

Limited multi-stage stochastic programming for managing water supply systems

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ABSTRACT

Decision-making processes often involve uncertainty. A common approach for modeling uncertain scenario-based decision-making progressions is through multi-stage stochastic programming. The size of optimization problems derived from multi-stage stochastic programs is frequently too large to be addressed by a direct solution technique. This is due to the size of the optimization problems, which grows exponentially as the number of scenarios and stages increases. To cope up with this computational difficulty, solution schemes turn to decomposition methods for defining smaller and easier to solve equivalent sub-problems, or through using scenario-reduction techniques. In our study a new methodology is proposed, titled Limited Multi-stage Stochastic Programming (LMSP), in which the number of decision variables at each stage remains constant and thus the total number of decision variables increases only linearly as the number of scenarios and stages grows. The LMSP employs a decision-clustering framework, which utilizes the optimal decisions obtained by solving a set of deterministic optimization problems to identify decision nodes, which have similar decisions. These nodes are clustered into a preselected number of clusters, where decisions are made for each cluster instead of for each individual decision node. The methodology is demonstrated on a multi-stage water supply system operation problem, which is optimized for flow and salinity decisions. LMSP performance is compared to that of classical multi-stage stochastic programming (MSP) method.

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1. Introduction

Decisions for water resources systems planning and management must often be taken in the face of the unknown, such as precipitation variability and water demands. Consequently, to address the uncertain parameters many stochastic programming (SP) methodologies (Birge and Louveaux, 1997) were applied to water resources systems models, including (a) Stochastic Dynamic Programming (SDP) for reservoirs management (Yeh, 1985; Faber and Stedinger, 2001) (b) implicit stochastic programming (ISP) for optimal management of multi-reservoir system with uncertain inflows (Hiew et al., 1989; Crawley and Dandy, 1993; Cai et al., 2001) (c) chance constrained programming (CCP) for water supply systems design under demand uncertainty (Lansley et al., 1989; Xu and Goulter, 1998; Babayan et al., 2005; Kapelan et al., 2005), for ground water remediation under uncertainty of aquifer

parameters (Morgan et al., 1993; Wagner, 1999), and for water allocation under inflow uncertainty (Sankarasubramanian et al., 2009) (d) multi-stage stochastic programming (MSP) and its variant- the two-stage stochastic programming (TSP) for a linear model of multi-reservoir system with uncertain inflows (Pereira and Pinto, 1985, 1991; Dupačová et al., 1991; Jacobs et al., 1995; Watkins et al., 2000; Seifi and Hipel, 2001; Kracman et al., 2006), and for water resources management (Li et al., 2009a,b; Guo et al., 2010)

In stochastic programming uncertain parameters are treated as random variables with a known probability density function (PDF). Hence, the consequences of the decisions taken at present are not known until the unknown data is realized. Still, the MSP can take into account some corrective decisions, which can be taken in the future when more data is revealed. To capture these corrective decisions, the MSP formulation permits recourse decisions in each time step based on realized data as further described in Section 2.

In most cases the MSP formulation results in large-scale optimization models, so that including many variables and constraints constitutes the key challenge for solving these models. This is particularly true if the mathematical model is nonlinear. Previous applications of TSP and MSP, as mentioned above (point-d), are of

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linear reservoir operation models. Because stochastic models result in large-scale optimization problems, most reservoir models were formulated as linear programs to ease for solving the optimization problem. But even with linear programs decomposition methods had to be applied to reduce the computational burden. Applications for stochastic nonlinear optimization models are rare, again due to the computational burden (Labadie, 2004).

Our study presents a methodology entitled Limited Multi-stage Stochastic Programming (LMSP), in which a smaller optimization problem approximates the MSP formulation. The LMSP solves a set of deterministic optimization problems to identify decision nodes, which have similar decisions. Once this classification is done, nodes with similar optimal decisions are assigned to a common cluster. The decision is then made for each cluster (group of decision nodes) rather than for each individual decision node, resulting in a smaller optimization problem which approximates the MSP formulation.

It is important to distinguish the approach of clustering on the basis of decisions (LMSP) from the reduction techniques (Dupačová et al., 2003) in which the stochastic process itself is clustered/aggregated to produce a smaller optimization model. These reduction techniques are applied *before* the model is introduced, independent of the optimization model that will be used. On the contrary, the LMSP uses a set of deterministic optimization problems to perform the decision clustering; moreover, the stochastic process is kept intact and only the decision is clustered. Details on the LMSP procedure and the distinctions between the two approaches are given in Section 3.

The rest of this paper is organized as follows. In the next Section, the traditional MSP approach is presented, to form a basis for the remainder of the paper. Section 3 then details the LMSP approach, and Section 4 presents the formulation for a water supply system (WSS) management model, its objective function and constraints. In Section 5, the MSP and LMSP are applied to the stochastic version of the WSS management model of Section 4.

2. Multi-stage stochastic programming (MSP)

A key aspect in formulating a stochastic optimization model is the sequence in which decisions alternate with the realization of uncertain data. Consider the general (nonlinear) optimization problem:

$$\begin{aligned} \min F(x, \omega) \\ \text{s.t. :} \\ g_i(x, \omega) \leq 0 \quad \forall i = 1 \dots m \end{aligned} \quad (1)$$

where x is the vector of decision variables and ω is a stochastic process (sequence of random variables representing a random time series). When we need to consider not just one decision and one observation but interactions between decisions and observations in a sequence of stages, then multi-stage stochastic programming (MSP) should be used (Dupačová, 1995). In the MSP over a time horizon $t = 1 \dots T_f$ the stochastic process $\omega = (\omega_1 \dots \omega_{T_f})$ and decision process $x = (x_1 \dots x_{T_f})$, are interlinked into a sequence of alternating decisions and observations: $x_1, \omega_1 \dots x_{T_f-1}, \omega_{T_f-1}, x_{T_f}, \omega_{T_f}$.

The MSP distinguishes between decisions that have to be made here-and-now, before future realizations of the random process are revealed, and the decisions that will be made at later stages, after some part of the information becomes known. The decisions at each stage are made while taking into account that will be opportunities for modification and corrections at later stages (recourse decisions). The decision process is *nonanticipative* or *implementable*, i.e. the decision vector x_t at time t depends only on previous information $\omega^{t-1} = (\omega_1 \dots \omega_{t-1})$. The mathematical formulation of the MSP is:

$$\begin{aligned} \min E \left[F \left(x_1, x_2(\omega_1), x_3(\omega_1, \omega_2), \dots, x_{T_f}(\omega^{T_f-1}), \omega \right) \right] \\ \text{s.t. :} \\ g_i \left(x_1, x_2(\omega_1), x_3(\omega_1, \omega_2), \dots, x_{T_f}(\omega^{T_f-1}), \omega \right) \leq 0 \quad \forall i = 1 \dots m \end{aligned} \quad (2)$$

where for example $x_3(\omega_1, \omega_2)$ indicates that a decision at stage three depends on the information revealed up to this stage. Recourse decisions can be based on past realizations, but they cannot be based on knowledge of the future before it happens. In the MSP terminology this means that the decision space X consists of all the functions $x_t(\cdot)$ which are *nonanticipative* (Flam, 1985) i.e., do not anticipate the future. The MSP solves for an optimal policy which contains the first-stage decisions (values) and the recourse decisions (functions of revealed information). Finding the functions $x_t(\cdot)$ which lead to optimal solution of the MSP is an intractable mathematical problem, since the possible futures constitute an infinite set, hence the need to discretize this set.

2.1. Scenario-based MSP

A common tool to present the stochastic process is to introduce scenarios (Dupačová et al., 2000), which are particular possibilities of how the process might be realized in the future. The stochastic process ω is discretized into a finite set of scenarios $\omega^s \in \Omega \quad \forall s = 1 \dots S_f$ with probability p^s where $\sum_{s=1}^{S_f} p^s = 1$, each scenario, ω^s , is defined as a possible realization of the stochastic process over the entire time horizon.

When the process is given by scenarios, the function $x_t(\cdot)$ is replaced by a corresponding finite number of vectors, equal to the different possibilities taken by the function $x_t(\cdot)$ as its input. For example, the function $x_2(\omega_1)$ is replaced by S_f vectors $x_2(\omega_1^s)$; $s = 1 \dots S_f$. Hence, the scenario-based formulation of (2) is:

$$\begin{aligned} \min E \left[F \left(x_1, x_2^s, \dots, x_{T_f}^s, \omega \right) \right] \\ \text{s.t. :} \\ g_i \left(x_1, x_2^s, \dots, x_{T_f}^s, \omega \right) \leq 0 \quad \forall i = 1 \dots m \quad \forall s = 1 \dots S_f \end{aligned} \quad (3)$$

where $x_2^s = x_2(\omega_1^s), \dots, x_{T_f}^s = x_{T_f}(\omega^s)$ is the nonanticipativity requirement. In the scenario-based formulation the non-anticipativity requirement means that every pair of scenarios s, s' which are indistinguishable up to stage t (share the same history) must fulfill $(x_2^s, \dots, x_t^s) = (x_2^{s'}, \dots, x_t^{s'})$.

Optimization problem (3) is an equivalent deterministic program of the uncertain optimization problem (1), in which the stochastic process is represented by the finite number of scenarios. The equivalent deterministic optimization problem is solved by a suitable optimization solver (based on the nature of the optimization problem i.e. linear, nonlinear, etc.)

A stochastic process ω which is represented by a finite number of scenarios $\omega^s \in \Omega \quad \forall s$, can be represented by a scenario tree (Fig. 1). A scenario tree consists of nodes and arcs; each node represents a possible realization of the stochastic process, where the root node represents the present time, and each node has a unique ancestor (origin node). The arcs represent the links between the nodes, and are associated with a conditional probability. Along the tree the uncertainty unfolds with the stages, where each path from the root to a leaf (end of the path) represents a scenario.

The general structure of the scenario tree can be captured as follows: (a) at each stage $t = 2 \dots T_f + 1$ there are $K_t - K_{t-1}$ nodes which are denoted by $k_t = K_{t-1} + 1 \dots K_t$ where the root of the tree is indexed by 1, i.e. $K_1 = 1$, (b) each specific value of $\omega^{t-1} = (\omega_1 \dots \omega_{t-1})$ corresponds to one particular node at stage $t = 2 \dots T_f + 1$, where ω_t

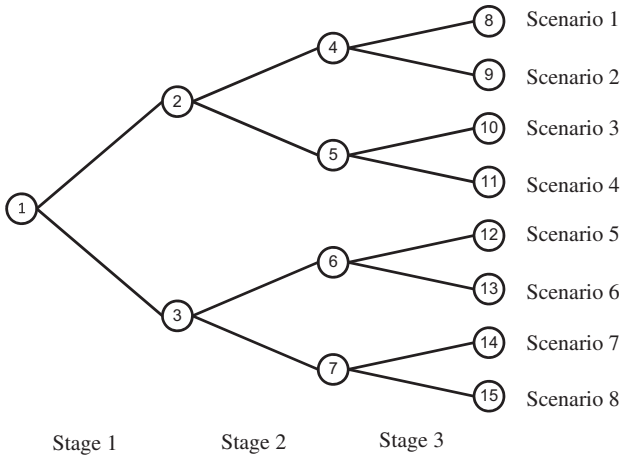


Fig. 1. Balanced scenario tree.

is the uncertain data at stage t , and ω^t is a vector of uncertain data up to year t .

The introduction of the scenario tree allows us to formulate the equivalent deterministic problem (3), where the nonanticipativity constraints are inherent explicitly in the shape of the scenario tree.

Another approach can be used to enforce the nonanticipativity without writing its constraints explicitly. In the implicit approach the nonanticipativity constraints are fulfilled automatically by introducing unique decision variables for each node of the tree. The implicit formulation is computationally cheaper than the explicit approach since it does not assign decision variables for each scenario at each stage.

When the objective and the constraints are separable functions, as in our case (Section 4), the equivalent deterministic problem with the implicit nonanticipativity approach leads to:

$$\min \left[\sum_{k_2=2}^{K_2} p_{k_2} f_1(x_1, \omega_1^{k_2}) + \sum_{k_3=K_2+1}^{K_3} p_{k_3} f_2 \left(\underbrace{x_1, x_2^{k_3}}_{x^{k_3}}, \underbrace{\omega_1^{k_3}, \omega_2^{k_3}}_{\omega^{k_3}} \right) + \dots + \sum_{k_{T_f+1}=K_{T_f}+1}^{K_{T_f+1}} p_{k_{T_f+1}} f_{T_f}(x^{k_{T_f+1}}, \omega^{k_{T_f+1}}) \right]$$

$$\text{s.t. :}$$

$$g_{j,t-1}(x^{k_t}, \omega^{k_t}) \leq 0 \quad \forall j = 1 \dots n, \quad \forall k_t = K_{t-1} + 1 \dots K_t, \quad \forall t = 2 \dots T_f + 1 \tag{4}$$

where ω_t^k is the data path up to node k_t at stage t ; x_t^k is a decision path leading to node k_t at stage t ; p_{k_t} is the data path probability up to node k_t at stage t and it is obtained by multiplication of the arcs' probabilities of the path.

An illustration of a scenario trees is given in Fig. 1. It shows a balanced scenario tree with three stages and two branches at each node; accordingly we have eight scenarios at the end of the three stages horizon.

In Fig. 1 scenarios 1–4 are indistinguishable up to stage 2 thus in the explicit nonanticipativity approach, formulation (3), we must impose nonanticipativity constraints: $x_2^{s=1} = x_2^{s=2} = x_2^{s=3} = x_2^{s=4}$. The implicit nonanticipativity approach, formulation (4), enforces the nonanticipativity without writing these constraints explicitly. Thus, instead of the decision variables $x_2^{s=1}, x_2^{s=2}, x_2^{s=3}, x_2^{s=4}$ we have $x_2^{\text{node}=2}$.

In the equivalent deterministic formulation of the MSP (4) we rely on a specified scenario tree which represents the stochastic process and the inter-correlation in it. In general a scenario tree can

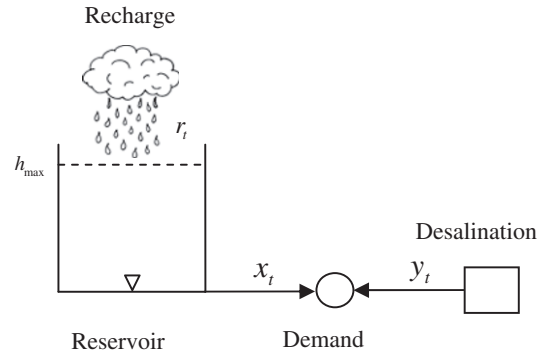


Fig. 2. Layout of the illustrative system.

accommodate any distribution and thus can account for any correlation within the random process. Various scenario tree generation methodologies such as: Monte-Carlo sampling, principal-component sampling, moment matching, and bootstrapping can be used to provide the scenario tree. A survey for such methods is given by Dupačová et al. (2000).

2.2. Illustrative example

Consider a hypothetical problem of managing a water supply system (WSS) comprised of a reservoir, in which the recharge is a stochastic process, and a desalination plant, both operated to supply one demand node (Fig. 2). The recharge is represented by the scenario tree (Fig. 3) and the objective is to minimize the desalination cost over the time horizon. The recharge values at nodes 2..15 takes the value 0 million cubic meters (MCM) if the node index is an odd number and 10 (MCM) if the node number is even. Each arc has an equal probability of 0.5. The mathematical model of the problem is:

$$\min \left(\sum_{t=1}^{T_f} C_t y_t \right)$$

$$\text{s.t. :}$$

$$x_t + y_t = D_t \quad \forall t = 1 \dots T_f \tag{5}$$

$$0 \leq \sum_{i=1}^t r_i - \sum_{i=1}^t x_i \leq V_{\max} \quad \forall t = 1 \dots T_f$$

$$x_t, y_t \geq 0 \quad \forall t = 1 \dots T_f$$

where x_t, y_t are extraction from reservoir and production of the desalination plant, respectively; C_t are desalination costs that rise with time $C_{t+1} \geq C_t \quad \forall t$; D_t is the demand; r_t is the stochastic recharge; and V_{\max} is the maximum water storage in the reservoir.

For example, when the reservoir is very large, i.e. $V_{\max} > 10$ (MCM), $T_f = 3$ years and $C_{t=1..3} = [1,2,3]^T$ (M\$/MCM), then substitution of the equality constraint in the objective function of Eq. (5) leads to the following mathematical model (one decision variable at each stage):

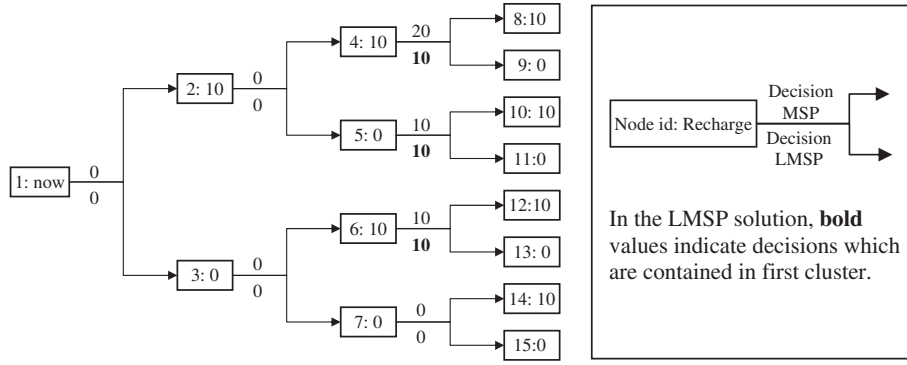


Fig. 3. Optimal solution (MCM) of the illustrative example.

$$\begin{aligned}
 & \min \left(- \sum_{t=1}^{T_f} C_t x_t \right) \\
 & \text{s.t.} \\
 & \sum_{i=1}^t r_i - \sum_{i=1}^t x_i \geq 0 \quad \forall t = 1 \dots T_f \\
 & x_t \geq 0 \quad \forall t = 1 \dots T_f
 \end{aligned} \tag{6}$$

Following the MSP formulation in (4) the optimization problem is:

$$\begin{aligned}
 & \min \left(-x_1 - 0.5 \sum_{k_2=2}^3 2x_2^{k_2} - 0.25 \sum_{k_3=4}^7 3x_3^{k_3} \right) \\
 & \text{s.t. :} \\
 & r_1^2 - x_1 \geq 0 \quad r_1^2 + r_2^4 + r_3^8 - x_1 - x_2^2 - x_3^4 \geq 0 \quad r_1^3 + r_2^7 + r_3^{14} - x_1 - x_2^3 - x_3^7 \geq 0 \\
 & r_1^3 - x_1 \geq 0 \quad r_1^2 + r_2^5 + r_3^9 - x_1 - x_2^2 - x_3^4 \geq 0 \quad r_1^3 + r_2^7 + r_3^{15} - x_1 - x_2^3 - x_3^7 \geq 0 \\
 & r_1^4 + r_2^4 - x_1 - x_2^2 \geq 0 \quad r_1^2 + r_2^5 + r_3^{10} - x_1 - x_2^2 - x_3^5 \geq 0 \quad x_1 \geq 0, x_2^{k_2=2,3} \geq 0, x_3^{k_3=4..7} \geq 0 \\
 & r_1^5 + r_2^5 - x_1 - x_2^2 \geq 0 \quad r_1^2 + r_2^5 + r_3^{11} - x_1 - x_2^2 - x_3^5 \geq 0 \\
 & r_1^6 + r_2^6 - x_1 - x_2^3 \geq 0 \quad r_1^3 + r_2^6 + r_3^{12} - x_1 - x_2^3 - x_3^6 \geq 0 \\
 & r_1^7 + r_2^7 - x_1 - x_2^3 \geq 0 \quad r_1^3 + r_2^6 + r_3^{13} - x_1 - x_2^3 - x_3^6 \geq 0
 \end{aligned} \tag{7}$$

where x_1 is the first-stage decision; $r_t^{\text{node}}, x_t^{\text{node}}$ corresponds to the recharge and the decision (reservoir withdrawal) for each of the scenario tree nodes 2...15 (Fig. 3).

The solution of this problem is given in Fig. 3, which has an objective value of 270 M\$.

3. Limited multi-stage stochastic programming

The size of the optimization problems derived from multi-stage stochastic programs is frequently too large to be tractable by a direct solution as the size of the optimization problem grows exponentially with the scenarios and stages number. The challenge of solving large-scale optimization problems with many variables and constraints is one of the main concerns of stochastic programming. Consequently, the common solution approach for these large-scale problems is based on decomposition of the original problem into an assembly of small and easier to solve sub-problems. Various decomposition methods have been suggested (e.g., Rockafellar and Wets, 1991; Mulvey and Ruszczyński, 1995). Decomposition in turn requires assumptions concerning the convexity of the objective function and the constraints. Decomposition of large-scale

optimization problem does not necessarily assure an efficient solution method (Mulvey and Ruszczyński, 1995).

We propose an approach different from previous ones, titled Limited Multi-stage Stochastic Program (LMSP). The LMSP is an attempt to solve without decomposition or scenario reduction techniques. In LMSP the number of decision variables in each stage remains constant and thus the total number of decision variables increases only linearly with the number of

scenarios and stages. The LMSP consists of four steps as outlined below.

Step 1: The clustering criterion is based on the scenarios' optimal decisions which are obtained by solving each scenario individually $v_t^s \forall t=1 \dots T_f, s = \arg \min F(x^s, \omega^s) \forall s = 1 \dots s_f$.

Step 2: The nodes' values \bar{v}^{k_t} corresponding to the scenarios' optimal decisions are calculated by:

$$\bar{v}^{k_t} = \frac{\left(\sum_{s \in k_t} p_s v_t^s \right)}{\left(\sum_{s \in k_t} p_s \right)} \tag{8}$$

for each $t = 2 \dots T_f$ and $k_t = K_{t-1} + 1 \dots K_t$ where $s \in k_t$ indicates the set of scenarios which pass through the node k_t . Eq. (8) transforms the optimal scenario decisions to match the structure of the scenario tree.

Step 3: Based on the values \bar{v}^{k_t} obtained at the nodes, for each stage $t > 1$ we perform a clustering based on these values. The K-means clustering method was used in this study, while other clustering methods could also be used. K-means is an iterative

clustering method in which N vectors are parted into M clusters where each vector belongs to the cluster with the nearest center, and the center of each cluster is defined as the mean of all its members (Seber, 1984).

Step 4: The LMSP model is solved, with the clustering scheme. Applying the clustering imposes more constraints in the equivalent deterministic formulation (4), for example $x_2^{k_2} = x_2^{k'_2}$ if node k_2 and node k'_2 are in the same cluster. These constraints can be eliminated by algebraic substitution of variables, thus reducing the number of variables in the problem. For instance, instead of the decision variables $x_2^{k_2}, x_2^{k'_2}$ we define $x_2^{\text{cluster}=1}$.

The LMSP answers the following questions: given the scenario tree of the uncertain future, what nodes are included in each cluster and what are the decisions associated with each cluster such that the objective is minimized. Ideally the two questions should be addressed simultaneously. However, a good approximation can be obtained by solving the problem in two phases; the first phase determines which nodes are included at each cluster, accomplished by solving each scenario individually and analyzing their separate results. The second phase determines the optimal solution for each cluster, using the optimization solver on the LMSP formulation.

Adding more constraints in the MSP will potentially raise the minimum value of the optimization problem. Hence, the minimum value obtained from the LMSP will always be larger or equal to the minimum value obtained by the MSP. However, this is not a significant drawback when one solves in a rolling/folding horizon, where the problem is solved repeatedly and only the first-stage decisions are implemented. Most important is the quality of the first-stage decision (as the others are still to be revisited in the following steps), and therefore the requirement is that the process of clustering must not change significantly the first-stage decisions, as compared to the original stochastic problem.

In fact, only effects on first-stage decisions need to be considered because these are the sole decisions to be made here-and-now with information that is currently available. Decisions at subsequent stages will be generated with the new information revealed by that time using a rolling horizon approach.

For the first-stage decisions of the MSP and the LMSP to be close they must take into account the future uncertainty and the possibility of recourse actions. The MSP considers the possibility for recourse action for each node in the scenario tree while the LMSP considers a smaller number of recourse actions, only for each cluster. These recourse actions will not be implemented in real-life and the only reason for their presence is to take into consideration the effect of the uncertain future on the first decision. The LMSP

reduces the number of the recourse action while preserving the effect of the uncertain future on the first decision.

The desired computational saving determines the number of clusters. The LMSP technique sets the same number of clusters for each stage, i.e., a constant clustering scheme, as this ensures linear increase in the number of decision variables with the number of scenarios and stages as opposed to exponential in the MSP formulation.

A special case is when there is only one cluster at each stage, so all decision nodes at each stage share the same decision. In this case the number of decision variables is minimal but this means solving a single-stage Here-and-Now problem where there is just one decision vector under all scenarios. At the other extreme, when the number of clusters is determined such that each cluster contains only a single node, the number of decision variables is maximal and the LMSP formulation will coincide with the MSP. Specifying the same number of clusters for each stage (for example two clusters at each stage) ensures linear increase in the number of decision variables with the number of stages. By this clustering scheme the model resulting from the LMSP approach is somewhere between the single-stage Here-and-Now approach and the classic MSP.

Noteworthy that the LMSP is not limited to constant clustering scheme, and the number of clusters can be changes at different stages. Consider the scenario tree in Fig. 1: one possible clustering scheme would be (1, 2, 4, 2) clusters corresponding to stages 1–4. In such clustering scheme decisions are made on the tree nodes up to stage 3 (up to stage 3 the formulation is the same as in MSP, since each cluster contains one node) and only in stage 4 we require clustering the 8 nodes into 2 clusters. Such a clustering scheme incorporates the detailed future up to stage 3 (say detailed representation of near future) while after stage 3 an aggregation is made on the decisions (less detailed representation of far future).

3.1. Illustrative example

The LMSP method is demonstrated on the hypothetical problem presented in Section 2.2 and its results are compared with the obtained in the MSP. Table 1 contains the optimal solution for each of the eight scenarios (Fig. 1). Table 2 contains the node values, according to Eq. (8).

If we choose to cluster the decisions into two clusters at each stage, after applying the K-means algorithm on nodes 4–7 to represent them in two sets we obtain: nodes {4, 5, 6} in the first cluster and node 7 in the second cluster. The decisions for the clusters are denoted as $x_3^{C=1}$ and $x_3^{C=2}$ for the first and second clusters, respectively. This results in the following optimization problem (9):

$$\begin{aligned} & \min \left(-x_1 - 0.5 \sum_{k_2=2}^3 2x_2^{k_2} - 0.25(9x_3^{C=1} + 3x_3^{C=2}) \right) \\ & \text{s.t.} \\ & r_1^2 - x_1 \geq 0 \quad r_1^2 + r_2^4 + r_3^8 - x_1 - x_2^2 - x_3^{C=1} \geq 0 \quad r_1^3 + r_2^7 + r_3^{14} - x_1 - x_2^3 - x_3^{C=2} \geq 0 \\ & r_1^3 - x_1 \geq 0 \quad r_1^2 + r_2^4 + r_3^9 - x_1 - x_2^2 - x_3^{C=1} \geq 0 \quad r_1^3 + r_2^7 + r_3^{15} - x_1 - x_2^3 - x_3^{C=2} \geq 0 \\ & r_1^2 + r_2^4 - x_1 - x_2^2 \geq 0 \quad r_1^2 + r_2^5 + r_3^{10} - x_1 - x_2^2 - x_3^{C=1} \geq 0 \quad x_1 \geq 0, x_2^{k_2=2,3} \geq 0, x_3^{C=1,2} \geq 0 \\ & r_1^2 + r_2^5 - x_1 - x_2^2 \geq 0 \quad r_1^2 + r_2^5 + r_3^{11} - x_1 - x_2^2 - x_3^{C=1} \geq 0 \\ & r_1^3 + r_2^6 - x_1 - x_2^3 \geq 0 \quad r_1^3 + r_2^6 + r_3^{12} - x_1 - x_2^3 - x_3^{C=1} \geq 0 \\ & r_1^3 + r_2^7 - x_1 - x_2^3 \geq 0 \quad r_1^3 + r_2^6 + r_3^{13} - x_1 - x_2^3 - x_3^{C=1} \geq 0 \end{aligned} \tag{9}$$

Table 1
Optimal solution for each scenario (MCM), illustrative example.

Scenario <i>s</i>	Recharge r_1^s, r_2^s, r_3^s	Scenarios' optimal decisions (Step 1)		
		v_1^s	v_2^s	v_3^s
1	10, 10, 10	0	0	30
2	10, 10, 0	0	0	20
3	10, 0, 10	0	0	20
4	10, 0, 0	0	0	10
5	0, 10, 10	0	0	20
6	0, 10, 0	0	0	10
7	0, 0, 10	0	0	10
8	0, 0, 0	0	0	0

The solution of this problem is given in Fig. 3. The objective value of the MSP (270 M\$) is somewhat lower than the LMSP (277 M\$) as expected because the LMSP contains additional constraints (according to the explanation in Section 3). At nodes 1, 2, 3, 6 and 7 both the MSP and the LMSP have the same decision. The LMSP decision at nodes 4, 5, 6 has the same value since the nodes are part of the same cluster.

3.2. LMSP vs. scenarios reduction

It is important to distinguish the approach of clustering on the basis of decisions (LMSP) from the scenarios reduction techniques in which the stochastic process itself, namely the scenario tree, is clustered/aggregated to produce more a compact tree and a smaller optimization model. These reduction techniques are applied *before* the model is introduced, independent of the optimization model that will be used. The size of the scenario tree is reduced by aggregating the tree nodes into separate sets to be later represented by a new node (Dupačová et al., 2003; Gülpinar et al., 2004; Latorre et al., 2007; Heitsch and Romisch, 2005; Sutiene et al., 2010). In contrast, the clustering in LMSP is intrinsic in the optimization scheme as LMSP solves a set of optimization problems to obtain the decision nodes clustering scheme. Consequently, the clustering depends on the optimization model, so different optimization objectives, constraints, or parameters will result in the LMSP formulating different clustering. We view this as an important and valuable property of LMSP over scenario reduction techniques.

We claim that different models should require different clustering schemes depending on the relationship between the stochastic process and the optimization model. This relationship should be discovered within the optimization process to make the clustering accordingly. Hence, the clustering should be within the optimization framework (as it done in the LMSP) and depend on the specific optimization model that is being solved.

Another notable difference is that the LMSP keeps the scenario tree intact without reducing its size. The clustering is made only on the decisions related to the tree. The scenario tree is already an approximation of the real stochastic process, thus reducing the size of the tree worsens its approximation the future.

Table 2
Aggregated nodes' values (MCM).

Node	Scenarios	Nodes' values (Step 2)	
		v_2^k	v_3^k
2	1, 2, 3, 4	0	
3	5, 6, 7, 8	0	
4	1, 2		25
5	3, 4		15
6	5, 6		15
7	7, 8		5

To demonstrate that different models require different clustering schemes, consider again the hypothetical problem in the Section 2.2. When the reservoir volume is set equal to the maximum annual recharge, i.e., $V_{\max} = 10$ (MCM), the reservoir cannot store water for more than one year, thus the optimal solution is to take the available water in each year from the reservoir and supply the rest of the demand from the desalination plant. Hence, scenarios that have the same recharge r_t at stage t will have the same decisions at stage t . On the other hand, when the reservoir is very large, i.e. $V_{\max} > 10$ (MCM), the optimal solution prefers to take more reservoir water at later stages since the desalination cost rises over time ($C_{t+1} \geq C_t \forall t$) and therefore the optimal solution stores the recharge in the early stages for use in later years. As a result, scenarios that have the same *cumulative* recharge $\sum_{i=1}^t r_i$ at stage t will have the same decisions at stage t (if two scenarios have the same cumulative recharge at stage t then it implies that they have the same water storage resulting in the same reservoir withdrawal, i.e. the same decision).

The conclusion to be drawn from this example is that different management models may require different clustering; the traditional reduction techniques would give the same clusters for both models (with and without storage capacity) because the reduction is made on the scenario tree, independent of the problem information and before the model is introduced. For example a reduction technique may result in averaging the 0, 10 (MCM) recharge (Fig. 3) into 5 (MCM) starting from stage 2, hence it reduces the total number of scenarios from 8 to 2.

A deterministic optimization management problem of a water supply system (WSS) is formulated in the following section. The recharge is then considered to be uncertain and the MSP and the LMSP methods are then applied.

4. Formulation of the management model of a large WSS

In this section, we present the deterministic formulation of a seasonal multi-year model for management of water quantities and salinity in a WSS. Long-term (years, decades) management of a large-scale WSS can be captured in a model of medium temporal and spatial aggregation that is used to manage simultaneously both the sources and the network (Fisher et al., 2002; Draper et al., 2003, 2004; Jenkins et al., 2004; Watkins et al., 2004; Zaide, 2006). The model used here was developed in Housh et al. (2012). It is comprised of sources (aquifers, reservoirs and desalination plants), a conveyance system (distribution network) and consumers (demand zones) who require certain quantities of water under specified salinity constraints. The objective is to operate the system with minimum multi-year total cost under technological, administrative and environmental constraints. The cost and the constraints of each year consist of seasonal desalination, pumping, delivery and an extraction levy from the aquifers. Therefore the decision variables are: water flow and water salinity distribution Q and C respectively, and the removal ratios in the desalination plants RR . Two sets of state variables describe the state of the system at the end of each season: water levels h_a in the natural resources (aquifers and reservoirs) and water salinities C_a in the natural resources.

The objective function and some of the constraints in the model are nonlinear, leading to a nonlinear optimization problem. The model does not include detailed hydraulics (the energy equations, Kirchhoff's second law); it is implicitly assumed that the short-term hydraulic operation is feasible for the seasonal quantities which are prescribed by our model. Still, the hydraulics is introduced in the objective function by the head loss equation. We present here only the basic derivation of the mathematical model. Full details can be found in Housh et al. (2012).

Let p, d, a, z, S, Y denote pipe, desalination plant, aquifer, demand zone, season and year, respectively.

4.1. Objective function

The objective is to operate the system with minimum total cost of (a) cost of desalination CD – a constant price per unit of desalinated water plus a variable cost which depends on the salinity removal ratio (b) extraction levy from the natural sources CE – which depends on the water level in the source (c) conveyance costs CC – which is related to the head loss in a link, given by the Hazen-Williams equation, and the topographical difference between its ends. A small WSS is shown in Fig. 4 and a larger one (Fig. 6) will be presented in the application section.

The objective of the multi-year model is to minimize the present value of the total cost of operation over the planning horizon T_f :

$$\text{cost} = \sum_Y \frac{\left(\sum_p CC_p^{S,Y} + \sum_d CD_d^{S,Y} + \sum_a CE_a^{S,Y} \right)}{(1+i)^Y} \quad (10)$$

where cost is the total operation cost (\$); i is the annual discount rate (-); $CC_p^{S,Y}$ is conveyance cost (\$/season); $CE_a^{S,Y}$ is extraction levy (\$/season); $CD_d^{S,Y}$ is desalination cost (\$/season).

4.2. Constraints

Natural sources – aquifers and reservoirs – are included in our model, thus hydrological water and salinity mass balances are imposed:

$$R_a^{S,Y} - Q_a^{S,Y} = SA_a (h_a^{S,Y} - h_a^{(S,Y)-1}) \quad (11)$$

$$(C_R)_a^{S,Y} R_a^{S,Y} - C_a^{(S,Y)-1} Q_a^{S,Y} = SA_a (C_a^{S,Y} h_a^{S,Y} - C_a^{(S,Y)-1} h_a^{(S,Y)-1}) \quad (12)$$

where $R_a^{S,Y}$ is recharge (m^3); SA_a is the storativity multiplied by area (m^2); $h_a^{S,Y}$, $C_a^{S,Y}$ are water level and salinity respectively (m), (mgcl/l); $h_a^{(S,Y)-1}$, $C_a^{(S,Y)-1}$ are water level and salinity in the previous season respectively (m), (mgcl/l); $(C_R)_a^{S,Y}$ is salinity of the recharge water (mgcl/l).

The salinity of the desalinated water is:

$$C_d^{S,Y} = C_{sea} \left(\frac{100 - RR_d^{S,Y}}{100} \right) \quad (13)$$

where $C_d^{S,Y}$ is desalinated water salinity (mgcl/l); C_{sea} is sea water salinity (27,000 mgcl/l); $RR_d^{S,Y}$ is removal ratio (%).

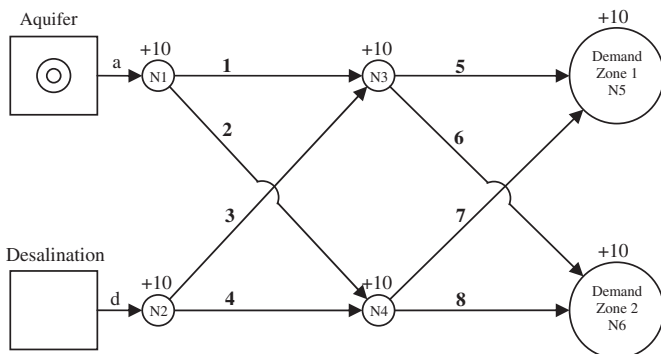


Fig. 4. Demonstration WSS.

The distribution system can be represented as a directed graph matrix $A \in \mathbf{R}^{N_2 \times M}$ where M is the number of edges (pipes) connecting the network nodes and N_2 is the number of intermediate and demand nodes.

For each season S in year Y , the following linear equation system insures water conservation at the network nodes:

$$A \cdot Q = b \quad (14)$$

where $Q = [Q_{source}, Q_{pipes}]^T$; $b = [0, Q_{demand}]^T$; Q_{source} is the vector of discharges leaving source nodes; Q_{pipes} is the vector of discharges in the links which are connected to intermediate nodes excluding the links which are connected to source nodes; Q_{demand} is the vector of outgoing discharges at demand nodes.

Salinity is also considered in the model so for each season S of year Y the following linear equation system insures salt mass conservation at network nodes:

$$\begin{aligned} A^0 D_Q C^0 &= 0, \quad A^0 \in \mathbf{R}^{N_2 \times (M+n_3)} \\ A^0 &= \begin{pmatrix} A & 0 \\ & -I \end{pmatrix} \\ C^0 &= [C_{source}, C_{pipes}, C_{demand}]^T \\ D_Q &\in \mathbf{R}^{(M+n_3) \times (M+n_3)} \quad \text{diagonal matrix} \\ D_Q &= \text{diag}([Q_{source}, Q_{pipes}, Q_{demand}]) \end{aligned} \quad (15)$$

where C_{source} is the salinity leaving source nodes; C_{pipes} is the salinity in the links which are connected to intermediate nodes, excluding the links which are connected to source nodes; C_{demand} is salinity supplied at demand nodes; n_3 number of demand nodes. Full mixing at nodes is assumed, so the salinities in all links leaving a node are equal. This dilution condition is given by the linear equation system:

$$B^0 \cdot C^0 = 0 \quad (16)$$

where each row of B^0 indicates equal salinity for two outgoing edges which share the same inflow node.

4.3. Operational bounds

Bounds are listed in Eq. (17): on water levels and salinities in the aquifer, reflecting both policy and physical/operational limits; on link discharges, representing maximum conveyance capacity and a fixed direction of flow; on water extraction from aquifers, representing hydrological and hydraulic considerations/fixed direction of flow; on desalinated water amounts, representing plants capacity, contract conditions and outward flow direction; on removal ratio, reflecting the plant's technology and design; on the demand salinity at demand nodes to ensure that the salinity of water supplied is within the required limits.

$$\begin{aligned} (h_{min})_a^{S,Y} < h_a^{S,Y} < (h_{max})_a^{S,Y} & ; \quad (C_{min})_a^{S,Y} < C_a^{S,Y} < (C_{max})_a^{S,Y} \\ 0 < Q_p^{S,Y} < (Q_{max})_p^{S,Y} & ; \quad 0 < Q_a^{S,Y} < (Q_{max})_a^{S,Y} \\ (Q_{min})_d^{S,Y} < Q_d^{S,Y} < (Q_{max})_d^{S,Y} & ; \quad (RR_{min})_d^{S,Y} \leq RR_d^{S,Y} \leq (RR_{max})_d^{S,Y} \\ (C_{min})_z^{S,Y} < C_z^{S,Y} < (C_{max})_z^{S,Y} & \end{aligned} \quad (17)$$

Water quantities are in (m^3 /season), salinities are in (mgcl/l) and elevations in (m); $()^{min}$ is minimum allowed value; $()^{max}$ is maximum allowed value.

In Housh (2011), sensitivity analysis was performed to check the model's performance and behavior under various conditions. The sensitivity analysis results show rational behavior of the model under changes of parameters and initial conditions.

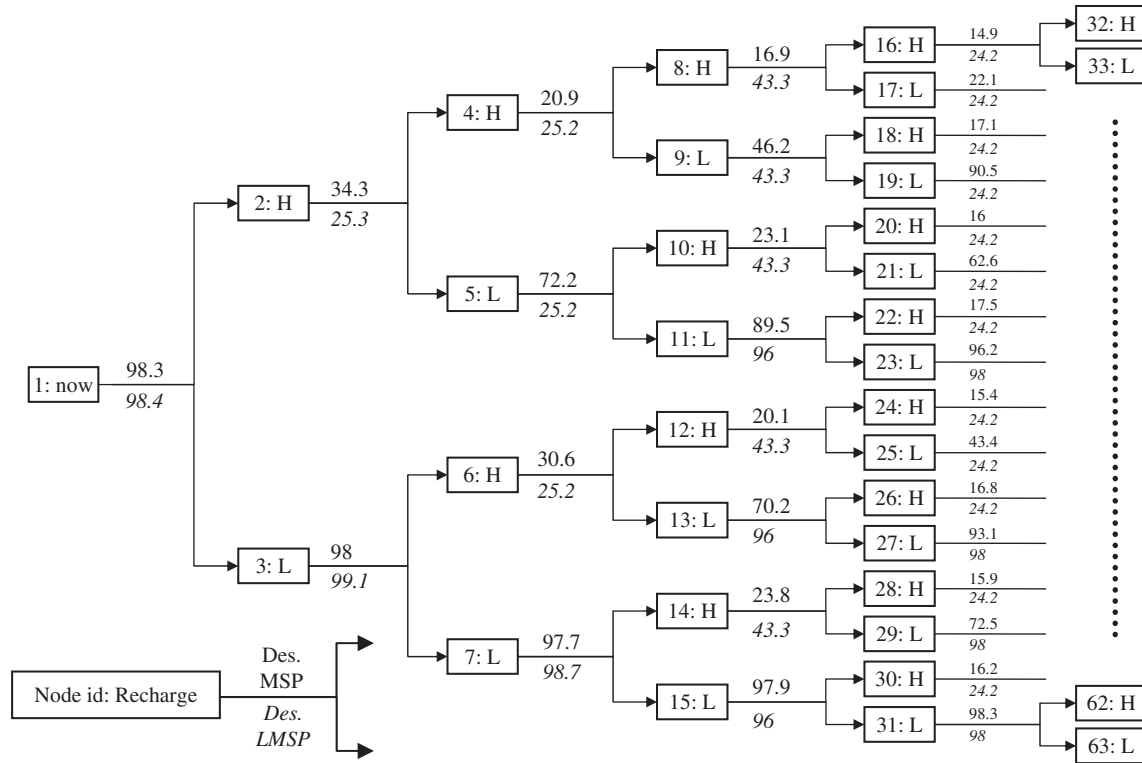


Fig. 5. Desalination amount, MSP vs. LMSP solution of the WSS in Fig. 4. Des., Desalination; H, High; L, Low according to Table 3.

4.4. Optimization problem

To reduce the model size we extract one dependent decision variable from each equality constraint. The dependent variables are then substituted into the objective function and the inequality constraints so a smaller model (fewer decision variables) is obtained. In the WSS model above, for fixed values of the flow variables Q and the removal ratio RR all the salinity variables C^0 are determined using Eqs. (11)–(13) and Eqs. (15) and (16). Parts of the flow variables Q are also dependent; the dependent variables could be extracted using the linear equation system (14):

$$Q_{dep} = A_1^{-1}(b - A_2 Q_{indep}) \tag{18}$$

where A_1 is a matrix of N_2 independent columns of A ; A_2 is a matrix of $M - N_2$ dependent columns of A ; $(\)_{dep}$ is the vector of dependent flows; $(\)_{indep}$ is the vector of independent flows. Hence, the independent decision variables in the WSS optimization problem are $x = [Q_{indep}, RR]$ and the model can be formulated as the following nonlinear optimization problem:

$$\begin{aligned} \min F &= \sum_{t=1}^{T_f} f_t(x_1, \dots, x_t, r_1, \dots, r_t) \\ \text{s.t. :} & \\ g_{j,t}(x_1, \dots, x_t, r_1, \dots, r_t) &\leq 0 \quad \forall j = 1 \dots n, \quad \forall t = 1 \dots T_f \end{aligned} \tag{19}$$

where t is the stage index $t \in [1, T_f]$; x_t is vector of decision variables; r_t is the recharge; $f, g, \forall j$ are linear/nonlinear functions. The stage can be defined as season or year, depending on the available recharge data.

5. Applications

In the WSS model there is uncertainty in the recharge r_t since it depends on climate variability, seasonal effects and climate change.

The following examples will focus on the uncertainty in the recharge, though it is recognized that there are often significant uncertainties in other variables such as demands and desalination costs.

Since the objective and the constraints of the WSS model are separable functions, the MSP formulation of the WSS optimization problem (19) with the recharge described as a stochastic process (depicted by the scenario tree) is given by formulation (4). The WSS formulation described above is solved using the MSP and LMSP formulations.

Results and comparisons are presented in the next sections. Due to the general structure of the objective function and constraints, a general nonlinear optimization solver is warranted. The MSP and the LMSP of the WSS model were programmed in MATLAB employing the interior-point solver of the FMINCON nonlinear optimization suite.

5.1. Application 1

A small WSS model (Fig. 4) has been solved in this example; minimum cost of operating the system which is fed from a one cell aquifer and a desalination plant to supply two customers over three years. The year has two seasons: the first has 265 days (“winter” with low demands and stochastic recharge given as scenario tree) and the second 100 days (“summer” with high demands and zero recharge).

Each stage (year) in the problem comprise of 10 decision variables. The yearly aquifers recharges $R = (r_1, \dots, r_{T_f})$ (MCM) are considered stochastic, given by the balanced scenario tree with two branches at each stage (Fig. 5). The conditional discrete probability in each year is given by the probability mass function (PMF) in Table 3, where r_t is the recharge in year t , and R^t is vector of recharges up to year t .

The scenario tree has 5 stages and 32 scenarios, where each of the nodes 2...63 takes the values of the low (L) recharge if the node

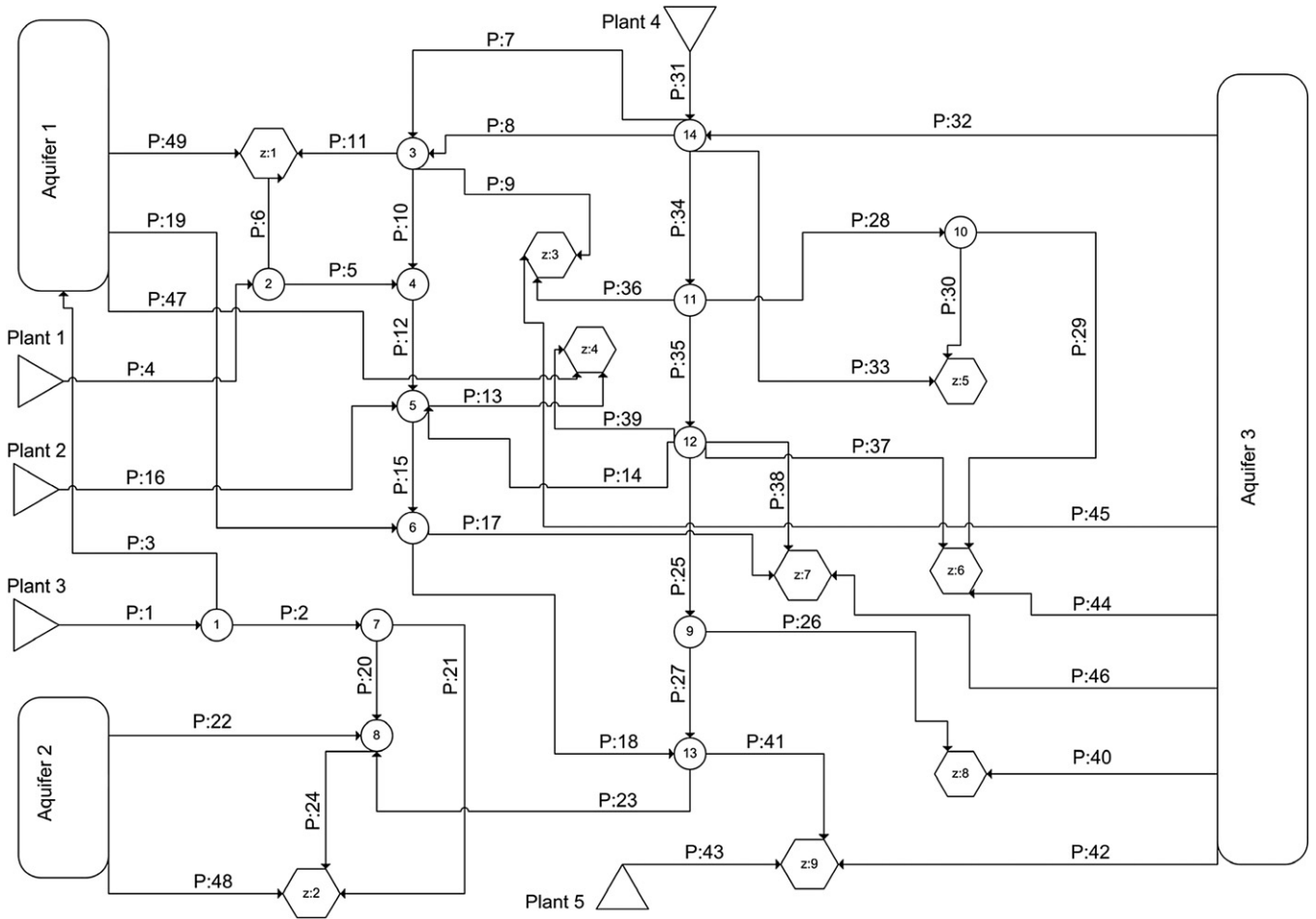


Fig. 6. Network layout of the large system.

index is an odd number, and the high (H) recharge if the node number is even (Fig. 5).

Each decision node 2...63 in Fig. 5 has its own decision variables; the independent decision variables for each of the node are the pipes flows $Q_{3,4,6,8}^1, Q_{3,4,6,8}^2$ (MCM) for season 1 and 2, respectively and the removal ratio RR^1, RR^2 (%) for season 1 and 2, respectively.

In MSP the number of decision nodes is 31, and the number of decision variables is $31 \times 10 = 310$ where 10 is the length of a single year decision vector. The minimum expected value of the MSP solution is 565 M\$. Applying the LMSP with two clusters at each stage reduces the problem size to 9 decisions nodes ($1 + 2 \times 4 = 9$), hence, the optimization problem has 90 decision variables. The minimum expected value of the LMSP is 570 M\$ (as expected somewhat higher than the MSP solution).

The clustering scheme to the 31 decision nodes is given in (Table 4) which is obtained after applying step 1 and 2 as described in Section 3.

Examining the decision clusters at year 4, nodes 8–15 show that the nodes which have history with L recharge at least twice are in

the same cluster, except node 14 which have H recharge only one time. This node has been added by the LMSP procedure to the first cluster in the first run, since the H recharge is realized in the near past of node 14. In year 5, nodes 16–31 the same rule is valid, where node 30 has H recharge only one time. This indicates that not only the cumulative amount matters, but also the lag from the decision node are important.

To compare the LMSP and the MSP we present the yearly desalination amount for each decision node. Note that each decision node has its own decision vector with 10 decision variables; however, the yearly desalination amount was chosen to present the decision characteristics. Fig. 5 compares the optimal desalination amount obtained from the MSP and the LMSP solution. For instance, in node 3 the MSP solution implies desalination amount of 98 (MCM) where the LMSP solution implies 99.1 (MCM).

The LMSP was applied with two clusters at each stage, hence at each stage the LMSP assign only two different values to the decision nodes e.g. at stage 5, the LMSP solution is 24.2 (MCM) of desalinated water for the decision nodes in the first cluster and 98 (MCM) of desalinated water for the decision nodes at the second stage.

As shown in Fig. 5, the MSP and LMSP provided very close first year decision for implementation which considers the future uncertainty and the possibility to recourse actions in the future. MSP considers the possibility for recourse action for each node in the scenario tree while the LMSP considers a smaller number of recourse actions (clusters). These recourse actions project the uncertain future effect on the first decision. In real-life application

Table 3
PMF of the recharge, example 1.

	Low recharge (L)	High recharge (R)
r_t (MCM)	0	100
$\text{Prob}(r_t R^t)$	0.5	0.5

Table 4
Clustering scheme, example 1.

Year	Cluster 1	Cluster 2
1	NA	NA
2	2	3
3	{4, 5, 6}	7
4	{8, 9, 10, 12, 14}	{11, 13, 15}
5	{16, 17, 18, 19, 20, 21, 22, 24, 25, 26, 28, 30}	{23, 27, 29, 31}

the MSP recourse action will not be implemented since it is solved in Rolling/Folding Horizon. The LMSP is considered a good approximation of the MSP when the first stage decisions are close to one another.

This is due to the fact that decision makers solve the model repeatedly and implement only the first stage decisions. As shown in Fig. 5, the desalination amount in the first stage obtained in the LMSP is very close to the obtained from the MSP; in fact all the decision variables are very close to each other, however, we chose not to present them to save space.

5.2. Application 2 (Large WSS)

The small system shown in Fig. 4 was used to test and verify the method’s performance, which was then applied to a larger and more realistic WSS. A water system with 9 demand zones, 3 aquifers, 5 desalination plants and 49 pipes (Fig. 6) has been solved in this example; the structure of this system mimics the central part of the Israeli National Water System. The year is divided into two seasons, which can be called "winter" (265 days, lower demands, stochastic recharge given as scenario tree) and "summer" (100 days, high demands, zero recharge).

The annual recharges to the aquifers $R_{a=1..3}$ (MCM) are considered stochastic, given by a balanced scenario tree with two branches at each stage. The conditional probability in each year is given in Table 5, where $r_{a,t}$ is the recharge in aquifer ‘a’ at year ‘t’, and R_a^t is vector of recharges up to year t.

The scenario tree in this case has 32 scenarios, where each of the nodes 2...63 takes the values of the low (L) recharge if the node index is an odd number, and the high (H) recharge if the node number is even.

Section 3.2 shows that the clustering scheme depends on the model formulation, thus we had different clustering scheme with and without storage capacity. The conclusion drawn from the hypothetical example is that different models may require different clustering. This example demonstrates how the clustering process is affected by different parameters of the model; particularly $(\overline{CE}^{\max})_a$ which determines the aquifers water cost (complete data are provided in Housh et al., 2012).

The large-scale network is solved twice, Run1: $(\overline{CE}^{\max})_a = 0$, Run2: $(\overline{CE}^{\max})_a = 0.5$ (M\$/MCM), respectively. In the MSP the number of the decision nodes is 31, thus the number of decision variables is $31 \times 62 = 1922$ where 62 is the length for each year’s decision vector. The minimum expected value of the MSP solution is Run1: 173.8 (M\$) and Run2: 230 (M\$) for the two problems Runs, respectively. Applying the LMSP with two clusters at each stage reduces the problem size to 9 decision nodes ($1 + 2 \times 4 = 9$), hence, the optimization problem has 558 decision variables. The minimum

Table 5
PMF of the recharge, example 2.

	Low recharge (L)	High recharge (H)
$r_{1,t}, r_{2,t}, r_{3,t}$ (MCM)	80, 80, 80	200, 200, 200
$\text{Prob}(r_{a=1..3,t} R_{a=1..3}^t)$	0.5	0.5

expected value of the LMSP is Run1: 175.8 (M\$) and Run2: 232.9 (M\$), not much higher than the MSP solutions.

Recalling that Run1 is set with no cost for aquifer water and Run2 is set with an aquifer water cost that depends on the level (lower cost at higher water levels) the optimal solutions of the two runs will have different characteristics. In Run1, since the aquifer water is free, once there is recharge to the aquifer the solution is to use it immediately, while in Run2 the solution will be to store the water by taking less from the aquifer to increase the aquifer water level and decrease the cost of its water. Water is stored in Run2 until aquifer water price competes with the price of desalinated water. Hence, aquifer withdrawals (decisions) in the two runs have different dependency on the recharge history.

The clustering in the LMSP procedure is made based on the decisions. Hence, decision nodes that imply using large amount of aquifer water would be in the same cluster and vice versa. However, since decisions in Run1 and Run2 have different characteristics and different dependency on recharge history the clustering scheme resulted from the LMSP is expected to be different in the two runs.

The clustering schemes for the 31 decision nodes in Run1 and Run2 are given in Fig. 7, which shows the history of the node in each year and the corresponding clustering suggested by the LMSP. Each node belonging to cluster 1 is in a "rich recharge cluster" while cluster 2 is a "low recharge cluster". For instance, node 8, which has H recharge along its history, is in the first cluster in both runs, whereas node 15, which has L recharge along its history, is in the second cluster in both runs.

Hence, decision node 8 is "more" aquifer-based operation in both runs (belongs to rich recharge cluster in both runs) and decision node 15 is "more" desalination-based operations in both runs.

Examining the clustering results at year 3 shows that nodes, which have a history with H recharge at least once, are placed in cluster 1 (high recharge history) in Run1. In contrast, Run2 clusters the nodes that have history with L recharge at least once in cluster 2 (low recharge history). Hence, nodes 5 and 6 are clustered differently, caused by different run parameters, indicating that nodes 5 and 6 are aquifer based in Run1 while in Run2 they are desalination based. This is because in Run2 the node history must contain more than one H recharge before it is considered in cluster 1 (rich recharge history).

The results for year 4 show that in both Runs nodes, which have history with at least one H recharge, are in cluster 1 except node 14, which has only one H recharge. This node has been added to the first cluster in the first Run, since the H recharge is realized in the near past of node 14. In year 5, nodes that have a history with H recharge at least three times are in cluster 1 in both Runs. Node 28, which has two H recharges in the near past, is also in cluster 1 in both Runs. This indicates that not only the cumulative amount matters, but also the lag from the decision node is important.

Nodes that have less than three times of H recharge history, but have H recharge at the later year are considered in cluster 1 in the Run1. In contrast, H recharge at the later year was not enough to make these nodes first cluster members in Run2.

The success of the LMSP approximation is determined by the difference in the first stage decisions between the LMSP and the MSP solution. Table 6 compares the total annual desalination amount in the first year, which is obtained from the MSP and the LMSP solution for both Runs.

As shown in Table 6, the MSP and LMSP provided very close first-year decisions for implementation; hence, solving the smaller optimization problem in LMSP (558 decision variables instead of 1922) has an advantage, especially when we are solving in rolling horizon mode. If we extend the time horizon in Application 2 from 5 years to 21 years (or if we increase the branches of the scenario

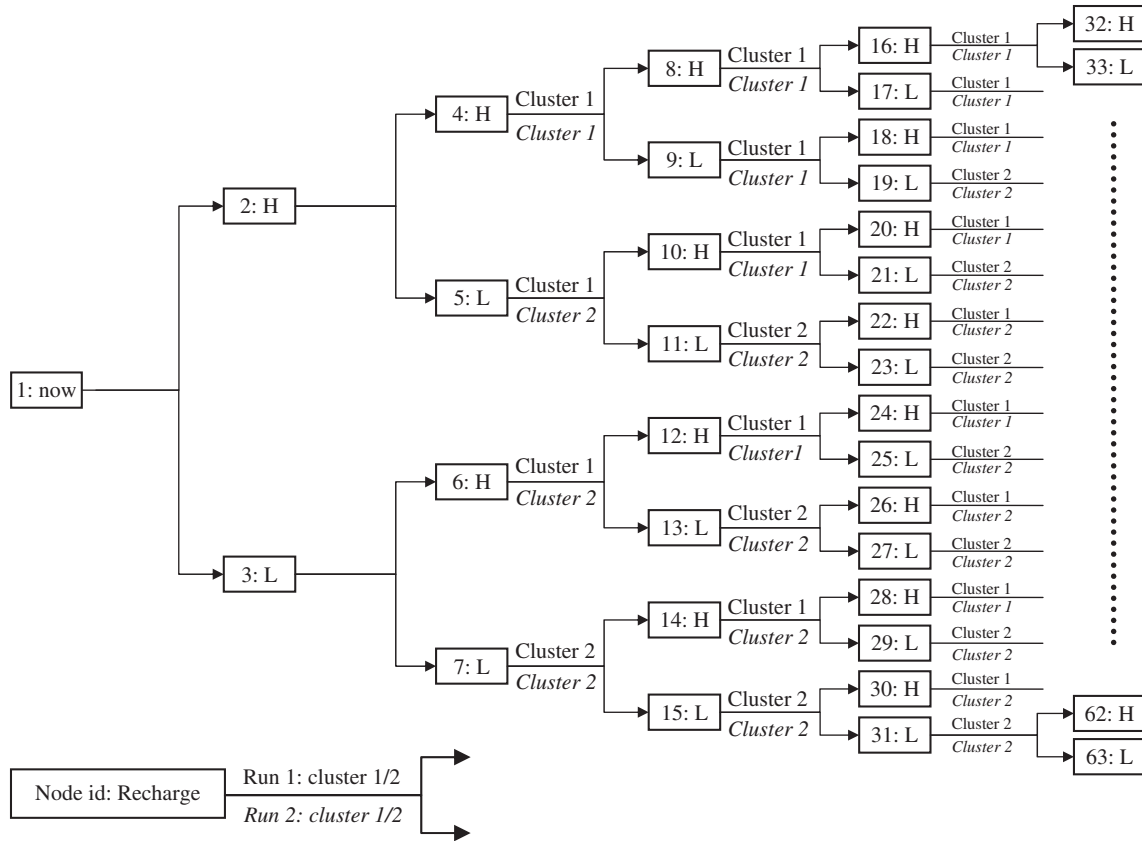


Fig. 7. Clustering scheme of the LMSP solution, Run1 vs. Run2. H, High; L, Low according to Table 5.

Table 6
Comparison between the MSP and the LMSP with different extraction levy.

Run	Approach	Desalination (MCM)				
		Plant 1	Plant 2	Plant 3	Plant 4	Plant 5
1	MSP	25.0	46.6	12.1	288.6	130.5
	LMSP	25.1	47.5	35.7	290.7	133.8
2	MSP	25.0	51.2	35.9	290.7	133.9
	LMSP	29.9	55.1	36.4	293.3	134.0

tree to 38 instead of 2), then the number of the decision variables in the MSP would be around 1.3 million while in the LMSP with two clusters the number is reduced to 2542 and to 3782 variables with three clusters. There does not seem to be a readily available optimization solver, which is capable of solving general nonlinear optimization problems with 1.3 million variables. On the other hand, we already reported solving optimization problem with 1922 variables.

6. Conclusions

A computationally cheaper approximation of the MSP has been developed. The LMSP approximates the MSP formulation by restricting the number of recourse decisions. This restriction is performed by means of a clustering procedure of the decision nodes of the scenario tree. The LMSP keeps the scenario tree intact without reducing its size; the clustering is made only on the decisions related to the tree. The process of clustering the decision variables did not change the first stage decision significantly, as compared to the original MSP, this is crucial when solving in folding/rolling mode.

In the applications we discussed the quality, the nature of this approximation, and the size of the optimization problem obtained as indicator for the computational time.

The results of using LMSP show that it provides a good approximation of the MSP solution, while solving much smaller optimization problem; 558 instead of 1922 decision variables in the large example (Application 2), the saving in the number of decision variables is as high as 70%. If we extend significantly the time horizon (or the number of scenarios) the MSP approach leads to a computationally intractable optimization problem as opposed to the LMSP, which leads to an optimization problem solvable by existing optimization solvers.

For the WSS model treated herein, the results show that not only the cumulative amount of the recharge effects the decisions but also the lag (distance) from the decision node is important. The large-scale example strengthens the claim that different models may require different clustering, depending on the relationship between the stochastic process and the optimization model, thus different model parameters require different clustering as obtained by the LMSP solution.

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Nomenclature

General

- t : decision stage
 T_f : total number of stages
 F : general nonlinear objective function
 f_t : general nonlinear objective function at stage t
 g_t : general nonlinear function (constraint)
 ω/R : stochastic process/recharge process of the entire horizon
 x : vector of decision variable for the entire horizon
 ω_t/R_t : stochastic process/recharge process at stage t
 x_t : vector of decision variables at stage t
 ω_t^j/R_t^j : stochastic process up to stage t
 x_t^j : vector of decision variables up to stage t
 s : scenario
 S_f : total number of scenarios
 p^s : probability of scenario

WSS model

- p, a, d : pipe, aquifer and desalination plant, respectively.
 z, S, Y : demand zone, season and year, respectively
 $CC_p^{S,Y}$: conveyance cost (\$/season)
 $(\overline{CE}_a^{max})^{S,Y}$: maximum extraction levy (\$/m³)
 $CE_a^{S,Y}$: extraction levy (\$/season)
 $CD_d^{S,Y}$: desalination cost (\$/season)
 $RR_d^{S,Y}$: removal ratio (%)
 Q_{source}/C_{source} : vector of discharges/salinity leaving source nodes
 Q_{pipes}/C_{pipes} : vector of discharges/salinity in the links
 Q_{demand}/C_{demand} : vector of discharges/salinity at demand nodes.
 $R_a^{S,Y}$: recharge (MCM)
 SA_a : storativity multiplied by area (m²)
 $h_a^{S,Y}/C_a^{S,Y}$: water level/salinity in aquifers (m)/(mgcl/l)
 $h_a^{(S,Y)-1}/C_a^{(S,Y)-1}$: water level/salinity in aquifers (m)/(mgcl/l)
 $(C_R)_a^{S,Y}$: salinity of the recharge water (mgcl/l)
 $C_d^{S,Y}$: desalinated water salinity (mgcl/l)
 C_{sea} : sea water salinity (27,000 mgcl/l)
 $(\#max)_{p,a,d}^{S,Y}$: maximum allowed value
 $(\#min)_{p,a,d}^{S,Y}$: minimum allowed value
 $(\#)_{dep}$: vector of dependent decision variables
 $(\#)_{indep}$: vector of independent decision variables