which make it possible to select one or more routes from the point under consideration to the inlet section. The total length of the line, L, and the total hills in it, h, have to satisfy the two-phase flow constraint

$$(\Delta p_L)\, L + (\Delta p_h)\, h \leq \Delta p \; .$$

For a given corridor, various routes satisfying this constraint can be selected. One may be shorter, but have more hills, whereas the other avoids some of the high points and is longer. The optimal route, which must also satisfy the two-phase flow constraint, uses the two resources—length and hills—in such a way that the objective function is minimized, while the constraint is satisfied. These two resources have to be allocated among the various segments so that they are best used.

Consider the following hypothetical example. The cost of laying the pipe is the same throughout the corridor, and a straight-line pipe from inlet to outlet is not feasible. There are too many "ups" along this route and the two-phase flow constraint cannot be satisfied. What one should do is find that hill along the straight-line route for which one obtains the maximum decrease in "ups" per unit increase in pipe length (and therefore over-all cost) by moving the pipe from the straight line. If the resulting route is feasible, it is optimal. If not, one has to find the next best reduction in hills and proceed until a feasible route is found. It is the desired optimal route. Matters are complicated when the cost is not uniform throughout the corridor. The dynamic programming algorithm ensures that these resources, length and hills, are allocated to the segments in a way which results in the *over-all* cost being minimized.

Note that if we were dealing with a single-phase pipeline with an available head difference of Δp, the constraint would be

$$(\Delta p_L)\, L \leq \Delta p$$

and the only resource to be allocated would be length, as hills do not affect the losses. Thus, the single-phase case is essentially the same as the two-phase case with $\Delta p_h = 0$.

For each Point (i,j), a table has to be set up, ranging over the possible values of ξ and η, the available length and hills. To do this, one must determine the lower and upper limits of both resources at the point and then divide each range into finite increments. The number of increments, and thus the size of the table, is fixed in the computer program for all points (for convenience) and is given by the user of the program. The limits are computed as follows:

$$\left.\begin{aligned}
\xi_{\min}(i,j) &= \min_{m \epsilon M} [\xi_{\min}(i-1,m) + \ell_{m,j}] \\
\xi_{\max}(i,j) &= \max_{m \epsilon M} [\xi_{\max}(i-1,m) + \ell_{m,j}] \\
\eta_{\min}(i,j) &= \min_{m \epsilon M} [\eta_{\min}(i-1,m) + h_{m,j}] \\
\eta_{\max}(i,j) &= \max_{m \epsilon M} [\eta_{\max}(i-1,m) + h_{m,j}]
\end{aligned}\right\} \; , \quad . \quad (7)$$

with

$$\xi_{\min}(1,j) = \xi_{\max}(1,j) = \eta_{\min}(1,j) = \eta_{\max}(1,j) = 0 \quad \ldots \ldots \quad (8)$$

for all j, where $i = 1$ denotes the inlet section. $m \epsilon M$ has the same meaning as before — m belongs to the group of points on Section $(i-1)$ which may be connected by a pipe segment to Point (i,j). Thus the table at (i,j) covers the entire range of the Points $(i-1,m)$, plus the additional length and hills required for the segment from them to Point (i,j), for all $m \epsilon M$.

While performing the minimization of the recursive equation for a fixed value of ξ and η, one also determines that $m\,(m \epsilon M)$ for which the minimum is achieved. This point, denoted by

$$m^*_{i,j}[\xi_k(i,j)\; ; \; \eta_\ell(i,j)] \, , \quad \ldots \ldots \quad (9)$$

is stored in the table together with the minimum value of the function

$$f^*_{i,j}[(i,j) \mid \xi_k(i,j)\; ; \; \eta_\ell(i,j)] \, . \quad \ldots \ldots \quad (10)$$

It is the point on Section $(i-1)$ from which, *given* the available $\xi_k(i,j)$ and $\eta_\ell(i,j)$ at Point (i,j), one should connect a segment to Point (i,j), and it is part of the suboptimal route from the inlet to Point

(i,j). That is, *if* (i,j) were the outlet point, this segment would be part of the optimal route. The global optimal route, however, cannot be determined until the recursive equation is used for all Sections $i=1, \ldots, n_i$. Then, at the outlet section, one finds the Point $j^*(n_i)$, which has the minimal value of $f^*_{n_i,j}$, by performing the minimization

$$\min_{j,k,\ell} \left\{ f^*_{n_i,j}[(n_i,j) \mid \xi_k(n_i,j)\ ;\ \eta_\ell(n_i,j)] \right\} . \quad . \quad . \quad (11)$$

over all k and ℓ for all j. $j^*(n_i)$ is the index of the point on the outlet section which is the end point of the optimal route. The values of ξ and η, for which this minimum was achieved, are the total length ξ^* and hills η^* of the optimal route. For these values one also finds the back-pointer

$$m^*_{n_i,j^*}[\xi^*;\ \eta^*] , \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (12)$$

which is the point on the (n_i-1)-st section through which the optimal route passes. Denoting this value for the moment simply as m^*, one can find the location in the table for that Point $(i-1,m^*)$, which led to the optimal route as being

$$\xi(i-1,m^*) = \xi^* - \ell_{m^*,j^*}$$

$$\eta(i-1,m^*) = \eta^* - h_{m^*,j^*} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (13)$$

The entries in the table for $(i-1,\ m^*)$ at these values of ξ and η are the cost of that part of the optimal route from the inlet to Point $(i-1,m^*)$ and the back-pointer. The back-pointer is the value of m on Section $(i-2)$, which is on the optimal route. Again, one subtracts the length of the segment of the optimal route from Section $(i-2)$ to $(i-1)$ and enters the table of the optimal point on Section $(i-2)$ with the new, reduced values of ξ and η. This procedure is followed all the way back to the inlet section. There the procedure will yield the point from which the route is to start for it to be optimal. During this part of backtracking, one also determines the contribution of each segment to the over-all cost, as well as to the over-all lengths and hills.

The dynamic programming problem has a single global constraint and, therefore, can be solved as having a single scarce resource — the pressure differential Δp. Doing this, the recursive equation is

$$f^*_{i,j}[(i,j) \mid \Delta p_k] = \min_{(i-1,m),\, m\epsilon M} \{ f[(i-1,m);\ (i,j)]$$

$$+ f^*_{i-1,m}[(i-1,m) \mid \Delta p_k - (\Delta p_L \cdot \ell_{m,j} + \Delta p_h\, h_{m,j})] \} .$$

Δp_k, $k=1, \ldots, K$ ranges over all possible values that satisfy the two-phase flow constraint for the Point (i,j).

The pressure differential available at Point $(i-1,m)$ is whatever was available at Point (i,j) minus the pressure drop along the segment. This approach results in a one-dimensional formulation (as it should be, for a single constraint) and will be tried at a later stage. The two-resource formulation was easier conceptually and was therefore used in the present work.

INTERPOLATION

Recall that at each Point (i,j) a table was set up having entries of f^* and m^* for discrete values of ξ and η over some range. In determining $f^*_{i,j}$ for some values of ξ and η, one has to use values of $f^*_{i-1,m}$ for computed values of the two resources: $(\xi - \ell)$ and $(\eta - h)$. These values, however, usually do not coincide with one of the discrete values of ξ and η used to construct the table at Point $(i-1,m)$. In such cases, the value of

$$f^*_{i-1,m}[(i-1,m) \mid \xi_k(i,j) - \ell_{m,j}\ ;\ \eta_\ell(i,j) - h_{m,j}]$$

has to be found in the table for Point $(i-1,m)$ by a two-dimensional interpolation. As this interpolation is done many times, it has to be simple and involve few computations.

If the discrete values of ξ and η used in a table are closely spaced, one can use the simplest interpolation procedure without much loss of accuracy. This procedure amounts to taking the table entry at the highest possible values of ξ and η that do not exceed the available amounts. For example, at $(i-1,m)$ for some $\xi_k(i,j)$ and $\eta_\ell(i,j)$, use the discrete values of $\xi(i-1,m)$ and $\eta(i-1,m)$ that satisfy

$$\xi(i-1,m) \le \xi_k(i,j) - \ell_{m,j}$$

and

$$\eta(i-1,m) \le \eta_\ell(i,j) - h_{m,j} .$$

Other interpolation procedures call for fitting a three-dimensional surface through the known values at the vertices of a rectangle (which correspond to table entries) and using the resulting equation of the surface to determine the value at any interior point.

The interpolation procedure has to be used twice: once in the "forward" stage of minimization performed in the recursive equation and the other in the "backward" determination of the optimal route.

Using any interpolation procedure, other than the one outlined in this section, another problem arises. Whereas the value of the function f^* is continuous over the space of ξ and η, the associated value of the optimal Point m^* (the back-pointer) is not. Each one of the vertices of the rectangle mentioned above may have a different value of m^*. In the forward phase of constructing the tables one may store in the table for Point (i,j) *all* the values of m^* (1 to 4) appearing at the vertices of the rectangle of the table for Point $(i-1,m)$.

In the backward stage one uses the four values of m^* (of which two or more may be equal) to determine the interpolated point on the previous section through which the optimal route passes. If this is indeed an interpolated point rather than one of the grid points, one does not have tables for it, as the point was not used in the forward phase. In this case one has to use the tables for the two adjacent grid points to construct an interpolated table for the new point.

These interpolation procedures are rather cumbersome and require a lot of computations. We have, therefore, adopted for the present work the first approach, that of always using an actual table entry from the previous section in performing the minimization of the forward stage. This results in a very simple procedure, and provided ξ and η increments are not too large, the accuracy is adequate.

SUBOPTIMAL ROUTES

Dynamic programming provides much useful information besides the optimal route. After the original problem has been solved, one may determine other routes, which are optimal under some added conditions. For example, say the outlet section has a number of points, and the algorithm picked the outlet point as that resulting in the lowest over-all cost. We may now impose the condition that the outlet be at some other point on the outlet section. The tables constructed for the original solution may be used to determine the optimal route for this new condition that, in the global sense, is called suboptimal. Moreover, the extra cost of this suboptimal route, over the cost of the optimal route, is the cost of the constraint that was imposed, namely, forcing the route to terminate at the specified point.

If after having the solution for the original problem we want to force the route to pass through some internal grid point, i.e., a point on a section other than the outlet section, some additional work has to be performed. The tables for the section from the inlet to the section in question are all valid, but the subsequent ones have to be recalculated with the new restriction.

In summary, the forward dynamic programming algorithm provides the optimal routes from the inlet section to every point on the grid for all possible values of the available resources. However, it does not provide the optimal route from each point to the outlet section. This is because we have selected a dynamic programming procedure which starts from the inlet section. The same procedure can be used starting from the outlet section. It would yield optimal routes from every point in the grid to the outlet section for all possible values of the available resources, but not the optimal routes from the inlet to the point. The computer program can be used twice, solving once from inlet to outlet and once in reverse. The two sets of tables would then provide directly the answers for all subsequent conditions being imposed on the route.

OPTIMAL PIPE DIAMETER

The objective of the optimal pipeline is to convey the required quantities of gas and oil at a minimum cost. One should therefore determine the most economical pipe diameter, as well as its optimal route. One might even consider changing the pipe diameter along the way if it proves economical. In two-phase flow lines, it is common procedure to have a uniform diameter throughout sections in which the discharges do not change. This is done for the purpose of "pigging", a procedure where "pigs"—cylindrical or spherical bodies—are sent with the flow down the line to clean the pipe walls from deposits, as well as to move fluid slugs over hills. For the time being, we shall consider only lines with a uniform diameter throughout.

Unfortunately, the two-phase flow constraint cannot be written as an explicit function of pipe diameter. The influence of the diameter is embodied in Δp_L and Δp_h. The diameter is an important parameter in the optimization problem, as the cost of the pipe is a very significant contribution to the over-all cost, and it varies greatly with diameter. The only way to select the most economical diameter is to run the same problem for a few diameters which seem reasonable, determine for each the optimal route (which will be different for each diameter), and then select that diameter that has the lowest over-all (optimal) cost.

SENSITIVITY ANALYSIS

The results of the optimization are only as good as the input data. There are inaccuracies and uncertainties in both the physical and the economic data. These are due to the use of a finite grid, to interpolation and, more than anything else, to the uncertainty in determining the unit prices before a route is selected. It is important to determine how sensitive the answers are to variations in the input data.

By the sensitivity of the results, we mean primarily changing of the optimal *route* due to a change in input data. It also means the change in the optimal *value* due to such changes.

To perform a sensitivity analysis, one has to rerun the program a number of times, vary one or a few parameters in each run, and compare the results. One thing can be done with respect to cost data, which eliminates the need to make many separate runs. During the original solution, one can easily divide the costs of the segments into the various components, such as the cost of pipe, clearing, ditching, etc., and add these separately along the route. In this way it is possible to tell which are the governing factors in the over-all cost, to which the optimal solution will probably be most sensitive.

A computer program was developed, using a time-sharing computer, for selecting the optimal route by dynamic programming.

DATA COLLECTION AND USE

Much thought was given to the questions of how to obtain the physical data needed in selecting the optimal route and how to transform it to computer-acceptable data. Automatic digitization of the topography from stereo aerial photographs on any convenient grid is now available at moderate cost. Aerial-photo interpretation, to yield soil types and tree cover, has to be done manually by experts. The areas delineated on the photos by the interpreter can be described digitally by the same equipment which is used for digitizing the topographic data. With the proper format, punched cards produced by digitizing equipment can be fed directly into the computer.

Initially we did not link this capability with the optimization program. It is intended that when the program is transferred from time-sharing to batch processing, the physical data will be supplied by the digitizing process.

Cost data will be assembled as it becomes available and incorporated in a cost file, which can be used as a reference. The actual cost data should be assembled for each project and included in the cost file. It is this specific data which will be used to find the optimal route.

SAMPLE PROBLEM

We have used an existing pipeline as an example for selecting an optimal route. The data used was for the pipeline as it operates today. The pipeline was designed, however, for a much larger discharge. The data are given in Table 1.

The coefficients for the two-phase flow constraint were computed by Flanigan's method,[1] using the computer program. The results, computed as outlined in the Appendix, are: $\bar{p}$ = 847.50 psia; $u = 15.729$ ft/sec; $F = 0.8832$; $C = 800.27$; $\Delta p_L = 6.26$ psi/mile = 0.00119 psi/ft; and Δp_h = 0.058 psi/ft of ups.

A system of local coordinates was chosen for convenience. The origin is at the inlet, Point A. The X-axis passes through the outlet, Point B, and the Y-axis is perpendicular to it. Terrain data was read from the topographical map. Aided by aerial photos of the area, soil types and tree cover were determined. These are shown in Fig. 2. A corridor was selected and a grid was established, as shown in Fig. 3. Note that the sections follow the major topographic features of the corridor, and the grid points are close enough to adequately represent the terrain and soil types. (We were restricted to a maximum of 10 sections and 10 points per section because of the limited capacity of the computer used in time-sharing. For the sample problem, however, this appears to provide sufficient accuracy.)

TABLE 1 — DATA FOR SAMPLE PROBLEM

Pipe diameter, in.	12.170
Distance from source to point of delivery, miles	5.8
Inlet pressure, psia	880
Outlet pressure, psia	815
Gas discharge, MMcf/D	73
Oil gravity, lb/cu ft	51.2
Gas gravity, air = 1	0.70
Average temperature, °F	90
Gas compressibility factor	0.867

To reduce computation time, we have restricted the number of segments which can be connected to each grid point. For example, for Point (5,4), which is Point 4 on the fifth section, only segments originating at Points (4,2), (4,3) and (4,4) were considered. It was considered unlikely that segments originating at other points on Section 4, and leading to Point (5,4), would be part of the optimal route, as they deviate greatly from the longitudinal direction of the corridor. The same was done at all other grid points; this was based entirely on intuition. Output from the program for one diameter and one set of cost data is shown in Table 2.

The program was run on CGE Time-Sharing Mark II. The cost of making a run and listing all the output is approximately \$20. The cost of the same run using batch should be approximately \$1.

The existing pipeline and the optimal route determined by the program are shown on Fig. 3. In this example, the cost of the optimal pipeline is \$199,359, while the cost of the existing pipeline is \$210,422.

CONCLUSIONS

Dynamic programming can be used to select the optimal route for pipelines designed to carry two-phase flow. Length and ups are treated as separate resources, although it is possible to view them as components of a single resource-pressure drop. When all the necessary physical and economic data are available, a computer program can be used to select the optimal route. For the sample problem presented, the optimal pipeline is about 5 percent cheaper than the actual pipeline which was constructed, based on the unit costs used in this paper.

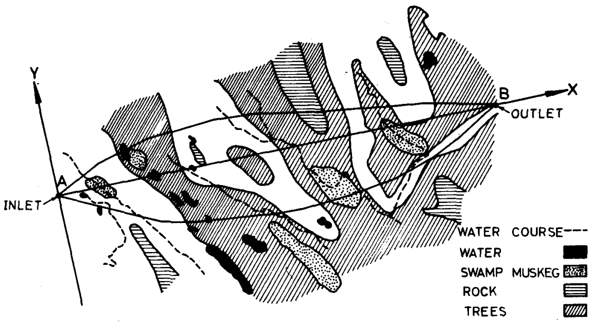

FIG. 2 — SOIL TYPES AND TREE COVER.

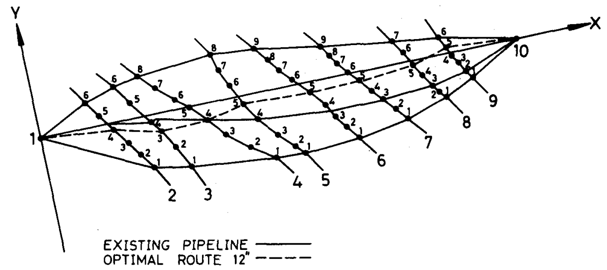

FIG. 3 — GRID, EXISTING AND OPTIMAL ROUTES.

TABLE 2—PROGRAM OUTPUT, TELETYPED INFORMATION

Sample Problem

Ten Sections. (4 × 4) Tables at Each Point
Diameter = 12.2 in.
Two-Phase Flow Constraint:
$0.00119L + 0.05800h \leq 65.00$
Four Soil Types. Two Tree Types.
Cost of Ditch in Different Soils in dollars/ft:
1 = rock, 2 = soft soil, 3 = muskeg or swamp, 4 = water
Cost (1) = 2.50
Cost (2) = 0.20
Cost (3) = 0.80
Cost (4) = 1.50
Cost of Clearing Trees in dollars/ft:
1 = no trees, 2 = small trees, 3 = large trees
Cost (1) = 0
Cost (2) = 0.55
Cost of pipe = $6.00/ft

Optimal Route Table

I	J	X (ft)	Y (ft)	Z (ft)	Cost to Point ($)
10	1	30,500.0	0	- 10.0	199,359.1
9	5	26,000.0	374.0	120.0	170,121.2
8	5	23,800.0	-249.0	70.0	155,258.8
7	5	20,200.0	-498.0	170.0	131,802.9
6	5	16,900.0	-581.0	110.0	110,428.7
5	5	12,800.0	-415.0	300.0	82,731.0
4	4	10,400.0	-995.0	200.0	66,743.7
3	3	7,395.0	-1,050.0	318.0	47,283.0
2	4	4,560.0	-332.0	413.0	28,346.8
1	1	0	0	395.0	0

Total length = 30,786.2 ft
Total hills = 228.0 ft
Total cost = $199,359.10
$0.00119 \times 30{,}786.2 + 0.05800 \times 228.0 = 49.9$

NOMENCLATURE

d = pipe diameter, in.
F = friction loss efficiency factor
h = sum of ups
$p1$ = inlet pressure, psia
$p2$ = outlet pressure, psia
$\bar{p}$ = average pressure, psia
Δp = pressure, psia
q_g = rate of gas flow, MMcf/D
R = gas-oil ratio, bbl/MMcf
T = average temperature along the line, °F
u = superficial gas velocity
z = gas compressibility factor (from tables)
γ_g = gas gravity (air = 1.0)
ρ_L = liquid gravity, lb/cu ft

ACKNOWLEDGMENTS

D. B. Horne, head of the Petroleum Engineering Div. at Underwood McLellan & Associates, was responsible for the initiation and execution of this project. The company made the funds available to institute this research. S. Sargent, C. Anderson, W. McIntyre and P. Makowichuck of Underwood McLellan & Associates, D. Morgan of Western Photogrammetry, and several pipeline contractors, all deserve acknowledgment for their discussion of the various aspects of this project.

REFERENCES

1. Flanigan, O.: "Two-Phase Gathering Systems", *Oil and Gas J.* (March 10, 1958) 132-141.
2. Baker, O.: "Designing Pipelines for Simultaneous Flow of Oil and Gas", *Pipeline Engineer* (Feb., 1960) Handbook Section, H-67-80.
3. Bellman, R.: *Dynamic Programming*, Princeton U. Press, Princeton, N. J. (1957).
4. Hadley, H. G.: *Non-Linear and Dynamic Programming*, Addison-Wesley Publishing Co., Inc., Reading, Mass. (1964) Chap. 10.

APPENDIX

FLANIGAN'S METHOD

Flanigan[1] outlined a simple procedure for computing pressure losses in two-phase flow pipelines. His procedures are used in this work, as follows.

Compute the average pressure in the line,

$$\bar{p} = (p1 + p2)/2 \quad \text{. . . . . (A-1)}$$

Compute the superficial velocity of the gas (ft/sec)

$$u = 31{,}194\,\frac{(q_g)\,(z)}{(\bar{p})\,(d)^2} \quad \text{. . . . . (A-2)}$$

Compute the friction loss efficiency factor,

$$F = \exp\left\{-0.07464\left[\log\left(\frac{u}{R^{0.32}}\right)\right]^2 + 0.4772\log\left(\frac{u}{R^{0.32}}\right) - 0.8003\right\} \quad \text{. . (A-3)}$$

This is a curve fit for Fig. 5 of Ref. 1.

Compute the friction factor for pressure loss,

$$C = \frac{20{,}500}{\gamma_g^{0.46}\,(T + 460)^{0.54}} \quad \text{. . . . . (A-4)}$$

Compute the pressure drop per unit length due to friction (psia/mile)*,

$$\Delta p_L = \frac{1}{2\,\bar{p}}\left[\frac{q_g \times 10^6}{C\,d^{2.6182}\cdot F}\right]^{1.853} \quad \text{. . . . (A-5)}$$

Compute the pressure drop per foot of hills (psia/ft),

$$\Delta p_h = \frac{\rho_L}{144}\left(\frac{3.06}{u + 3.06}\right) \quad \text{. . . . . (A-6)}$$

★★★

*The term in brackets (Eq. A-5) is a curve fit of Fig. 6 of Ref. 1.