Design of Optimal Reliable Multiquality Water-Supply Systems

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Abstract: A methodology which integrates the optimal design and reliability of a multiquality water-supply system is presented and demonstrated. The system designed is able to sustain prescribed failure scenarios, such as any single random component failure, and still maintain a desired level of service in terms of the quantities, qualities, and pressures supplied to the consumers. In formulating and solving the model, decomposition is used. The decomposition results in an "outer" nonsmooth problem in the domain of the circular flows, and an "inner" convex quadratic problem. The method of solution includes the use of a nonsmooth optimization technique for minimizing the outer problem, for which a member of the subgradient group is calculated in each iteration. The method allows reversal of flows in pipes, relative to the direction initially assigned. The methodology is applied to a system with 33 pipes, five pumps, and 16 nodes (two source nodes with treatment facilities and 14 consumer nodes) for a single loading condition and one quality parameter.

INTRODUCTION

This paper deals with three aspects of water-supply systems that are incorporated into a single framework for designing optimal reliable multiquality water-supply systems: reliability, optimal design, and quality.

Reliability is a measure of performance. A system is said to be reliable if it functions properly for a specified time interval under prescribed conditions. The conditions and the time interval define the boundaries of the system, while the word "properly" is translated into reliability measures.

Reliability analysis of water-distribution systems is not yet adequately developed, nor accepted in practice. At present, reliability of water-distribution systems is provided in most cases by following prescribed heuristic guidelines, like ensuring more than one path between the system nodes, or having all the pipe diameters greater than some minimum value. It is implicitly assumed that by following these guidelines reliability will be assured, but the level of reliability is neither quantified nor measured. Therefore, only limited confidence can be placed on these guidelines, since reliability is not considered explicitly. The challenge is to define meaningful reliability measures that can be computed, so they can be used in design.

Design is the phase in which the sizes and characteristics of the components are determined for a given system layout. The optimal design problem is to find the component characteristics (e.g., pipe diameters, pump heads and maximum power, reservoir volumes) that minimize the total system cost, such that constraints at the consumer nodes are fulfilled and hydraulic laws are satisfied.

Four different approaches have been taken in optimization algorithms to incorporate reliability: (1) techniques which link simulation with a general optimization algorithm; (2) methods based on the decomposition approach of Alperovits and Shamir (1977); (3) methods relying on graph theory procedures; and (4) methods relying on entropy as a surrogate for reliability. An extensive literature review on the inclusion of reliability considerations in the management and simulation of water-distribution systems can be found in Ostfeld (1994).

If reliability is to be incorporated into models for the optimal design of water distribution systems, we must first define reliability and make sure that the definition is such that it can be accommodated in an efficient algorithm. Previous work has mostly used measures defined on the system itself—such as connectivity and reachability (Wagner et al. 1988a)—and did not consider the perspective of the consumers affected by the system's failure to meet their demands.

Multiquality water-supply systems are distribution systems in which waters of different qualities are taken from sources, possibly treated, conveyed, and supplied to consumers. Such systems can be described as a graph with the links representing the pipes, and the nodes representing connections between pipes, hydraulic control elements, consumers, and sources.

Ostfeld and Shamir (1993) classified the problems of multiquality water-distribution systems according to the physical laws that are considered explicitly as constraints: (1) Discharge-head (QH) models—quality is not considered. The network is described only by its hydraulic behavior; (2) Discharge-quality (QC) models—the physics of the system are included only as continuity of water and of pollutant mass at nodes. Quality is described essentially as a transportation problem, in which pollutants are carried in the pipes and mass conservation is maintained at the nodes. Such a model can account for the decay of pollutants within the pipes and even chemical reactions, but does not satisfy the continuity of energy law (Kirchoff's law No. 2), and therefore there is no guarantee of hydraulic feasibility and of maintaining head constraints at nodes; (3) Discharge-quality-head (QCH) models—quality constraints and the hydraulic laws that govern the system behavior are all considered.

The methodology presented here is for the design of optimal reliable multiquality water-supply systems in which reliability is cast as an inherent property of the system. The system we construct is able to sustain prescribed failure scenarios—in our case any single random component failure—and still maintain a desired level of service in terms of the quantities, qualities, and pressures supplied to the consumers.

METHODOLOGY

When a system component fails (e.g., pump, pipe) there are two results: (1) isolation of the failed component by valve closure to allow its repair or replacement; and (2) redistribution of the flows in the remaining system (and perhaps other changes in pump operation or in the removal ratios at the treatment facilities). The ability of the system to meet its consumers demands after a component failure has occurred depends on two interrelated system attributes: its inherent redundancy (i.e., the existence of more than one way to fulfill the

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consumers requirements), and the hydraulic capacity of its remaining components. Our approach takes these two attributes into account explicitly.

Four major stages constitute the methodology for the optimal design of reliable multiquality water-supply systems:

1. Formulation of an optimal design problem of a multiquality water-distribution system under a number of loadings (i.e., demand patterns), in which the objective function is minimization of system cost. The constraints are on the continuity of flow and energy; pressure heads at consumption nodes; length of each pipeline [this is a result of the mathematical formulation of the model, in which each pipe is made of a number of segments, see Alperovits and Shamir (1977)]; power of pumping stations; and threshold concentrations at consumption nodes. The decision variables are the vector of flows in all pipes for each loading condition; pumping heads for each pumping station and loading condition; the pipe segment lengths; the maximum power of each pumping station; and the treatment facilities capacities and removal ratios.
2. Identification of backup subsystems, which will maintain a prescribed level of service, under each of the loadings, when a failure occurs. A backup is a subset of links of the full system; two backups can be defined such that if any one link in the full network fails, then one of these two backups "survives," i.e., remains intact. More than two backups may have to be defined if one wishes to consider failures of more than one link at a time. Since this has a much lower probability of occurrence, we restrict our analysis to single-link failures and, therefore, require two backups.
3. The hydraulic laws and consumer demands are formulated separately for each of the backups and loading conditions, to define for each the level of service required.
4. The models for the backups are added to the model of the complete system, and the optimization model is solved.

This formulation creates an expanded design model that minimizes the total system cost, subject to explicit constraints on residual system performance. Explicit inclusion of the required redundancy of the system is addressed by inclusion of the backup subsystems as part of the constraints.

This approach raises two main issues: how to select the backup subsystems, and how to examine the trade-offs between cost and reliability.

A set of backups is "good" if it has the following properties:

1. It gives a proper meaning to reliability, i.e., the backups are able to supply the required service to the consumers for the prescribed set of failure scenarios.
2. The optimal solution of the design model with this set of backups is competitive (with respect to cost) to other sets of backups that satisfy property 1.

The selection of a good set of backups is analogous to the selection of a good system layout, for which there is to date no quantitative model, and is thus based on engineering judgement and experience.

The method we have used to select the backups in the case study presented later is based on topological considerations. It defines two backups with the following properties:

1. The set of nodes for each of the backups equals the set of the system nodes.
2. The union of the sets of arcs of the two backups equals the set of the system arcs.
3. Each of the backups is a connected graph.
4. The number of common arcs of the two backups is minimal.

The backups are found by: (1) Searching for two spanning trees in the system whose distance is maximum. The distance between two trees is defined as the number of arcs contained in one tree but not in the other. These are found by using the algorithm of Kishi and Kajitani (1969), or the algorithm of Kameda (1976); (2) addition of other arcs, if such exist, which were not used in the aforementioned first step, to either of the spanning trees, creating the two backups.

The tradeoff between cost and reliability can be evaluated by changing the selection of the backup subsystems for given consumers demands, or by reducing the consumers requirements for a given set of backups. The case study presented later describes the trade-offs between cost and reliability for a set of two backups and one loading condition, by reducing the consumers requirements and by changing the time the backups are designed to operate.

MATHEMATICAL FORMULATION

In formulating the optimal design problem of a multiquality water-supply system, we adopt the decomposition idea of Alperovits and Shamir (1977), and expand it to multiquality water-supply systems. The decomposition principle is based on the following reasoning. Given the flows throughout a looped multiquality water-supply system, its optimal design is the solution of a QCH quadratic convex programming formulation, which can be separated into two problems: a QH and a QC problem. The QH is a linear programming problem, similar to that of Kessler and Shamir (1989), and the QC has a quadratic convex objective function subject to a set of linear inequality constraints.

The system includes: pipes, pumps, treatment facilities, sources, and consumers. It has N pipes, NSO source nodes (all with treatment facilities), NNC internal nodes, NEI internal pipes (all the pipes, excluding the pipes connected to the sources), and $NPUMP$ pumping stations. NL loops and NP paths are considered. ND commercial pipe diameters are assigned to each link so the total number of pipe segments is $NS = ND \times N$.

The QCH problem formulation (termed the P1 formulation), for NLO loading conditions (indexed k) is

$$\text{(P1) } \underset{\mathbf{q}\in\mathbf{Q}}{\text{minimize}} \left\{ \phi(\mathbf{q}) = wc(\mathbf{q}) + \underset{\mathbf{X}_p\geq\mathbf{0};\,\mathbf{RR}\geq\mathbf{0}}{\text{minimize}} \left[\mathbf{a}_p^T(\mathbf{q})\mathbf{X}_p + \frac{1}{2}\mathbf{RR}^T\mathbf{H}(\mathbf{q})\mathbf{RR} \right] \right\} \quad (1)$$

$$\text{subject to: } [\mathbf{L}_p^k \quad \bar{\mathbf{I}}_p \quad \mathbf{J}_p^k(\mathbf{q}^k)]\mathbf{X}_p = \mathbf{b}^k \quad \forall\, k \quad (2)$$

$$[\mathbf{P}_p^k \quad \bar{\mathbf{I}}_p \quad \mathbf{J}_p^k(\mathbf{q}^k)]\mathbf{X}_p \leq \Delta\mathbf{H}_{\max}^k \quad \forall\, k \quad (3)$$

$$\bar{\mathbf{I}}_a\mathbf{X}_p = \mathbf{a}; \quad \mathbf{A}(\mathbf{q})\mathbf{X}_p \leq \mathbf{0}; \quad \mathbf{B}(\mathbf{q})\mathbf{RR} \leq \bar{\mathbf{c}}(\mathbf{q}) \quad (4\text{–}6)$$

All terms will be explained and detailed later, but first we decompose the P1 formulation into two independent models: P1-QH and P1-QC, whose union is termed the inner problem. This is done with the assumption that the quality distribution in the system has no influence on the hydraulics.

The resulting models are

$$\text{(P1-QH) } \underset{\mathbf{X}_p\geq\mathbf{0}}{\text{minimize}}\ \mathbf{a}_p^T(\mathbf{q})\mathbf{X}_p \quad (7)$$

$$\text{subject to: } [\mathbf{L}_p^k \quad \bar{\mathbf{I}}_p \quad \mathbf{J}_p^k(\mathbf{q}^k)]\mathbf{X}_p = \mathbf{b}^k \quad \forall k \tag{8}$$

$$[\mathbf{P}_p^k \quad \bar{\mathbf{I}}_p \quad \mathbf{J}_p^k(\mathbf{q}^k)]\mathbf{X}_p \le \Delta\mathbf{H}_{\max}^k \quad \forall k \tag{9}$$

$$\bar{\mathbf{I}}_a\mathbf{X}_p = \mathbf{a}; \quad \mathbf{A}(\mathbf{q})\mathbf{X}_p \le \mathbf{0} \tag{10, 11}$$

and

$$(\text{P1-QC}) \; wc(\mathbf{q}) + \underset{\mathbf{RR}\ge 0}{\text{minimize}} \frac{1}{2}\mathbf{RR}^T\mathbf{H}(\mathbf{q})\mathbf{RR} \tag{12}$$

$$\text{subject to: } \mathbf{B}(\mathbf{q})\mathbf{RR} \le \bar{\mathbf{c}}(\mathbf{q}) \tag{13}$$

where $\mathbf{q}^T(N \times NLO) = (\mathbf{q}^1, \mathbf{q}^k, \mathbf{q}^{NLO})$ = vector of flows in the network pipes for all the loading conditions; $\mathbf{q}^k$ = vector of flows in the network pipes for loading condition k; and $\phi(\mathbf{q})$ = optimal value function.

The vector $\mathbf{q}$ belongs to $\mathbf{Q}$, the set of all the flows for all the loading conditions which maintain continuity (Kirchoff's law No. 1) at all system nodes. The minimization of $\phi(\mathbf{q})$ over $\mathbf{Q}$ is termed the outer problem. We shall show later that the dimension of the decision vector can be reduced significantly by using the circular flows (flows in loops and pseudoloops) instead of the flows in all links, $\mathbf{q}$.

The P1-QH Model

The purpose of the P1-QH model is to determine the least cost pipes and pumping stations and their operation, while satisfying the consumers requirements for quantities and pressures, in compliance with the physical laws that govern the system.

The decision variables (dimensions of vectors and matrices given in parentheses) are $\mathbf{X}_p[NS + NPUMP \times (\text{NLO} + 1)]$ = vector (size NS) of lengths of all the candidate segments, where each pipe is allowed to be made of segments from all the possible diameters (Karmeli et al. 1968); the pumping stations pressure heads (size $NPUMP \times NLO$), and their maximum power (size $NPUMP$).

The constraints are divided into four blocks, as follows:

Eq. (8)—continuity of energy constraints (Kirchoff's law No. 2) for each loading condition, where $\mathbf{L}_p^k[NL, N + NPUMP \times (\text{NLO} + 1)] = (\mathbf{L} \quad \mathbf{L}_{pu}^k \quad \mathbf{0})$:

$\mathbf{L}(NL, N)$ = the loop matrix. The term "loops" also includes open paths between two nodes at which the heads are fixed. The rows and columns correspond to the loops and pipes, respectively. Defining a positive direction of circulation for each loop, its (i, j) term is $+1$ if pipe j is in loop i, same direction; -1 if pipe j is in loop i, opposite direction; 0 otherwise.

$\mathbf{L}_{pu}^k(NL, NPUMP \times NLO)$ = a matrix defining the pumping station locations with respect to each loop and loading condition k. The rows correspond to the loops, and the columns to the pumping stations heads. Its (i, j) term is -1 if pump j is in loop i and loading condition k, same direction; $+1$ if pump j is in loop i and loading condition k, opposite direction; 0 otherwise. The remaining terms, which do not belong to $\mathbf{L}$ or $\mathbf{L}_{pu}^k$(sizes NL, $NPUMP$) are zeros.

$$\bar{\mathbf{I}}_p[N + NPUMP \times (NLO + 1), NS + NPUMP \times (NLO \times 1)] = \begin{bmatrix} \bar{\mathbf{I}} & \mathbf{0} \\ \mathbf{0} & \mathbf{I} \end{bmatrix}$$

$\bar{\mathbf{I}}(N, NS)$ = a matrix representing the internal arrangement of the pipe segments (j) within the set of links (i). Its (i, j) term is $+1$ for $(i - 1) \times ND < j \le (i) \times ND$, and 0 otherwise.

$\mathbf{I}[NPUMP \times (NLO + 1), NPUMP \times (NLO + 1)]$ = an identity matrix.

$$\mathbf{J}_p^k(\mathbf{q}^k)[NS + NPUMP \times (NLO + 1), NS + NPUMP \times (NLO + 1)] = \begin{bmatrix} \mathbf{J}^k(\mathbf{q}^k) & \mathbf{0} \\ \mathbf{0} & \mathbf{I} \end{bmatrix}$$

$\mathbf{J}^k(\mathbf{q}^k)(NS \times NS)$ = a diagonal matrix which attaches, for a given loading condition k, a hydraulic gradient to each pipe segment. The hydraulic gradients are calculated using the Hazen Williams headloss equation.

$\mathbf{b}^k(NL)$ = a vector of the head differences for the loop equations, for loading condition k. $\mathbf{b}^k = \mathbf{0}$ for closed loops, and b^k = the known head difference between the end nodes of an open path. Eq. (8) generates for each loading condition, NL equality constraints.

Eq. (9)—constraints on minimum pressure heads at selected internal nodes (usually at consumer nodes) for each loading condition, where $\mathbf{P}_p^k[NP, N + NPUMP \times (NLO + 1)] = (\mathbf{P} \quad \mathbf{P}_{pu}^k \quad \mathbf{0})$; $\mathbf{P}(NP, N)$ = the path matrix. The rows and columns correspond to the paths and pipes, respectively. Defining a positive direction for each path, its (i, j) term is $+1$ if pipe j is in path i, same direction; -1 if pipe j is in path i, opposite direction; 0 otherwise.

$\mathbf{P}_{pu}^k[NP, NPUMP \times NLO)$ = a matrix defining the pumping station locations with respect to each path and loading condition k. The rows correspond to the paths, and the columns to the pumping stations heads. Its (i, j) term is -1 if pump j is in path i and loading condition k, same direction; $+1$ if pump j is in path i and loading condition k, opposite direction; 0 otherwise. The remaining terms, which do not belong to $\mathbf{P}$ or $\mathbf{P}_{pu}^k$(sizes NP, $NPUMP$) are zeros.

$\Delta\mathbf{H}_{\max}^k(NP)$ = a vector of the maximum admissible headlosses for loading condition k, along (NP) paths which connect reference nodes with the internal nodes. Eq. (9) generates for each loading condition one inequality constraint for every path over which a hydraulic energy loss restriction is imposed.

Eq. (10)—link length constraints, where $\bar{\mathbf{I}}_a[N, NS + NPUMP \times (NLO + 1)] = (\bar{\mathbf{I}}\,\mathbf{0})$

$\mathbf{a}(N)$ = a vector of the link lengths. Eq. (10) has N equations, each corresponding to one link. The ith equation is the sum of the candidate pipe segment lengths, which must equal the link length.

Eq. (11)—power constraints, where

$$\mathbf{A}(\mathbf{q})[NPUMP \times NLO, NS + NPUMP \times (NLO + 1)] = \begin{bmatrix} & & -\mathbf{I} \\ \mathbf{0} & \mathbf{A}_{pu}(\mathbf{q}) & -\mathbf{I} \\ & & -\mathbf{I} \end{bmatrix}$$

The first $(NPUMP \times NLO, NS)$ terms of $\mathbf{A}(\mathbf{q})$ are zeros; the others are

$\mathbf{A}_{pu}(\mathbf{q})(NPUMP \times NLO, NPUMP \times NLO)$ = a diagonal matrix, which assigns the power per unit head coefficients for each pumping station and loading condition, given a flow distribution $\mathbf{q}$.

$-\mathbf{I}(NPUMP, NPUMP)$ = identity matrix multiplied by -1. The ith equation of (11) states that the maximum power of pump j under loading condition k is bounded by the maximum power of the station, which itself is a decision variable.

The objective of the P1-QH model is to minimize the overall cost of the system components (pipes and pumping stations) and operation, where $\mathbf{a}_p(\mathbf{q})[NS + NPUMP \times (NLO + 1)]$ = the vector of costs per unit of length of the pipe segments (size NS); the energy cost per unit head of operating the pumping stations (size $NPUMP \times NLO$); and the cost per unit power of installing the pumping stations (size $NPUMP$).

The P1-QC Model

The purpose of the P1-QC model is to determine the least treatment costs of the system (capital and operation), while fulfilling the consumers requirements for qualities, and the physical laws describing the water-quality distributions.

The decision variables are $\mathbf{RR}[NSO \times (NLO + 1)]$ = the vector of the removal ratios (size $NSO \times NLO$), and the maximum removal ratios at the treatment facilities (size NSO).

The constraints are made of three blocks, included in (13): The first block ($NNC \times NLO$ constraints) are restrictions on maximum allowable concentrations at the internal nodes (usually consumer nodes) for steady-state quality conditions.

The second block ($NSO \times NLO$ constraints) are restrictions on the removal ratios. The ith equation states that the maximum removal ratio at treatment facility j and loading condition k is bounded by the maximum removal ratio of the treatment facility, which is itself a decision variable.

The third block (NSO constraints) are restrictions on the maximum possible removal ratios at the treatment facilities. In our model we assume that the maximum removal ratio can reach 100% (or 1.0), i.e., the concentrations of the quality parameters can be lowered to zero.

The definitions of $\mathbf{B}(\mathbf{q})$ and $\bar{\mathbf{c}}(\mathbf{q})$ are

$$\mathbf{B}(\mathbf{q})\,[NNC \times NLO + NSO \times (NLO + 1),\ NSO \times (NLO + 1)] = \left[\begin{array}{ccc|c} \mathbf{JRR}^1(\mathbf{q}^1) & & & \\ & \bullet & \mathbf{0} & \\ & \mathbf{JRR}^k(\mathbf{q}^k) & & \mathbf{0} \\ \mathbf{0} & & \bullet & \\ & & \mathbf{JRR}^{NLO}(\mathbf{q}^{NLO}) & \\ \hline & & & -\mathbf{I}_{\mathbf{RRmax}} \\ & \mathbf{I}_{\mathbf{RR}} & & \bullet \\ & & & -\mathbf{I}_{\mathbf{RRmax}} \\ \hline & \mathbf{0} & & \mathbf{I}_{\mathbf{RRmax}} \end{array}\right]$$

where $\mathbf{JRR}^k(\mathbf{q}^k)(NNC, NSO)$ = Jacobian matrix, for loading condition k, of the concentrations at the internal nodes with respect to the removal ratios; $\mathbf{I}_{\mathbf{RR}}(NSO \times NLO, NSO \times NLO)$, $\mathbf{I}_{\mathbf{RRmax}}(NSO, NSO)$ = identity matrices; and the remaining terms, which do not belong to $\mathbf{JRR}^k(\mathbf{q}^k)$ ($k = 1, \ldots, NLO$), $\mathbf{I}_{\mathbf{RR}}$, and $\mathbf{I}_{\mathbf{RRmax}}$, are zeros.

$$\bar{\mathbf{c}}^T(\mathbf{q})[NNC \times NLO + NSO \times (NLO + 1)] = [\mathbf{c}^1_{\max} - \mathbf{c}^1(\mathbf{q}^1), \ldots,$$
$$\mathbf{c}^k_{\max} - \mathbf{c}^k(\mathbf{q}^k), \ldots, \mathbf{c}^{NLO}_{\max} - \mathbf{c}^{NLO}); \mathbf{0}, \ldots, \mathbf{0}; \mathbf{1.0}]$$

where $\mathbf{c}^k_{\max}(NNC)$ = vector of the maximum admissible concentrations, for loading condition k, at the internal nodes; $\mathbf{c}^k(\mathbf{q}^k)(NNC)$ = vector of the concentrations, at the internal nodes, for loading condition k, steady-state quality conditions, and zero removal ratios.

The difference between the vector of the maximum admissible concentrations and the vector of the concentrations at the internal nodes, for all the loading conditions, comprises the first ($NNC \times NLO$) terms of $\bar{\mathbf{c}}^T(\mathbf{q})$. The remaining terms are $NSO \times NLO$ zeros, and NSO terms of 1.0.

The objective of the P1-QC model [(12)] is to minimize the cost of purchasing the water at the sources, plus the capital and operating costs of treating these waters.

The cost of water (before treatment) at the sources may not be equal due to differences in quality, or other reasons. The total cost of purchasing the water is

$$wc(\mathbf{q}) = PV\left[\sum_{k=1}^{NLO} \Delta T^k \left(\sum_{i=1}^{NSO} wc_i q_i^k\right)\right] \qquad (14)$$

where PV = present value factor; wc_i (\$/m^3) = cost of water at source i; ΔT^k (h/yr) = duration of loading condition k in a year; and q_i^k (m^3/h) = discharge supplied from treatment facility i, during loading condition k.

Since $wc(\mathbf{q})$ is a constant for a given $\mathbf{q}$, it does not appear explicitly as part of the optimization model P1-QC or P1-QH. However, the flows $\mathbf{q}$ are altered during the solution process and, therefore, $wc(\mathbf{q})$ influences the solution of the QCH model. This will be considered in the section on solution technique.

The capital and operating costs of the treatment plants, for a single quality parameter, are approximated by quadratic functions of the removal ratios. The capital cost is

$$TCC_i = \varepsilon_i Z_i^*(\mathbf{q})(RR\max_i)^2 = \gamma_i(\mathbf{q})(RR\max_i)^2 \qquad (15)$$

where TCC_i (\$) = construction cost of treatment facility i; ε_i (\$/m^3) = construction cost coefficient of treatment facility i, which is determined from analysis of cost data; $Z_i^*(\mathbf{q})$ (m^3) = optimal volume of treatment facility i for a given flow distribution $\mathbf{q}$: $Z_i^*(\mathbf{q}) = \max(\Delta t_i q_i^1, \ldots, \Delta t_i q_i^k, \ldots, \Delta t_i q_i^{NLO})$, where Δt_i (h) = minimum detention time required in treatment facility i; $RR\max_i$ = designed (maximum) removal ratio at treatment facility i; and $\gamma_i(\mathbf{q})$ (\$) = construction treatment cost coefficient at treatment facility i for a given flow $\mathbf{q}$.

The operation cost of the treatment plants is

$$TOC_i^k = (\Delta T^k q_i^k PV \alpha_i)(RR_i^k)^2 = \beta_i^k (RR_i^k)^2 \qquad (16)$$

where TOC_i^k (\$) = operation cost at treatment facility i and loading condition k; q_i^k (m^3/h) = the discharge supplied from treatment facility i, during loading condition k; α_i (\$/m^3) = a treatment cost coefficient at treatment facility i, which is determined from analysis of cost data; RR_i^k = removal ratio at treatment facility i and loading condition k; and β_i^k (\$) = operation cost coefficient at treatment facility i for loading condition k.

The capital and operation cost coefficients are put in the positive definite diagonal matrix $\mathbf{H}(\mathbf{q})[NSO \times (NLO + 1), NSO \times (NLO + 1)]$ whose first $NSO \times NLO$ diagonal terms are $2\beta_i^k$ ($i = 1, \ldots, NSO$; $k = 1, \ldots, NLO$), and the remaining NSO terms are $2\gamma_i(\mathbf{q})$ ($i = 1, \ldots, NSO$).

After formulating the optimal design model, reliability is incorporated through the following stages: (1) identification of the backup subsystems for each of the loading conditions considered, which will be responsible for retaining the desired level of service for the prescribed failure scenarios; (2) specifications of the blocks (i.e., the expressions of) (8), (9), (11), (13) for each of the backup subsystems; and (3) addition of the blocks so defined to the optimal design model.

MODEL PROPERTIES

1. The inner problem is convex.
2. The optimal value function is nonconvex and nonsmooth (Ben-Tal et al. 1992). This will require the use of a nondifferentiable optimization technique to handle minimization of the outer problem.
3. The dimension of the outer problem is much smaller than that of the inner one.

4. For a given flow distribution, the solution of the inner problem is a global minimum.
5. There is always a feasible solution of the P1-QC model since we assume treatment facilities at all the sources, and a maximum removal ratio of 100%. This requires the designer to decide whether the solution is physically possible, and, if necessary, to change the system layout and/or consumers' requirements.
6. The final solution is a local minimum.

SOLUTION TECHNIQUE

The minimization of the optimal value function requires the use of a nonsmooth technique, which in turn depends on the ability to compute for a given flow distribution an arbitrary member of the subgradient group of the optimal value function.

The computation of a member of the subgradient group at each iteration is based on the gradient of the Lagrangian of the problem with respect to the circular flows. The nonsmooth technique adopted to minimize the outer problem is the *r*-algorithm of Shor (1985).

The theoretical background and mathematical conditions needed for calculating a subgradient of the optimal value function, using the following, were developed by Ben-Tal et al. (1992) for inner problems that are linear or convex quadratic. We use their essential result [theorem 2.3 in Ben-Tal et al. (1992)] and apply it to P1

$$\nabla_{\mathbf{q}} \mathrm{Lg}(\mathbf{q}_0; \mathbf{X}_p^*, \mathbf{RR}^*; \mu_p^*, \lambda_{\mathbf{RR}}^*) \in \partial\phi(\mathbf{q}_0) \quad (17)$$

This is the gradient of the Lagrangian of the P1 formulation with respect to $\mathbf{q}$, at the point $(\mathbf{q}_0; \mathbf{X}_p^*, \mathbf{RR}^*; \mu_p^*, \lambda_{\mathbf{RR}}^*)$, where Lg = Lagrangian; $\mathbf{q}_0$ = a given feasible flow distribution in the pipes; $\mathbf{X}_p^*$, $\mathbf{RR}^*$ = optimal primal values of the decision variables of the P1-QH and P1-QC models, respectively; and μ_p^*, $\lambda_{\mathbf{RR}}^*$ = optimal dual values of the decision variables of the P1-QH and P1-QC models, respectively.

A substantial reduction in the dimension of the outer problem can be achieved by using the circular flows (Alperovits and Shamir (1977), i.e., flows in loops, instead of the flows in all pipes. This allows any flow distribution to be written

$$\mathbf{q}^k = \mathbf{q}_0^k + \mathbf{L}^T \Delta \mathbf{q}^k \quad \forall\, k \quad (18)$$

where $q_0^k(N)$ = any (initial or subsequent) flow distribution, which satisfies node continuity; and $\Delta\mathbf{q}^k(NL)$ = circular flow changes. Eq. (18) holds for each loading k, and guarantees that

$$\text{If } \mathbf{q}_0^k \in \mathbf{Q} \text{ then also } \mathbf{q}_0^k + \mathbf{L}^T\Delta\mathbf{q}^k \in \mathbf{Q} \quad \forall\, k \quad (19)$$

where $\mathbf{Q}$ = set of feasible flow distributions, in the sense that they maintain node continuity.

The *r*-algorithm of Shor (1985) is based on successive space dilations in the direction of the difference between two successive subgradients, and the use of the subgradient in the transformed space as the search direction. This idea overcomes the main difficulty of the subgradient method, in which at each iteration the subgradient is almost perpendicular to the direction towards the minimum, resulting in a poor rate of convergence.

Tailoring the *r*-algorithm to our problem yields an iterative procedure for minimizing the outer problem. To simplify the presentation we show the algorithm for one loading condition. The generalization for multiple loading conditions is straightforward.

Initialization

1. Set: $m = 0$, where m is the iteration counter.
2. Given an initial feasible flow distribution $\mathbf{q}_0 \in \mathbf{Q}$, compute $\phi(\mathbf{q}_0)$ and the subgradient $\mathbf{g}_0 \in \partial\phi(q_0)$ using (17).
3. Choose a step size $h_1 > 0$ such that

$$h_1 \mathbf{L}_s^T \frac{\mathbf{g}_0}{\|\mathbf{g}_0\|} \le \mathbf{q}_{0,s} \quad (20)$$

where $\mathbf{L}_s$ = rows of the fundamental loop matrix, which correspond to the pipes leaving the sources; and $\mathbf{q}_{0,s}$ = vector of flows from the sources. Eq. (20) ensures that the direction of the outgoing flows from the sources remains positive (i.e., the flow will not reverse). The direction of flow in all other pipes is *not* restricted; this is an important property of our methodology.

4. Compute

$$\Delta\mathbf{q}_1 = -h_1 \frac{\mathbf{g}_0}{\|\mathbf{g}_0\|} \quad (21)$$

where $\Delta\mathbf{q}_1$ = vector of changes of the circular flows.

5. Compute

$$\mathbf{q}_1 = \mathbf{q}_0 + \mathbf{L}^T\Delta\mathbf{q}_1; \quad \phi(\mathbf{q}_1); \quad \mathbf{g}_1 \in \partial\phi(\mathbf{q}_1) \quad (22)$$

6. Set a matrix denoted $\mathbf{B}_1$ to be an identity matrix.

Main Scheme

For $m \ge 1$ the $m + 1$ iteration is

1. Compute the direction vector

$$\mathbf{r}_m = \mathbf{B}_{m+1}^T[\partial\phi(\mathbf{q}_m) - \partial\phi(\mathbf{q}_{m-1})] \quad (23)$$

2. Compute the unit direction vector

$$\xi_{m+1} = \frac{\mathbf{r}_m}{\|\mathbf{r}_m\|} \quad (24)$$

3. Compute the updated matrix

$$\mathbf{B}_{m+1} = \mathbf{B}_m \left[\mathbf{I} + \left(\frac{1}{\delta} - 1\right) \xi_{m+1}\xi_{m+1}^T\right] \quad (25)$$

where δ = space dilation parameter of the *r*-algorithm; and $\mathbf{I}$ = an identity matrix.

4. Compute a step size
Choose $h_{m+1} > 0$ such that

$$h_{m+1} = h_0\theta^m \quad (26)$$

where h_0 = a prescribed step size; and θ = a coefficient between zero and 1. Check if h_{m+1} violates (27)

$$h_{m+1}\mathbf{L}_s^T\mathbf{B}_{m+1} \frac{\mathbf{B}_{m+1}^T\partial\phi(\mathbf{q}_m)}{\|\mathbf{B}_{m+1}^T\partial\phi(\mathbf{q}_m)\|} \le \mathbf{q}_{m,s} \quad (27)$$

then

$$h_{m+1} = \min_i \left\{ \frac{q_{m,s}^i}{\left[\mathbf{L}_s^T\mathbf{B}_{m+1} \dfrac{\mathbf{B}_{m+1}^T\partial\phi(\mathbf{q}_m)}{\|\mathbf{B}_{m+1}^T\partial\phi(\mathbf{q}_m)\|}\right]^i} \right\} \quad (28)$$

for all i such that $h_{m+1} > \dfrac{q_{m,s}^i}{\left[\mathbf{L}_s^T\mathbf{B}_{m+1} \dfrac{\mathbf{B}_{m+1}^T\partial\phi(\mathbf{q}_m)}{\|\mathbf{B}_{m+1}^T\partial\phi(\mathbf{q}_m)\|}\right]^i}$

where $q_{m,s}^i$ = outgoing flow from source i; and the denominator of (28) is the ith component of the vector $\mathbf{L}_s^T\mathbf{B}_{m+1}[\mathbf{B}_{m+1}^T\partial\phi(\mathbf{q}_m)/\|\mathbf{B}_{m+1}^T\partial\phi(\mathbf{q}_m)\|]$.
Do an approximate line search.

Set $n_{\max}$, where $n_{\max}$ is the maximum number of iterations for the approximate line search, and set an internal iteration counter denoted $n = 1$.

Compute

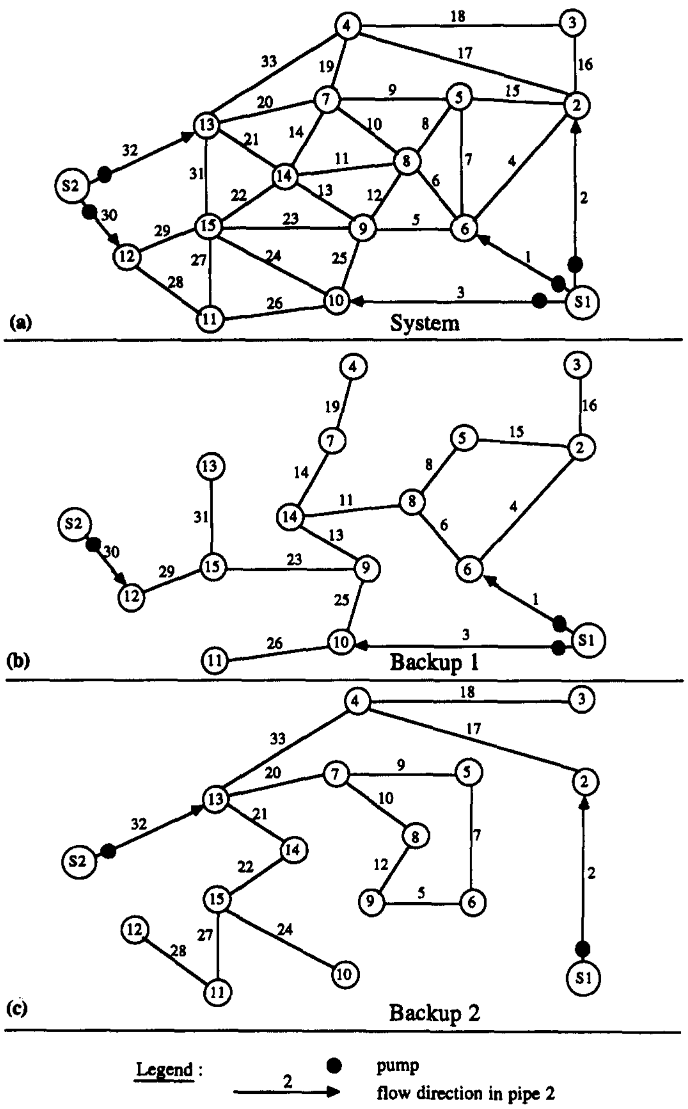

FIG. 1. Multiquality Water-Supply System with Two Backup Subsystems

TABLE 1. Consumers Data

Node (1)	Elevation (m) (2)	Consumption (m^3/h) (3)	Threshold concentration (mg/L) (4)
2	+140	400	250
3	+155	300	400
4	+160	200	300
5	+120	200	400
6	+130	500	200
7	+130	200	400
8	+130	300	450
9	+140	300	400
10	+150	400	200
11	+110	200	350
12	+100	200	250
13	+120	200	400
14	+145	200	300
15	+130	300	400

$$\Delta_{\mathbf{q}_{m+1}} = -h_{m+1}\mathbf{B}_{m+1}\frac{\mathbf{B}_{m+1}^T\partial\phi(\mathbf{q}_m)}{\|\mathbf{B}_{m+1}^T\partial\phi(\mathbf{q}_m)\|} \quad (29)$$

Compute

$$\mathbf{q}_{m+1} = \mathbf{q}_m + \mathbf{L}^T\Delta\mathbf{q}_{m+1} \quad (30)$$

Compute $\phi(\mathbf{q}_{m+1})$ and set $n = n + 1$.

If $\phi(\mathbf{q}_{m+1}) < \phi(\mathbf{q}_m)$ or $n > n_{\max}$ then go to the next step. Otherwise reduce the step size h_{m+1} by a prescribed coefficient, and go back to step leading to (29).

5. Compute $\partial\phi(\mathbf{q}_{m+1})$.
6. Check optimality. If at least two of the following three criteria are met, stop:

$$|\phi(\mathbf{q}_{m+1}) - \phi(\mathbf{q}_m)| \le \bar{\varepsilon}_1; \quad \|\mathbf{q}_{m+1} - \mathbf{q}_m\| \le \bar{\varepsilon}_2;$$

$$\frac{\mathbf{B}_{m+1}^T\partial\phi(\mathbf{q}_m)}{\|\mathbf{B}_{m+1}^T\partial\phi(\mathbf{q}_m)\|} \le \bar{\varepsilon}_3 \quad (31)$$

where $\bar{\varepsilon}_1$, $\bar{\varepsilon}_2$, $\bar{\varepsilon}_3$ = prescribed small values. Declare $\mathbf{q}_m$ as the optimal flow, and $\phi(\mathbf{q}_m)$ as the optimal value of the objective function. Otherwise set

$$m \to m + 1; \quad \mathbf{q}_m \leftarrow \mathbf{q}_{m+1}; \quad \mathbf{B}_m \leftarrow \mathbf{B}_{m+1} \quad (32a)$$

$$\partial\phi(\mathbf{q}_{m-1}) \leftarrow \partial\phi(\mathbf{q}_m); \quad \partial\phi(\mathbf{q}_m) \leftarrow \partial\phi(\mathbf{q}_{m+1});$$

$$\phi(\mathbf{q}_m) \leftarrow \phi(\mathbf{q}_{m+1}); \quad (32b)$$

and go back to step 1.

CASE STUDY

The optimization model was applied to the system shown in Fig. 1, which is based on Walski et al. (1987). The system

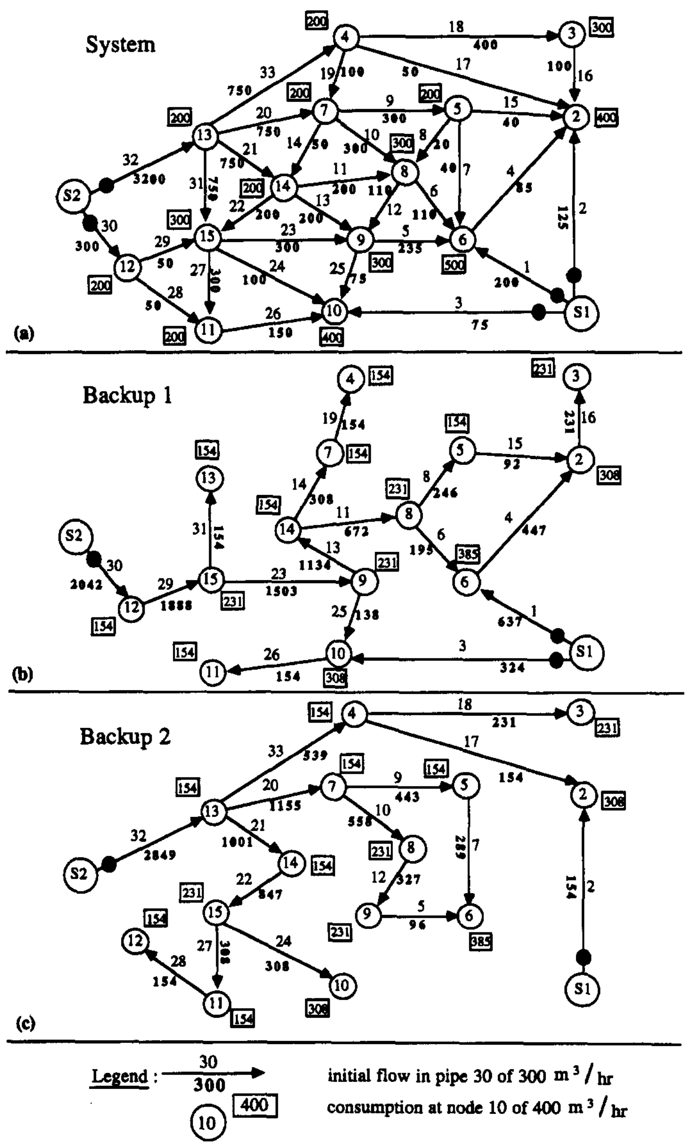

FIG. 2. Initial Flows and Consumptions for Base Run

TABLE 2. Base Run: Optimal Pipe Lengths and Diameters for Initial Flow Distribution

Pipe (1)	Flow (m³/h) System (2)	Backup 1 (3)	Backup 2 (4)	Lengths and Diameters [m (in.)] (5)
1	200.0	637.0	—	3,000 (16)
2	125.0	—	154.0	3,000 (10)
3	75.0	324.0	—	3,000 (14)
4	85.0	447.0	—	155.29 (16), 844.71 (20)
5	235.0	—	96.0	1,000 (20)
6	110.0	195.0	—	778.6 (14), 221.4 (16)
7	40.0	—	289.0	12.72 (6), 987.28 (10)
8	20.0	246.0	—	1,000 (20)
9	300.0	—	443.0	878.39 (10), 121.61 (12)
10	300.0	—	558.0	878.77 (10), 121.23 (12)
11	200.0	672.0	—	756.14 (10), 243.86 (12)
12	110.0	—	327.0	1,000 (20)
13	200.0	1,134.0	—	775.34 (10), 224.66 (12)
14	50.0	308.0	—	32.8 (4), 967.2 (6)
15	40.0	92.0	—	29.06 (6), 970.94 (10)
16	100.0	231.0	—	252.34 (6), 747.66 (10)
17	50.0	—	154.0	200.66 (4), 799.34 (6)
18	400.0	—	231.0	412.3 (12), 587.7 (14)
19	100.0	154.0	—	407.72 (14), 592.28 (16)
20	750.0	—	1,155.0	49.65 (16), 950.35 (20)
21	750.0	—	1,001.0	347.32 (14), 652.68 (16)
22	200.0	—	847.0	604.76 (10), 395.24 (12)
23	300.0	1,503.0	—	196.14 (16), 803.86 (20)
24	100.0	—	308.0	110.25 (10), 889.75 (12)
25	75.0	138.0	—	1,000 (14)
26	150.0	154.0	—	1,000 (16)
27	300.0	—	308.0	1,000 (20)
28	50.0	—	154.0	963.5 (10), 36.5 (12)
29	50.0	1,888.0	—	1,000 (20)
30	300.0	2,040.0	—	3,000 (20)
31	750.0	154.0	—	26.45 (12), 973.55 (14)
32	3,200.0	—	2,849.0	3,000 (20)
33	750.0	—	539.0	1,000 (20)

TABLE 3. Base Run: Optimal Heads and Power of Pumps for Initial Flow Distribution

Pump (1)	Optimal Head (m) System (2)	Backup 1 (3)	Backup 2 (4)	Maximum power (hp) (5)
1	101.41	240.09	—	708.03
2	106.02	—	99.02	70.60
3	100.46	379.43	—	569.14
30	121.72	450.31	—	4,257.13
32	216.95	—	260.85	3,440.60

is optimized for a single loading condition and one quality parameter with two backups. It contains 33 pipes, five pumps and 16 nodes: two source nodes with treatment facilities (S1, S2), and 14 consumer nodes (2, . . . , 15). The first subsystem (backup 1) consists of a spanning tree plus two additional pipes, 4 and 6, and the second subsystem (backup 2) consists of another spanning tree, with the additional pipe 5 (see Fig. 1).

Table 1 shows the consumers data: elevation, consumption, and threshold concentration. The minimum pressure head required at each consumption node is 30 (m). The initial concentrations at sources S1 and S2 are 300 and 600 (mg/L), respectively. The total head at source S1 is 80 (m) and at source S2 it is 60 (m).

The length of all the pipes is 1,000 (m), excluding the pipes connected to the sources (1, 2, 3, 30, 32) whose length is 3,000 (m). The Hazen-Williams coefficient for all the pipes is 130, and each pipe is allowed to be made from the candidate diameters (in in.): 4, 6, 10, 12, 14, 16, 20, whose associated unit costs ($/m) are 11, 16, 32, 50, 60, 90, 170.

In addition, we used the following data:

- Water costs ($/m³) (before treatment) at sources S1 and S2 are 0.05, 0.03, respectively.
- Detention times (h) at the treatment facilities at sources S1 and S2 are 8, 7, respectively.
- Treatment cost coefficients ($/m³) at the treatment facilities at sources S1 and S2 are 0.03, 0.02, respectively.
- Construction cost coefficients ($/m³) for the treatment facilities at sources S1 and S2 are 30, 20, respectively.
- Energy cost ($/kW·h): 0.1.
- Cost of installing the pumping stations ($/hp): 3,200.
- Pump efficiency: 0.8.
- Present value coefficient: 10.04 (for an annual interest of 5.5% and planning horizon of 15 years).
- The system is expected to operate without failure 90% of the time, while each of the backups operates 5% of the time.

BASE RUN

As a detailed illustrative example we present the solution from the initial flow distribution shown in Fig. 2. This distribution has provided the least cost solution.

The total consumption of the system (3,900 m³/h) is reduced uniformly by 23% for each of the backups at all the consumer nodes, giving a total demand of 3,003 (m³/h). The consumers' requirements for pressure and quality are the same for the system and the backups. The system is optimized with the parameters $h_0 = 30$; $\theta = 0.85$, and $\delta = 2.0$ on a CONVEX computer. The routines E04MBF and E04NCF from the NAG library are used to solve the P1-QH and P1-QC models, respectively, at each iteration. The average central processing unit (CPU) time for an inner iteration of the P1 model is 18 (s).

TABLE 4. Base Run: Optimal Concentrations at Consumer Nodes for Initial Flow Distribution

Node (1)	Optimal Concentration (mg/L) System (2)	Backup 1 (3)	Backup 2 (4)
2	194.82	204.73	250.0
3	204.91	204.73	200.0
4	204.91	250.00	200.0
5	204.91	250.00	200.0
6	195.95	195.41	200.0
7	204.91	250.00	200.0
8	204.91	250.00	200.0
9	204.91	250.00	200.0
10	200.00	200.00	200.0
11	204.91	200.00	200.0
12	204.91	250.00	200.0
13	204.91	250.00	200.0
14	204.91	250.00	200.0
15	204.91	250.00	200.0

TABLE 5. Base Run: Optimal Removal Ratios and Volumes of Treatment Facilities for Initial Flow Distribution

Source (1)	Optimal Removal Ratio ( ) System (2)	Backup 1 (3)	Backup 2 (4)	Maximum removal ratio ( ) (5)	Optimal volume of treatment facility (m³) (6)
S1	0.4043	0.4043	0	0.4043	7,688
S2	0.6585	0.5833	0.6667	0.6667	24,500

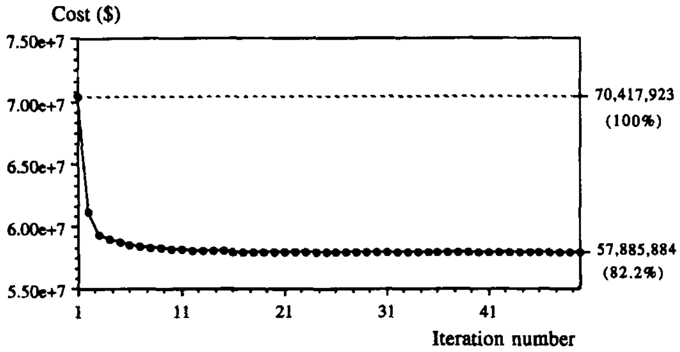

FIG. 3. Cost versus Iteration for Base Run

TABLE 6. Base Run: Optimal Pipe Lengths and Diameters for Optimal Flows

Pipe (1)	Flow (m^3/h) System (2)	Backup 1 (3)	Backup 2 (4)	Lengths and Diameters [m (in.)] (5)
1	184.46	633.98	—	3,000 (16)
2	113.87	—	170.67	3,000 (10)
3	141.75	339.26	—	3,000 (14)
4	111.53	445.43	—	1,000 (16)
5	186.27	—	93.47	599.76 (12), 400.24 (14)
6	87.37	196.46	—	31.7 (6), 968.3 (10)
7	153.44	—	291.53	115.14 (10), 884.86 (12)
8	14.35	247.57	—	457.95 (12), 542.05 (14)
9	411.95	—	445.53	1,000 (20)
10	212.48	—	555.47	184.81 (14), 815.19 (16)
11	206.46	675.02	—	718.14 (16), 281.86 (20)
12	45.92	—	324.47	1,000 (16)
13	181.49	1,137.02	—	121.03 (14), 878.97 (16)
14	67.95	308.00	—	640.87 (12), 359.13 (14)
15	47.17	93.57	—	396.47 (6), 603.53 (10)
16	90.71	231.00	—	1,000 (14)
17	39.73	—	137.33	124.64 (4), 875.36 (6)
18	390.71	—	231.00	470.43 (12), 529.57 (14)
19	130.86	154.00	—	30.93 (6), 969.07 (10)
20	761.52	—	1,155.00	764.85 (16), 235.15 (20)
21	643.80	—	1,001.00	221.85 (14), 778.15 (16)
22	123.80	—	847.00	1,000 (20)
23	316.50	1,490.76	—	1,000 (20)
24	51.22	—	308.00	160.8 (6), 839.2 (10)
25	57.64	122.74	—	38.81 (6), 961.19 (10)
26	149.39	154.00	—	1,000 (16)
27	258.94	—	308.00	1,000 (16)
28	90.44	—	154.00	533.66 (10), 466.34 (12)
29	56.63	1,875.76	—	1,000 (20)
30	347.07	2,029.76	—	3,000 (20)
31	746.24	154.00	—	890.53 (16), 109.47 (20)
32	3,112.85	—	2,832.33	3,000 (20)
33	761.29	—	522.33	1,000 (20)

TABLE 7. Base Run: Optimal Heads and Power of Pumps for Optimal Flow Distribution

Pump (1)	Optimal Head (m) System (2)	Backup 1 (3)	Backup 2 (4)	Maximum power (hp) (5)
1	106.23	120.99	—	355.11
2	109.94	—	102.3	80.83
3	107.14	137.10	—	215.33
30	128.13	206.37	—	1,939.26
32	211.77	—	232.11	3,066.30

The optimal solution of the inner problem, in the first iteration, is shown in Tables 2–5. The optimal total cost of the QCH problem in the first iteration is: $70,417,923. It is made of $56,626,479 (80.4%) from the QH problem, and of $13,791,444 (19.6%) from the QC problem (including the costs of purchasing the water at the sources).

The heads at nodes 3, 4, 10, for the system; node 4 for backup 1; and nodes 2, 10, for backup 2 are at their minimum level. The concentrations at nodes: 10 for the system; 10, 12 for backup 1; and 2 for backup 2 reach their maximum limit.

The objective function values of the P1 model, as a function of the iteration number, are shown in Fig. 3. The optimal solution of the P1 model, at the final iteration is shown in Tables 6–9. Fig. 4 shows the optimal flow distribution in the system and the backups (note that the accurate values of the flows are listed in Table 6).

The optimal total cost of the QCH problem at the final iteration is 57,885,884 ($). It is made up of $44,140.781 (76.3%) from the QH problem, and of $13,745,103 (23.7%) from the QC problem (including the costs of purchasing the water at the sources).

The heads at nodes 3, 4 for the system; node 3 for backup 1; and node 10 for backup 2 are at their minimum level. The concentrations at node 6 for the system; nodes 10, 12 for backup 1; and nodes 2, 6, 10 for backup 2 reach their maximum limit.

Both the QH and QC problems are improved at the final solution. The QH is reduced by $12,485,698 (22.0%) and the QC by $46,341 (0.34%), relative to their costs in the optimal solution for the initial flows.

The major improvement in system cost is due to the reduction in the required heads for operating backup 1, which yields a significant decrease in the maximum power needed for the pumps (compare Table 3 to Table 7). This is because the cost of constructing the system (pipes and pumps) is three to four times higher than the cost of purchasing and treating the water. Therefore, the major reduction is governed by the QH problem.

TABLE 8. Base Run: Optimal Concentrations at Consumer Nodes for Optimal Flow Distribution

Node (1)	Optimal Concentration (mg/L) System (2)	Backup 1 (3)	Backup 2 (4)
2	193.19	207.04	250.0
3	230.61	207.04	200.0
4	230.61	250.00	200.0
5	230.61	250.00	200.0
6	200.00	198.02	200.0
7	230.61	250.00	200.0
8	230.61	250.00	200.0
9	230.61	250.00	200.0
10	194.65	200.00	200.0
11	230.61	200.00	200.0
12	230.61	250.00	200.0
13	230.61	250.00	200.0
14	230.61	250.00	200.0
15	230.61	250.00	200.0

TABLE 9. Base Run: Optimal Removal Ratios and Volumes of Treatment Facilities for Optimal Flow Distribution

Source (1)	Optimal Removal Ratio ( ) System (2)	Backup 1 (3)	Backup 2 (4)	Maximum removal ratio ( ) (5)	Optimal volume of treatment facility (m^3) (6)
S1	0.5695	0.3936	0.0326	0.5695	7,786
S2	0.6157	0.5833	0.6667	0.6667	24,219

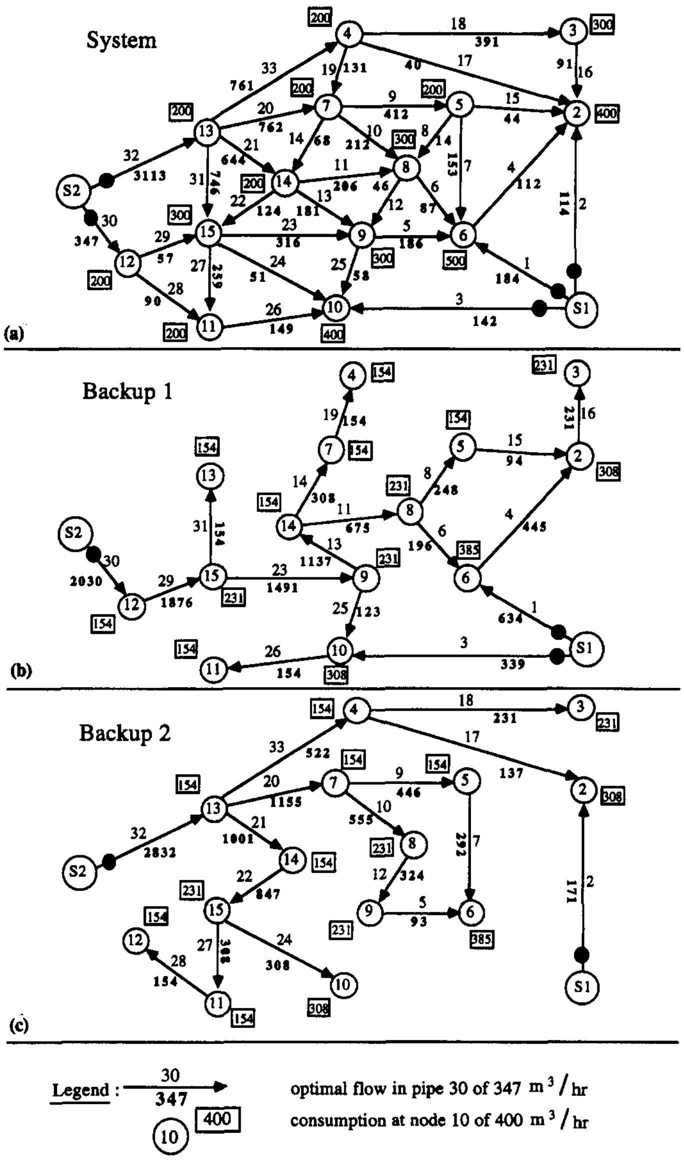

FIG. 4. Optimal Flows and Consumptions for Base Run

TRADE-OFF BETWEEN COST AND RELIABILITY

The trade-off between cost and reliability is examined by changing the consumers requirements in the cases of failure scenarios in which backup 1 or backup 2 operate.

The trade-off is evaluated with respect to the requirements of the consumers for flow, quality, and pressure, and to the percentage of time during which the backups operate. The resulting trade-off curves can be used to determine the system cost associated with a desired system service.

The trade-off between total supply and cost is investigated by uniformly reducing the total system consumption (3,900 m^3/h) for the backups, while keeping the demands for quality and pressure unchanged. The resulting curve is shown in Fig. 5 (note that the base run is at the point where 77% of the full demand is supplied).

The trade-off between quality and cost is investigated by uniformly increasing the threshold concentration requirements for the backups, while keeping the demands for flow and pressure unchanged. The resulting curve is shown in Fig. 6. There is almost no reduction in system cost due to the increase in the threshold concentration requirements for the backups. This is because the capacities (volumes and maximum removal ratios) of the treatment facilities are determined by the system. The saving in system cost is achieved by reduction of the

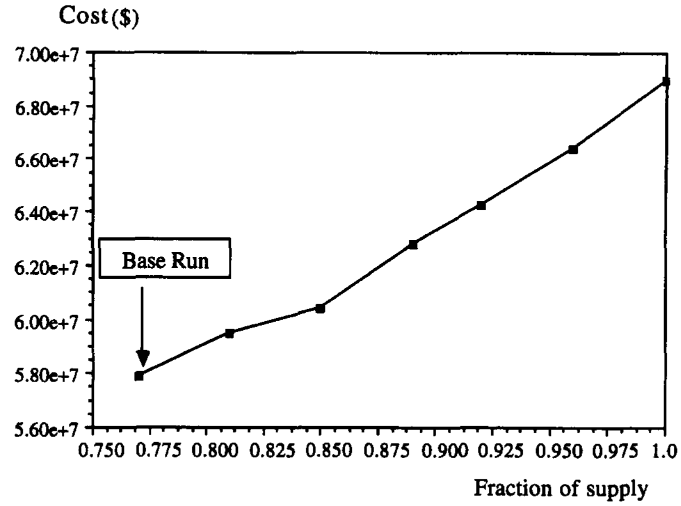

FIG. 5. Trade-Off between Supply and Cost for the Same Head and Quality Constraints at the System and the Backups

treatment needed for the backups, which is negligible in comparison to total cost. The lower bound of the cost, with no quality constraints for the backups, is $68,697,752.

In analyzing the system behavior with respect to changes in consumption, and with respect to quality, we assumed that a consumer demand for flow is met if its pressure head is at least 30 (m). In examining the trade-off between pressure and cost we relax this assumption by using the supply response head relationship suggested by Wagner et al. (1988b), which has the following parts: (1) consumption is zero when the pressure is below 20 (m); (2) for pressures between 20 and 30 (m) the reduction in consumption is given by

$$F_r = 1 - 0.01(P_r - 30)^2 \quad (33)$$

where F_r = fraction of full consumption at a node; and P_r (m) = pressure head at a node; and (3) full consumption for pressures above 30 (m). The trade-off between pressure and cost is shown in Fig. 7. The shape of the curve is concave. This is because the reduction in system cost is dominated by the reduction of supply to the consumers, and not by the saving in energy cost.

The trade-off between the fraction of time during which the backups operate and system cost is shown in Fig. 8, for flow and quality at the system and the backups as listed in Table 1, and for minimum pressure head at all the consumer nodes of 30 (m).

CONCLUSIONS

A methodology which integrates the optimal design and reliability of a multiquality water-supply system was developed and applied to a system based on Walski et al. (1987).

Reliability is incorporated directly into the design phase. The system is required to sustain prescribed failure scenarios, in our case any single random component failure, and still maintain a desired level of service to the consumers in terms of the quantities, qualities, and pressures.

The decomposition idea of Alperovits and Shamir (1977) for the design of optimal water-distribution systems is expanded for the design of multiquality water-supply systems. The formulation of the model (i.e., the P1 for formulation) results in an outer nonsmooth problem in the circular flow domain, and an inner convex quadratic problem. The inner problem can be further split into a linear programming QH part (the P1-QH) and a convex quadratic QC part (the P1-QC). This is an inherent attribute of a multiquality water-supply system that can be used in other formulations, such as the

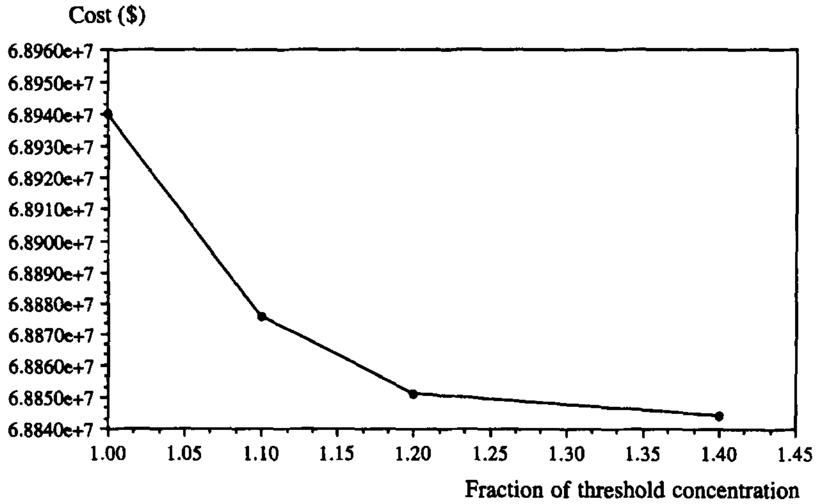

FIG. 6. Trade-Off between Quality and Cost for the Same Head and Consumption Constraints at the System and the Backups

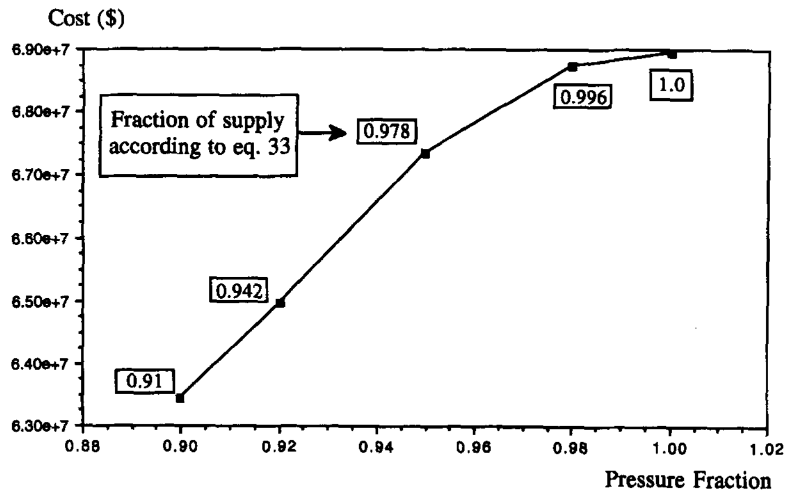

FIG. 7. Trade-Off between Pressure Head and Cost for the Same Consumption and Quality Constraints at the System and the Backups

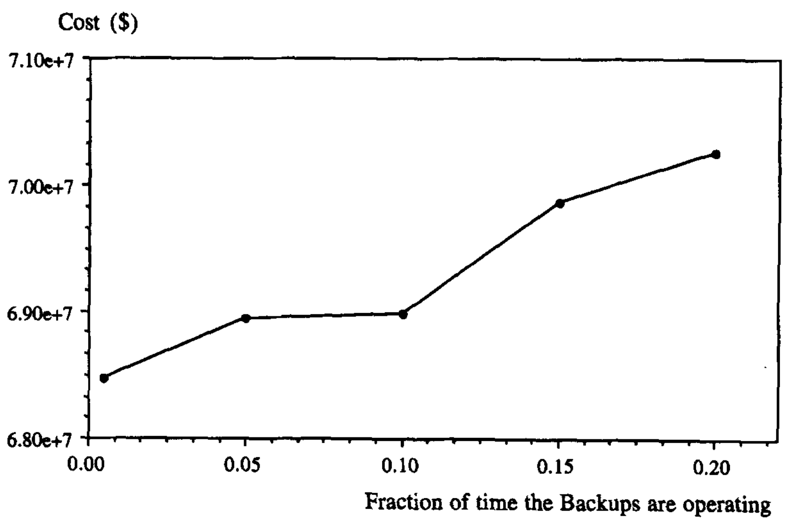

FIG. 8. Trade-Off between Different Durations in Which Backups are Operating and Cost for the Same Flow, Quality, and Head Constraints at the System and the Backups

optimal operation problem. The solution is obtained using the nonsmooth r-algorithm of Shor (1985).

The backup subsystems are determined for a given set of prescribed failure scenarios. As such, their selection is basically part of the planning phase, in which the system layout is determined. There is no quantitative model for selection of the "best" layout, nor of the backup subsystems. The layout selection is heuristic and is based on engineering judgement and experience.

The selection of the set of backups in our work has only considered the connectivity of the system. A suggestion for further research is the development of a screening model aimed at grading given candidate sets of backups. This grading can be a tool to assist the designer in selecting backups for the optimization model. The screening model can be based on the following stages:

1. Selection of candidate sets of backups, following topological considerations. A possible way of selecting candidate sets can be by the use of a matrix whose elements describe the frequency of the pipes appearing in the paths that connect sources to consumers. To build this matrix, the paths between the sources and consumers should be scanned. This can be determined by using path enumeration algorithms like those of Misra (1970) or Aggarwal et al. (1973).
2. Definitions of a goal programming model in which the objective function is the minimization of the damages incurred to the consumers for not supplying their required demands, and the constraints are simplifications of the constraints of the P1-QH and P1-QC models.
3. Solution of the model defined in stage 2 for each of the sets defined in stage 1.

The value of the optimal solution will be used for grading the sets.

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APPENDIX I. REFERENCES

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APPENDIX II. NOTATION

The following symbols are used in this paper:

$\mathbf{A(q)}$ = a matrix defined as

$$\begin{bmatrix} & & -\mathbf{I} \\ \mathbf{0} & \mathbf{A}_{pu}(\mathbf{q}) & -\mathbf{I} \\ & & -\mathbf{I} \end{bmatrix};$$

$\mathbf{A}_{pu}(\mathbf{q})$ = diagonal matrix made of the power per unit head coefficients;
$\mathbf{a}$ = vector of the link lengths;
$\mathbf{a}_p(\mathbf{q})$ = vector of unit costs of the objective function of the P1-QH model;
$\mathbf{B(q)}$ = matrix in Eqs. (6) and (13) that is made of $\mathbf{JRR}^k(\mathbf{q}^k)$, $\mathbf{I}_{RR}$, and $\mathbf{I}_{RRmax}$;
$\mathbf{B}_1$ = identity matrix for the initial iteration;
$\mathbf{B}_m$ = matrix at the mth iteration computed by Eq. (25);
$\mathbf{b}^k$ = vector of the head differences for the loop equations for loading condition k;
$\bar{\mathbf{c}}(\mathbf{q})$ = vector of the differences between the threshold concentrations and the concentrations at the internal nodes, for zero removal ratios at the treatment plants;
$\mathbf{c}^k(\mathbf{q}^k)$ = vector of the concentrations at the internal nodes for loading condition k and zero removal ratios;
$\mathbf{c}^k_{max}$ = vector of the maximum admissible concentrations for loading condition k at the internal nodes;
F_r = fraction of full consumption at a node;
$\mathbf{g}_m$ = subgradient at the mth iteration;
$\mathbf{H(q)}$ = positive definite diagonal matrix made of β^k_i and γ_i;
h_m = step size at the mth iteration;
h_0 = a prescribed step size;
$\mathbf{I}$ = identity matrix;
$\bar{\mathbf{I}}$ = matrix of the internal arrangement of the pipe segments;
$\bar{\mathbf{I}}_a$ = matrix defined as $[\bar{\mathbf{I}} \quad \mathbf{0}]$;
$\bar{\mathbf{I}}_p$ = matrix defined as

$$\begin{bmatrix} \bar{\mathbf{I}} & \mathbf{0} \\ \mathbf{0} & \mathbf{I} \end{bmatrix};$$

$\mathbf{I}_{RR}$, $\mathbf{I}_{RRmax}$ = identity matrices in $\mathbf{B(q)}$;
$\mathbf{J}^k(\mathbf{q}^k)$ = diagonal matrix which attaches for a given loading condition k the hydraulic gradient to each pipe segment;
$\mathbf{J}^k_p(\mathbf{q}^k)$ = matrix defined as

$$\begin{bmatrix} \mathbf{J}^k(\mathbf{q}^k) & \mathbf{0} \\ \mathbf{0} & \mathbf{I} \end{bmatrix};$$

$\mathbf{JRR}^k(\mathbf{q}^k)$ = Jacobian matrix for loading condition k of the concentrations at the internal nodes with respect to the removal ratios;
k = loading conditions index;
$\mathbf{L}$ = loop matrix;
Lg = Lagrangian of the P1 formulation;
$\mathbf{L}^k_p$ = matrix defined as $[\mathbf{L} \quad \mathbf{L}^k_{pu} \quad \mathbf{0}]$;
$\mathbf{L}^k_{pu}$ = matrix defining the pumping station locations with respect to each loop and loading condition k;
$\mathbf{L}_s$ = rows of the loop matrix which correspond to the pipes leaving the sources;
m = main scheme iteration counter;
N = number of pipes;
ND = number of commercial pipe diameters;
NEI = number of internal pipes;
NL = number of loops;
NLO = number of loading conditions;
NNC = number of internal nodes;
NP = number of paths;
$NPUMP$ = number of pumping stations;
NS = number of pipe segments;
NSQ = number of source nodes;

n = iteration counter for the approximate line search;
n_{max} = maximum number of iterations or the approximate line search;
$\mathbf{P}$ = path matrix;
$\mathbf{P}_p^k$ = matrix defined as $[\mathbf{P} \quad \mathbf{P}_{pu}^k \quad \mathbf{0}]$;
$\mathbf{P}_{pu}^k$ = matrix defining the pumping station locations with respect to each path and loading condition k;
P_r = pressure head at a node;
PV = present value factor;
$\mathbf{Q}$ = set of flows that maintain continuity at the system nodes;
$\mathbf{q}$ = vector of flows in the network pipes;
$\mathbf{q}_0$ = given feasible flow distribution in the pipes;
$\mathbf{q}_0^k$ = given feasible flow distribution for loading condition k;
$\mathbf{q}^k$ = vector of flows in the network pipes for loading condition k;
$q_{m,s}^i$ = outgoing flow from source i in Eq. (28);
q_m = vector of flows at the mth iteration;
$\mathbf{q}_{m,s}$ = vector of the outgoing flows from the sources at the mth iteration;
$\mathbf{RR}$ = vector made of the removal ratios, and the maximum removal ratios at the treatment plants;
$\mathbf{RR}^*$ = vector of the optimal primal values of the P1-QC model;
$RR\max_i$ = designed (maximum) removal ratio at the ith treatment plant;
RR_i^k = removal ratio at the ith treatment plant and loading condition k;
r_m = direction vector at the mth iteration;
TCC_i = construction cost of the ith treatment plant;
TOC_i^k = treatment operational cost of the ith treatment plant and loading condition k;
$wc(\mathbf{q})$ = total cost of purchasing the water at the sources;
wc_i = cost of water at source i;
$\mathbf{X}_p$ = vector of the decision variables of the P1-QH model;
$\mathbf{X}_p^*$ = vector of the optimal primal values of the P1-QH model;
$Z_i^*(\mathbf{q})$ = optimal volume of the ith treatment plant;
α_i = treatment cost coefficient at the ith treatment plant;
β_i^k = operational treatment cost coefficient at the ith treatment plant for loading condition k;
$\gamma_i(\mathbf{q})$ = construction treatment cost coefficient at the ith treatment plant;
δ = space dilation parameter of the r-algorithm;
$\Delta\mathbf{H}_{max}^k$ = vector of the maximum admissible headlosses for loading condition k;
$\Delta\mathbf{q}^k$ = circular flow changes vector for loading condition k;
$\Delta\mathbf{q}_m$ = vector of changes of the circular flows at the mth iteration;
ΔT^k = duration of loading condition k in a year;
Δt_i = minimum detention time at the ith treatment plant;
$\bar{\varepsilon}_1, \bar{\varepsilon}_2, \bar{\varepsilon}_3$ = prescribed small values;
ε_i = construction cost coefficient at the ith treatment plant;
ξ_m = unit direction vector at the mth iteration;
θ = coefficient between zero and 1;
λ_{RR}^* = vector of the optimal dual values of the P1-QC model;
μ_p^* = vector of the optimal dual values of the P1-QH model; and
$\phi(\mathbf{q})$ = optimal value function.