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Water quality management in regional systems

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ABSTRACT The management of regional water resources encompasses allocation of water to users, expansion of facilities, transportation of water and water inventory management. These activities are distributed over time and space, and some may depend on hydrological conditions. They are also interdependent in many ways. A mathematical linear programming model is used to find optimal solutions in a budget and water resources constrained environment. Incorporating water quality management in the model makes the problem nonlinear. A successive linear approximation iterative scheme is applied to cope with nonlinearities. Its application is demonstrated to a problem of finding a minimum cost solution, considering water quantities, salinization of a source, saline return flows, desalination and salinity damages.

La gestion de la qualité de l'eau dans des systèmes régionaux
RESUME La gestion des ressources en eau régionales comprend l'allocation d'eau aux consommateurs, l'expansion des installations et l'adduction de l'eau. Ces activités varient dans le temps et l'espace et quelque-unes peuvent dépendre des conditions hydrologiques. Ces activités dépendent aussi les unes des autres de plusieurs manières. Un modèle mathématique à programmation linéaire est utilisé pour trouver les solutions optimales, les ressources en eau étant soumises à certaines restrictions. L'incorporation de la qualité de l'eau dans ce modèle rend le problème non linéaire. Ce problème est alors attaqué en utilisant une technique itérative d'approximations linéaires successives. La mise en application de cette méthode est démontrée en trouvant la solution à coût minimal, en tenant compte des volumes d'eau, de la salinisation d'une source ainsi que des dommages engendrés par la salinisation et la désalinisation.

WATER RESOURCES MANAGEMENT MODELLING

Management of regional water resources systems requires decisions on four groups of control or decision variables:

(a) allocation of water among users;

(b) construction and expansion of facilities, such as well fields, intakes, treatment plants, storage reservoirs, conveyance, pumping and distribution pipelines;

(c) extractions from the various sources and transportation of water throughout the system in each time period (e.g. month, season, year);

(d) operation of seasonal and over-year storage for management of supply, as well as a variety of other purposes, such as hydroelectric power generation, flood protection and low flow augmentation.

For long range planning consideration must be given to the range of hydrological conditions which may occur (dry, wet). Decisions on capacity expansion are made in each of several periods, each a few years long. The operational decisions however are for each season of the year, and for each hydrological condition.

TAHAL (Schwarz, 1983) has developed a model - TEKUMA - which determines the plan of allocation, capacity expansion, production, transportation and operation that maximizes the net benefit - the sum of all water-related values minus the sum of all investment and operating costs and losses incurred by insufficient supply. The plan satisfies a variety of constraints - hydrological, hydraulic, budgetary, water allocations, and others. TEKUMA uses linear programming as its core, and has a matrix generator and report writing routines which make it user friendly for water resources management.

The TEKUMA model has been used for long-range (20-years) planning of Israel's national water resources system. It has also been used for planning several major regional systems, as well as to plan the operation of smaller but complex systems, and also for short time periods (e.g. days).

With the increasing importance of water quality, the model has recently been expanded to include consideration of such factors as:

(a) use of lower quality waters, such as reclaimed sewage and brackish groundwater;

(b) increasing salinization of aquifers, due primarily to the encroachment of sea water;

(c) the impact of return flows on groundwater quality;

(d) greater tolerance of certain crops to salinity;

(e) increasing pollution of surface water sources;

(f) the possibility of desalination, although at a high cost.

Extension of the model required the addition to the decision variables of salinity (or some other conservative, non reacting, water quality parameter), its variation over time in sources and at nodes, and the introduction of desalination as an additional operation and development variable. Constraints of maximum tolerable salinities are imposed as upper bounds on water supply to consumers. The losses incurred by surpassing given salinity thresholds are added to the objective function.

Mass balance equations are introduced to supplement the water balance equations of the original TEKUMA model. These nonlinear equations are incorporated in the linear programming model by means of a successive linear approximations iterative scheme.

Further refinement has permitted analysis of the impact on groundwater quality of return flows from irrigation and other uses, and of the variations in groundwater balance resulting from increased exploitation.

The model has been tested and used to plan the annual operation of the multi-quality water supply system serving the southern Arava region in Israel and the water supply to Eilat (Fig.1).

MATHEMATICAL MODEL FORMULATION

A partial description of the structure of TEKUMA I model is shown below. The time dimensions of the model are defined in the following:

Period A group of years (say 5 to 10) which are considered to be essentially identical to each other and thus be represented by one single year. Capacity expansion variables are defined for each period, and are assumed to be developed at the beginning of the period, and made available for all years in that period, and in the following.

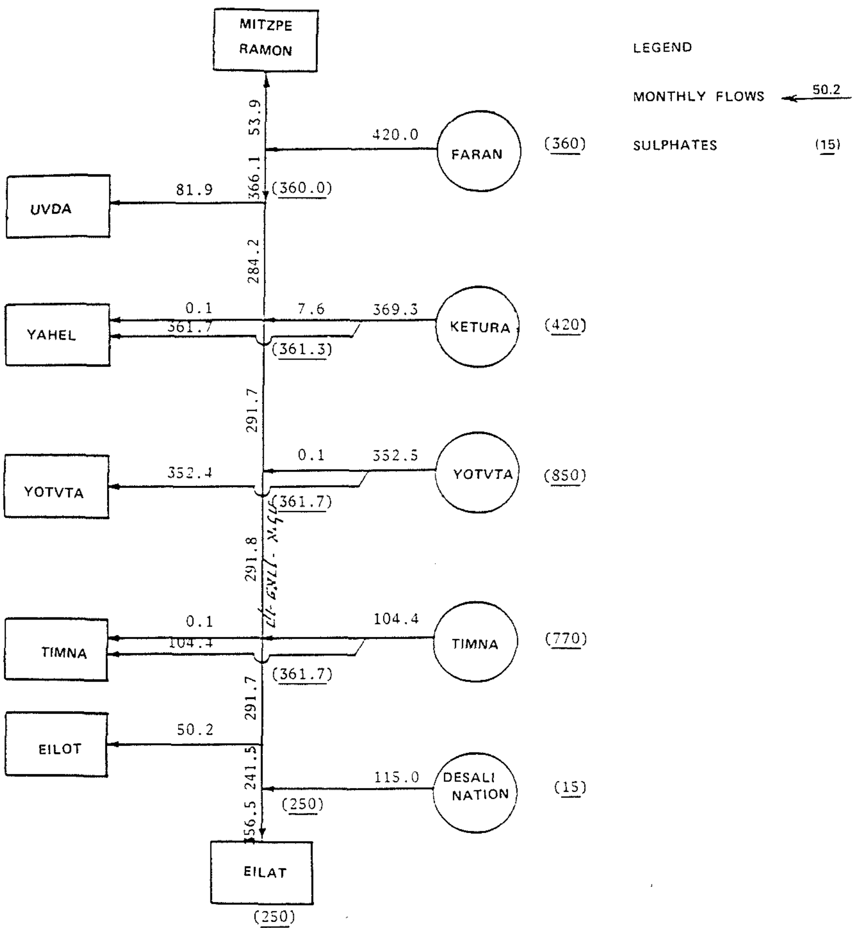

FIG.1 Southern Arava - January 1984 operation.

Season A group of months (or days), contiguous or not, considered "homogeneous" in terms of water availability in the source, consumer demands, and operational decisions. For the Israel national water resources system we have defined:
winter - December, January, February, March
summer - June, July, August
interim - all other months

The climate is defined by availability of water in the sources, and corresponds to a certain percentile of the distribution of annual water yields. For the Israel national water resources system we have used the means of years located around the 5, 50 and 95% points on the cumulative probability histogram. They represent the top and bottom 10% fractions and the central 80% years, and are called "dry", "wet" and "regular", respectively.

Decision variables

S_{njl} target water supply in period n in region j to consumer sector l ($m^3 year^{-1}$)

K_{npjl} temporary curtailment of water supply to consumer sector l in period n, climatic state p, and region j ($m^3 year^{-1}$)

M_{nk} capacity expansion in period n for link k ($m^3 day^{-1}$), including intake and pumping capacity expansion

T_{npkm} conveyance of water in period n, climatic state p, link k and season m ($m^3 day^{-1}$)

D_{npim} amount used from storage from source i in period n, climatic state p, and season m ($m^3 day^{-1}$)

C_{npim} salinity in period n, climatic state p, source i and season m ($mg\ l^{-1}$)

The objective function

$$\min \Sigma_n - \rho'_n \Sigma_j \Sigma_l S_{njl} * CS_{njl} + \rho'_n \Sigma_p P_{np} \Sigma_j \Sigma_l K_{npjl} * CK_{npjl}$$

$$+ \rho''_n \Sigma_k M_{nk} * CM_{nk}$$

$$+ \rho'_n \Sigma_p P_{np} \Sigma_m m_{nm} \Sigma_k T_{npkm} * [CT_{nk} + (C_{npim} - CTR_{nj})^+ * CC_{nj}]$$

where

CS_{njl} unit (worth/product) value of water ($\$m^{-3}$);

CK_{npjl}, CM_{nk}, CT_{nk} unit costs ($\$m^{-3}$, or $\$m^{-3}$ day for capacity expansion);

CC_{nj} unit salinity damage cost ($\$m^{-3} mg\ l^{-1}$;

CTR_{nj} threshold salinity, above which damages occur ($mg\ l^{-1}$);

$(.)^+$ positive when (.) > 0, zero otherwise;

P_{np} probability of climatic state p in period n;

$\rho'_n \rho''_n$ present worth discount factors for period n. The first includes discounted summing within the period to its beginning and then discounting to the present. The second includes only the second effect.

Some of the major constraints

Water requirement bound

$$LDMND_{njl} < S_{njl} < UDMND_{njl}$$

where $LDMND_{njl}$ is the minimum requirement, and $UDMND_{njl}$ is the maximum demand.

Water quality limits

$$C_{npim} < CMAX_{nj}$$

where $CMAX_{nj}$ is the maximum salinity in region j, which is supplied from node i.

Regional water balance (for period n, climatic state p, region j, season m)

$$\Sigma_{l}(S_{njl} - K_{npjl}) * \alpha_{nmjl} < \Sigma_{K}^{*}T_{npmk}$$

where

α_{nmjl} (daily/annual) demand fraction in season m:
Σ_{K}^{*} sum of links supplying to the region under consideration (j).

Capacity expansion

$$T_{npkm} < \Sigma_{n'=1}^{n'=n} M_{n'k}$$

Annual source water balance (for period n, climatic state p, source i)

$$\Sigma_{m}m_{nm}[\Sigma_{k}^{out}T_{npkm} - \Sigma_{k}^{in}T_{npkm} - D_{npim}]$$

$$- \Sigma_{j}\Sigma_{l}(S_{njl} - K_{npjl}) * \beta_{nijl} < YREPL_{npi}$$

where

m_{nm} length of season in days;
$\Sigma_{k}^{out}\Sigma_{k}^{in}$ sum of outflowing and inflowing links connected to the source;
β_{nijl} return flow from user sector l in region j to source i;
$YREPL_{npi}$ yearly replenishment in period n, climatic state p, to source i.

Annual source mass balance (for period n and source i)

$$\Sigma_{m}M_{nm}\Sigma_{p}P_{np}\Sigma_{k}^{in}T_{npkm} * [(1 - R_{K})\, C_{npim}^{up} - C_{npim}]$$

$$+ \Sigma_{j}\Sigma_{l}\Sigma_{p}P_{np}\,(S_{njl} - K_{npjl}) * \beta_{nijl} * (C_{nijl} - C_{npim})$$

$$+ \Sigma_{p}P_{np} * YREPL_{npi} * (CR_{npi} - C_{npim})$$

$$= V_{ni} * (C'_{ni} - C'_{n-1,i})/n_{n}$$

where

C_{npim}	salinity in period n, climatic state p, and season m at node i;
C^{up}_{npim}	salinity at the upstream node of a link which flows to node i;
R_k	fraction of salinity removed by desalination;
C_{nijl}	salinity of return flow from sector l in region j to source i;
CR_{npi}	salinity of natural replenishment;
C'_{ni}	salinity (mean over climatic states) at the end of period n in source i;
V_{ni}	mean mixing volume of source i in period n;
n_n	length of period n (years).

Node mass balance (for period n, climatic state p, region j, season m)

$$\Sigma^{in}_{k} T_{npkm} + [(1 - R_k)\, C^{up}_{npim} - C_{npim}] = 0$$

The iterative solution The node mass balance and source mass balance include products of the two unknown decision variables, e.g. $T_{npkm} * C_{npim}$. The linearization of these products is based on the following iterative scheme of successive linear approximations. The product of the two decision variables, X and Y appearing in a constraint of the form:

$$XY = C$$

is replaced by

$$X'Y + XY' = X'Y' + C$$

where X,Y are new values, and X',Y' are values from the previous iteration, or weighted means from several previous iterations. The new form is now linear in X and Y.

Demonstration problem

The application of the model is demonstrated in a simplified problem (Fig.2) which includes:

1. a source with storage and mixing capacity;
2. a consumer with a fixed water requirement and a given salinity damage function;

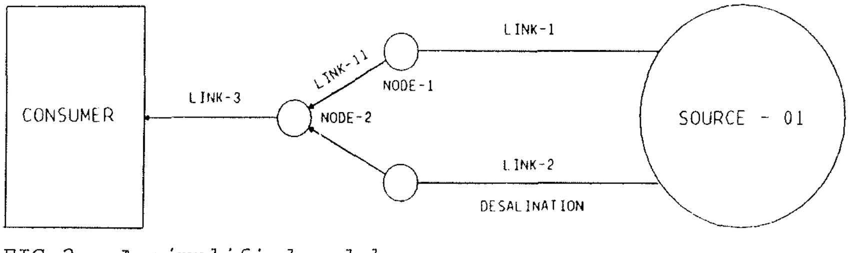

FIG.2 A simplified model.

3. a conveyance system;
4. a desalination plant;
5. return flows from the consumer to the source with a fixed salinity.

The problem data are:

number of periods: 9
length of each period: 1 year
initial water volume: $10^6 m^3$
annual replenishment: 0.2 $10^6 m^3$
initial salinity: 100 mg l^{-1}
replenishment salinity: 100 mg l^{-1}
return flow salinity: 1600 mg l^{-1}
annual water requirement: 0.6 $10^6 m^3$
threshold salinity: 500 mg l^{-1}
salinity damage: 1 \$/mg $l^{-1} m^{-3}$
desalination: salinity removal - 90%
water losses - 2%
costs: desalination - 700 \$ m^{-3}
parallel link - 0.7 \$ m^{-3}

The results are shown in four tables generated by TEKUMA. Table 1 shows the flows by links. Table 2 shows annual water balances of the source and at the nodes. Table 3 shows the resulting salinities, and Table 4 presents the shadow prices of the water balance constraints.

The results demonstrate some of the typical quality management processes and planning variables:

(a) The salinity of the source increases steadily as a result of saline return flows. It rises from the initial value of 100 mg l^{-1} to the final level of 1000 mg l^{-1}.

(b) The salinity of the water supply surpasses the threshold of 500 mg l^{-1} in the second and third period. Desalination costs exceed salinity damages.

(c) Desalination is economically justified after the third period. The cost of removal of 1 mg l^{-1} in the higher salinity levels (above 785 mg l^{-1}) is smaller than the damage of 1 mg l^{-1} for salinities above threshold.

(d) Water extraction from the source exceeds the sum of natural replenishment and return flows. The volume of water in storage decreases from 1.0 to 0.057 $10^6 m^3$ at the end of the last period.

(e) The shadow price of water in the source is initially similar to water supplied to the consumer, but it gradually declines to zero at the end. The shadow price here indicates the value of water quality. The high shadow price of the water supplied reflects the indirect damage incurred to the source by saline return flows.

The problem described here is not representative in its size and dimensions. TEKUMA generated a relatively small matrix with 109 rows, 236 columns and 711 elements. The solution required eight successive salinity approximations and 102 LP iterations.

The application of TEKUMA to a real problem, like the Israel's national water resources system, consists of 59 consumer groups in nine regions, 31 water sources, 77 links, three planning horizons, three different climatic zones and three seasons. Regional or national water resources management problems can usually be reduced to such a size. The resulting matrix for the national

TABLE 1 Flow by links (in $10^3 m^3$)

SOURCE / LINK	STATE	1985 YEAR	1986 YEAR	1987 YEAR	1988 YEAR	1989 YEAR	1990 YEAR	1991 YEAR	1992 YEAR	1993 YEAR
SOURCE-01										
****OUT(TOTAL)		600	600	600	606	606	607	607	607	607
LINK-1		600	600	600	305	282	272	268	267	267
LINK-2		0	0	0	301	325	335	339	340	340
NODE - 1										
****OUT**** LINK-11		600	600	600	305	282	272	268	267	267
**** IN**** LINK-1		600	600	600	305	282	272	268	267	267
NODE - 2										
****OUT**** LINK-3		600	600	600	600	600	600	600	600	600
**** IN(TOTAL)		600	600	600	606	606	607	607	607	607
LINK-2		0	0	0	301	325	335	339	340	340
LINK-11		600	600	600	305	282	272	268	267	267
TOTAL										
****OUT		1800	1800	1800	1511	1488	1478	1475	1474	1473
**** IN		1200	1200	1200	911	888	878	875	874	873

TABLE 2 Water balances (in $10^3 m^3$)

SOURCE	FACTOR	1985	1986	1987	1988	1989	1990	1991	1992	1993
SOURCE-01	REPLENISHMENT	200	200	200	200	200	200	200	200	200
	RETURN FLOW	300	300	300	300	300	300	300	300	300
	OUTFLOWS	600	600	600	606	606	607	607	607	607
	SPILLS	0	0	0	0	0	0	0	0	3
	RESERVES USED	100	100	100	106	106	107	107	107	110
	CUN.STO.CHANGE	100	200	300	406	513	619	726	833	943
	RESERVE ACCNT.	100	200	300	406	513	619	726	833	943
NODE - 1	INFLOWS	600	600	600	305	282	272	268	267	267
	OUTFLOWS	600	600	600	305	282	272	268	267	267
NODE - 2	INFLOWS	600	600	600	606	606	607	607	607	607
	OUTFLOWS	600	600	600	600	600	600	600	600	600
	SPILLS	0	0	0	6	6	7	7	7	7
TOTAL BALANCE	REPLENISHMENT	200	200	200	200	200	200	200	200	200
	RETURN FLOW	300	300	300	300	300	300	300	300	300
	INFLOWS	1200	1200	1200	911	888	878	875	874	873
	OUTFLOWS	1800	1800	1800	1511	1488	1478	1475	1474	1473
	THROUGH FLOWS	1200	1200	1200	905	882	872	868	867	867
	LOSSES	0	0	0	0	0	0	0	0	0
	SPILLS	0	0	0	6	6	7	7	7	10
	RESERVES USED	100	100	100	106	106	107	107	107	110
	CUN.STO.CHANGE	100	200	300	406	513	619	726	833	943
	SAFETY RESERVE	0	0	0	0	0	0	0	0	0
	RESERVE ACCNT.	100	200	300	406	513	619	726	833	943

TABLE 3 Salinities

SOURCE	STATE	1985 YEAR	1986 YEAR	1987 YEAR	1988 YEAR	1989 YEAR	1990 YEAR	1991 YEAR	1992 YEAR	1993 YEAR
SOURCE-01		287	594	785	897	957	985	996	1000	1000
NODE - 1		287	594	785	897	957	985	996	1000	1000
NODE - 2		287	594	785	500	500	500	500	500	500
CONSUMER		287	594	785	500	500	500	500	500	500

TABLE 4 Shadow price of water at sources (10^{-2} US$ m^{-3})

SOURCE	STATE	1985 YEAR	1986 YEAR	1987 YEAR	1988 YEAR	1989 YEAR	1990 YEAR	1991 YEAR	1992 YEAR	1993 YEAR
SOURCE-01		47978	25487	10948	4532	1825	611	146	17	0
NODE - 1		48051	25560	11021	4605	1899	685	221	91	74
NODE - 2		48051	25560	11021	41267	41386	41371	41333	41292	41251

system consists of 2000 rows, 3000 columns, and 25 000 elements, requiring about 3000 iterations for its solution. The CPU required by an IBM 370/168 machine, was about 1000 s.

The methodology presented in this paper seems therefore to be applicable to solving regional or national water resources management problems, including quality management.

REFERENCE

Schwarz, J. (1983) Use of mathematical programming in the management and development of Israel's water resources. In: *Ground Water in Water Resources Planning* (Proc. UNESCO-IAH-IAHS Koblenz Symp., August 1983), 917-929. IAHS Publ. no. 142.