Optimal multi-year management of a regional water supply system under uncertainty: the Affine Adjustable Robust Counterpart (AARC) approach

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Abstract

In this study a regional Water Supply System (WSS), fed from natural sources which depend on uncertain recharge, and from desalination plants with fixed capacity, is to be operated over years. The water is transported through a network to meet consumers' demands. The requirement is to decide dynamically on the optimal operating policy, based on the revealed uncertainty up to the decision point, for minimizing the total operation cost of the system while fulfilling operational constraints at multiple time decision points. The Robust Counterpart (RC) methodology [Ben-Tal et al., 2009] is adopted, which uses a min-max approach assuming that the uncertain parameters reside within a user-defined uncertainty set. The dynamic version of RC is called Adjustable Robust Counterpart (ARC). One of its special tractable versions is the Affine Adjustable Robust Counterpart (AARC) in which the dependence of future decision variables on revealed uncertain data is restricted to be linear. The AARC solution provides a non-probabilistic analysis for multiyear management of WSS under uncertain conditions.

1. Introduction

Decision making under uncertainty is a key challenge in water resources management. Deterministic models can yield solutions which may become heavily infeasible as a result of data perturbations [Watkins and McKinney, 1997].

One of the most adapted approaches for optimization under uncertainty is stochastic programming, in which the uncertain data are assumed to have a known probability density function (PDF). A variety of stochastic methods have been applied to water resources

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management, including stochastic dynamic programming [Yeh, 1985; Faber and Stedinger, 2001], implicit stochastic optimization [Lund and Ferreira, 1996; Labadie, 2004], scenarios-based optimization [Pallottino et al., 2005;Kracman et al., 2006], and chance constraint methods [Lansey et al., 1989; Sankarasubramanian et al., 2009].

However, despite its intuitive formulation, stochastic programming faces three significant drawbacks. Firstly, stochastic programs, requires a perfectly known PDF in each of the uncertain data while such information is rarely available in practice. Secondly, given PDFs the mathematical stochastic formulation, especially multistage formulation, is extremely difficult to solve since it requires optimization over a space of functions. Thus, in all the applications above the scenario based stochastic programming was applied. In scenario based programs a finite set of scenarios is considered as an approximation of the PDFs this process exempts the optimization over a space of functions. However, working with scenario would increase the size of the optimization problem significantly. Furthermore, building a representative scenario sets out of the PDFs is not a trivial task.

Thirdly, in the classical stochastic programming, the models are aimed at optimize the expected value of the objective function and do not allow to evaluate the trade-offs between the risks of infeasibility and the losses in optimality.

Watkins and McKinney [1997] show that the scenario-based Robust Optimization (RO) methodology proposed in Mulvey et al. [1995] as applied to urban water transfer planning and ground-water quality management allows for finding solutions hedging against inherent parameter uncertainty. Nevertheless, this approach shares some of the disadvantages with stochastic programming (For distinction in the scenario based RO and the RO considered in this stud the reader is referred to Housh et al. [2011]).

To cope with some of the challenges faced by stochastic programming, over a decade ago a new paradigm for optimization under uncertainty, the Robust Counterpart (RC) approach, was proposed [Ben-Tal and Nemirovski 1998, 1999, 2000a] and used in a number of applications which includes portfolio models [Ben-Tal et al., 2000b; Lobo and Boyd, 2000], inventory theory [Bertsimas and Thiele, 2006; Bienstock and Ozbay, 2008], process scheduling [Li and Ierapetritou, 2008], and network models [Mudchanatongsuk et al., 2005]. This methodology seems to be attractive for water management problems as well [Housh et al., 2011].

The RC is distribution free min-max oriented approach which solves the uncertain optimization problem by assuming that the uncertain parameters can only reside within a known uncertainty set.

Housh et al. [2011] applied “static” version of the RC approach for water recourses system management. In the “static” RC the decisions for all future stages are determined at the beginning of the time horizon, “here and now”, before the uncertain data are revealed. To capture the dynamic nature of the problem Housh et al. [2011] adapted the ‘‘folding RC’’ approach (FRC) [Ben-Tal et al., 2000b].

In this study we consider the “dynamic” version of the RC, the Affine Adjustable RC (AARC), for WSS management. This work could be seen as a natural continuation of the study in Housh et al. [2011]. We will rely on the theoretical and methodological description of the RC which previously presented there.

The rest of the paper is organized as follows. Section 2 presents the motivation for the AARC method along with the basics of the AARC methodology. Section 3 presents mathematical formulation of the WSS model. Section 4 describes the application of the AARC to the WSS model. Section 5 presents applications of the AARC.

2. The Adjustable Robust Counterpart (ARC)

2.1. Motivation

The RC approach was originally designed for problems where all decision variables are “here and now” and should be determined at the beginning of the horizon, before the uncertain data is revealed [Ben-Tal and Nemirovski 1998].

Many real-world problems can be more accurately modeled using multistage decision environment. In such decision environment decisions which should be implemented after the first stage, $t > 1$, may depend on the reveled uncertain data up to stage t.

The only way to apply the original RC approach (which is “static” in nature) for a multistage decision making problem, is to use it within a folding horizon framework.

In the folding horizon framework instead of solving the problem once at the beginning of the entire planning horizon and adapt the resulted decision for the entire horizon, the problem is sequentially re-solved over the stages. At time “now” the RC solution for all stages is computed, and the first stage decision is implemented. At the beginning of the next stage, we solve a new problem with reveled uncertain data from the previous stages and reduced time horizon. This is repeated over all of the stages. Following [Ben-Tal et al., 2000b] the RC within the folding framework, namely, “folding RC” approach (FRC) was applied in Housh et al. [2011].

In that study the FRC approach with other multistage decision making approaches and shows that the results are of the FRC are very competitive with those obtained by stochastic and deterministic methods.

Despite the promising results obtained the preference of the FRC over inherently dynamic methods was not apparent. The FRC solution was a quite conservative, as it inherently “static” and does not explicitly takes into the computation the fact that decision at the next stages will be determined with additional information about the uncertainty.

In this study we implement the dynamic version of the RC, namely, the Affine Adjustable RC (AARC). Ben-Tal et al. [2004] presented the AARC to cope with the need for non-probabilistic multistage decision making under uncertainty. The AARC extends the RC approach for a problems in which part of the variables must be determined before the uncertain data is revealed ("nonadjustable variables" as called in the robust optimization terminology or "here and now decisions" as called in the stochastic programming terminology), while the other part is for variables that can be chosen after the some of the data is reveled ("adjustable variables" or "wait and see decisions").

2.2. Methodology

To extend the robust optimization methodology to dynamic settings, Ben-Tal et al. [2004] proposed the AARC, in which –just like- the RC, the uncertainties are assumed to vary within a predefined uncertainty set. While the RC corresponds to the case when all the variables represent decisions that must be made before the actual realization of the uncertain data is reveled. The AARC recognizes the dynamic decision environment by allowing some decisions to be adjustable ("recourse variables" or "wait and see" variables) which can be made after the realization of the uncertainties and be adjusted to its actual realization.

To simplify the mathematical presentation, in this section the methodology is presented for two-stage problem. The approach can be straightforwardly extended to multistage uncertain linear program. Consider the following two-stage uncertain linear program:

$$\min_{x,y}\left\{c^T x : Ax + By \le b\right\} \tag{1}$$

where $c \in \mathrm{R}^{n_1 x1}$, $A \in \mathrm{R}^{mxn_1}, b \in \mathrm{R}^{mx1}$ are uncertain data; $B \in \mathrm{R}^{mxn_2}$ is known data (without uncertainty); $x \in \mathrm{R}^{n_1 x1}$ represents "here and now" decisions; $y \in \mathrm{R}^{n_2 x1}$ represents "wait and see" or "recourse decisions".

Following the terminology of the two-stage stochastic programming the matrix B is called recourse matrix. When B is not uncertain such as the case above, we call the corresponding uncertain LP a fixed recourse one. Hence, in this paper we only focus on a fixed recourse multistage uncertain problem.

Note that in the above formulation we assume without any loss of generality that the objective is a function of the first stage decisions x. If the objective depends also on y we can use auxiliary variable to formulate the objective as a constraint, however, to keep the problem fixed recourse the coefficients of y in the objective must not be uncertain.

The AARC explicitly parameterizes the recourse decisions as affine functions of the revealed uncertainty. Hence, the AARC restricts the adjustable/recourse variables to be affine functions of the corresponding revealed data where in the original problem the function itself is unknown. For the two-stage above the second stage variable, $y(c, A, b)$, is parameterized as follows:

$$y = z_0 + Z_1 c + \sum_{i=1}^{n_1} Z_{2i} A_i \tag{2}$$

where A_i corresponds to i^{th} column of A. After substituting (2) in (1), the problem becomes:

$$\min_{x, z_0, Z_1, Z_{2i \forall i}} \left\{ c^T x : Ax + Bz_0 + BZ_1 c + \sum_{i=1}^{n_1} BZ_{2i} A_i \leq b \right\} \tag{3}$$

The resulting optimization problem (3) is a single stage uncertain linear program in which all the decision variables are determined at the first stage before any uncertain data is reveled. Thus, the problem could be solved with the "static" RC where the decision variables are the elements of the vectors $x \in \mathrm{R}^{n_1 x 1}$, $z_0 \in \mathrm{R}^{n_2 x 1}$ and the elements of the matrices $Z_1 \in \mathrm{R}^{n_2 x n_1}$, $Z_{2i} \in \mathrm{R}^{n_2 x m}$ $\forall i = 1..n_1$.

To apply the RC for (3) let us consider the reformulation:

$$\min_{x_f} \left\{ c_f^T x_f : A_f x_f \leq 0 \right\} \tag{4}$$

Problem (3) can be reformulated as (4), where all the decision variables are in the vector x_f and where only part of A_f are uncertain. We assume without any loss of generality that (3) can be reformulated as (4), for more detailed see Section 2 in Housh et al. [2011].

According to the RC methodology [Ben-Tal and Nemirovski, 1998, 2000a], the RC of uncertain problem (4) is:

$$\min_{x_f} \left\{ c_f^T x_f : A_f x_f \leq 0 \ \forall A_f \in U \right\} \tag{5}$$

where U is a user-defined uncertainty set. To address over-conservativeness, the RC method introduces ellipsoidal uncertainty sets to reflect the fact that the coefficients of the constraints are not expected to be simultaneously at their worst values.

The deterministic equivalent of problem (5) when U ellipsoidal uncertainty set of the form $U_i = \{a_{fi} : \hat{a}_{fi} + \Delta\varsigma, \|\varsigma\| \leq \theta\} \ \forall i$, is given as:

$$\begin{aligned} &\min_{x_f} c_f^T x_f \\ &\text{Subject to} \\ &\hat{a}_{fi} x_f + \theta \left\| x_f^T \Delta \right\| \leq 0 \quad \forall i \end{aligned} \tag{6}$$

where a_{fi}^T is the i^{th} row in the matrix A_f; $\hat{a}_{fi}$ is the nominal value of the a_{fi}; Δ is the mapping matrix and θ is a subjective the parameter chosen by the decision-maker to reflect his attitude toward risk.

For short introductory on the RC method and the transition from (5) to (6) the reader is referred to Section 2 in Housh et al. [2011] and the references therein for specific details/proofs.

Optimization problem (6) is Second Order Conic Program which is a convex tractable optimization problem that can be solved by polynomial time interior point algorithms.

3. Management model of a Water Supply System (WSS)

We consider the same WSS management model from Housh et al. [2011] where water is taken from sources, which include aquifers, reservoirs and desalination plants, conveyed through a distribution system to consumers who require certain quantities of water.

The time horizon covers several years, with an annual time step. The operation is subject to constraints on water levels in the aquifers, annual carrying capacities of the conveyance pipes and production capacities of the desalination plants. The objective is to operate the system with minimum total cost of desalination and conveyance, plus a penalty/reward related to the final state of the aquifers at the end of the planning horizon. This model could be classified as a medium aggregation model such as the developed by a model of medium aggregation [Draper et al., 2003; Jenkins et al., 2004; Watkins et al., 2004; Zaide, 2006].

Despite that the mathematical model was fully presented by the authors in previous paper in what follows a concise formulation is presented since it is crucial step for applying the new methodology. For more details and for the reasoning of the model formulation the reader is referred to Housh et al. [2011].

$$\sum_{t=1}^{T_f}[\underbrace{\sum_d des_{d,t}\cdot Q_{d,t}}_{\text{Desalination}}+\underbrace{\sum_l C_{l,t}\cdot Q_{l,t}}_{\text{Conveyance}}]+\underbrace{\sum_{a=1}^{a_f}[(\hat{h}_a-h_{a,T_f})\cdot E_a]}_{\text{Penalty/Reward}}\rightarrow \min \tag{7}$$

Subject to

$$h_{a,t}=h_{a,0}+\left(\sum_{i=1}^{t}R_{a,i}-\sum_{i=1}^{t}Q_{a,i}\right)/SA_a \qquad \forall a\forall t \tag{8}$$

$$G_t\cdot Q_t\geq S_t \qquad \forall t \tag{9}$$

$$\begin{aligned}&h_{a,t}^{\min}\leq h_{a,t}\leq h_{a,t}^{\max} \quad ; \quad 0\leq Q_{l,t}\leq Q_{l,t}^{\max}\\ &0\leq Q_{a,t}\leq Q_{a,t}^{\max} \quad ; \quad Q_{d,t}^{\min}\leq Q_{d,t}\leq Q_{d,t}^{\max} \qquad \forall a\forall d\forall l\forall t\end{aligned} \tag{10}$$

where a, d, l and t denote aquifer, desalination, link and year, respectively. Water quantities are in (MCM/year), elevations in (m), and costs in ($\text{M\$/MCM}$).

$des_{d,t}$ is the cost of desalinated water, $Q_{d,t}$ is the annual quantity of water desalinated, $C_{l,t}$ is the unit cost of transportation and $Q_{l,t}$ the annual transport in the link, $h_{a,t}$ is the water level in the aquifer at the end of year t, $\hat{h}_a$ the desired final water level, and E_a is the penalty per meter of deviation from this level ($\text{M\$/m}$), $Q_{a,t}$ is the extraction amount from the aquifer, $R_{a,t}$ is the aquifer recharge; SA_a (MCM/m) is the storativity multiplied by area, G_t is the graph matrix of the network, $Q_t = [Q_{\forall a,t}, Q_{\forall d,t}, Q_{\forall l,t}]^T$, $S_t = [0, Q_{demand,t}]^T$, $Q_{demand,t}$ is the vector denotes water demands, $(\ )^{\min}$ is minimum allowed value; $(\ )^{\max}$ is maximum allowed value.

4. Applying the AARC

4.1. Modeling uncertainty

In this study we only focus on the uncertainty in the recharge vector $R = [R_{\forall a,t=1}, \cdots, R_{\forall a,t=T_f}]^T$ to demonstrate the methodology. Though, the authors recognize there is often significant uncertainty in other variables, such as demands uncertainty. The application of the AARC can be straightforwardly extended to deal with such uncertainty.

Housh et al., [2011] showed the advantage of choosing ellipsoidal uncertainty set in the RC methodology. In section 2.2 Housh et al. [2011] proved that if we choose ellipsoidal uncertainty set $U = \{R : \hat{R} + \Delta\varsigma, \|\varsigma\| \le \theta\}$ with the center taking as the expected value of the uncertain recharge $\hat{R}$ and the mapping matrix Δ as the Cholesky decomposition of the covariance matrix Σ, then it is the same as saying that we are immunized against θ standard deviations of the constraint. Hence, simple probabilistic arguments such as first and second moments of the uncertain recharge can be utilized to construct the ellipsoidal uncertainty set.

In this application we consider the annual recharge values are independent random variables, where the spatial recharge vector of the aquifers $R' = R_{\forall a,t'}$ in each year t' is correlated with covariance matrix $\Sigma_{R'}$ and expectation vector $\mu_{R'}$, Hence, the expectation vector of the overall recharge is $\hat{R} = [\mu_{R'}, ..., \mu_{R'}]^T$ and covariance matrix Σ is a diagonal block matrix with $\Sigma_{R'}$ in the main diagonal.

4.2. Formulation of the AARC

To apply the AARC it is necessary to eliminate the uncertain equality constraints(8), thus coming to the inequality constrained problem. Additionally, it is convenient to work on the vectorized version of the model:

$$
\begin{aligned}
& K \to \min \\
& \text{Subjecte to} \\
& Coff^T Q - R^T v + P_0 - K \le 0 \\
& h_{a,0} + R^T \delta_{a,t} - \gamma_{a,t}^T Q - h_{a,t}^{\max} \le 0 \qquad \forall a \forall t \\
& -h_{a,0} - R^T \delta_{a,t} + \gamma_{a,t}^T Q + h_{a,t}^{\min} \le 0 \qquad \forall a \forall t \\
& G \cdot Q \ge S \\
& Q^{\min} \le Q \le Q^{\max}
\end{aligned}
\tag{11}
$$

where $P_0 = \sum_a (\hat{h}_a - h_{a,0}) E_a$ is a certain constant, $Q = [Q_{\forall t}]^T$,

$Q_t = [Q_{\forall a,t}, Q_{\forall d,t}, Q_{\forall l,t}]^T$, $Coff = [Coff_{\forall t}]^T$, $Coff_t = [\left(E \cdot SA^{-1}\right)_{\forall a}, des_{\forall d,t}, C_{\forall l,t}]^T$,

$R=[R_{\forall a,t=1},\cdots,R_{\forall a,t=T_f}]^T$, $v=[\left(E\cdot SA^{-1}\right)_{\forall a},\cdots,\left(E\cdot SA^{-1}\right)_{\forall a}]^T$, G is diagonal block matrix with the matrices $G_{\forall t}$ in the main diagonal, $S=[S_{\forall t}]^T$, $\delta_{a,t}$ and $\gamma_{a,t}$ has 0 and SA_a^{-1} values according to a and t in order to extract the elements corresponding to the constraints from the elements of R and Q, respectively. Thus, the dimension of $\delta_{a,t}$ equal to the dimension of R while the dimension of $\gamma_{a,t}$ is equal to the dimension of Q.

In our model the decision vector at each stage is $Q_t=[Q_{\forall a,t},Q_{\forall d,t},Q_{\forall l,t}]^T$. Hence, Q_1 is "here and now" decision and $Q_{t=2..T_f}$ are "wait and see" decisions. The "wait and see" decisions of stage t are function of all the realized data up to stage t:

$$Q_t = f(V_t\cdot R) \qquad \forall t=2..T_f \tag{12}$$

where V_t is given diagonal matrix, includes 0, 1 values in the main diagonal according to t, i.e. V_t will have the value 1 in the main diagonal corresponding to the elements of R at times $\forall t=1...(t-1)$ of all the reveled uncertainty contributed to the decision at time t. If only part of the reveled uncertainty contributes (e.g. only revealed data within 2 time gaps) then the elements of V_t are adjusted accordingly. In this study we will consider that all the revealed uncertain data up to stage t contributes to the decision of stage t.

If we restrict the former dependency in (12) to affine functions (linear function) as the AARC methodology implies we obtain:

$$Q_t = f(V_t\cdot R)=q_t+M_t\cdot V_t\cdot R \qquad \forall t=2..T_f \tag{13}$$

Thus at each stage t the decision variables are the elements of the vector q^t and the non-zero elements in the matrix $M_t\cdot V_t$. Hence, the overall flow vector Q in (11) is defined as:

$$Q=q+MV\cdot R \tag{14}$$

where $q=[q_{\forall t}^T]^T$ and $MV=[(M\cdot V)_{\forall t}^T]^T$. Substituting (14) in (11) and applying the RC as described in section 2.2 results in:

$$\begin{aligned}
&K\rightarrow \min\\
&\text{Subjecte to}\\
&Coff^Tq+Coff^TMV\cdot\hat{R}-\hat{R}^Tv+\theta\left\|\left(Coff^TMV-v^T\right)\Delta\right\|+P_0-K\le 0\\
&h_{a,0}+\hat{R}^T\delta_{a,t}-\gamma_{a,t}^Tq-\gamma_{a,t}^TMV\cdot\hat{R}+\theta\left\|\left(\delta_{a,t}^T-\gamma_{a,t}^TMV\right)\Delta\right\|-h_{a,t}^{\max}\le 0 && \forall a\forall t\\
&-h_{a,0}-\hat{R}^T\delta_{a,t}+\gamma_{a,t}^Tq+\gamma_{a,t}^TMV\cdot\hat{R}+\theta\left\|\left(\gamma_{a,t}^TMV-\delta_{a,t}^T\right)\Delta\right\|+h_{a,t}^{\min}\le 0 && \forall a\forall t\\
&-e_i^T\cdot G\cdot q-e_i^T\cdot G\cdot MV\cdot\hat{R}+\theta\left(\left\|e_i^T\cdot G\cdot MV\cdot\Delta\right\|\right)+e_i^T\cdot S\le 0 && \forall i\\
&k_j^T\cdot q+k_j^T\cdot MV\cdot\hat{R}+\theta\left(\left\|k_j^T\cdot MV\cdot\Delta\right\|\right)-k_j^T\cdot Q^{\max}\le 0 && \forall j\\
&-k_j^T\cdot q-k_j^T\cdot MV\cdot\hat{R}+\theta\left(\left\|k_j^T\cdot MV\cdot\Delta\right\|\right)+k_j^T\cdot Q^{\min}\le 0 && \forall j
\end{aligned} \tag{15}$$

where e_i and k_j are unit vectors having the value 1 at the i^{th} and the j^{th} elements respectively.

Optimization problem (15) is Second Order Conic Program where the decision variables are the elements of the vector q and the non-zero elements in the matrix MV .

5. Demonstrated example

In this section we consider a simplified example of the WSS model presented in Section 3, the model does not include the module of the conveyance system this to limit the number of decision variables and facilitate the presentation and the discussion of the whole solution components. We would like to stress that this was done only for this presentation purposes, the application of the methodology for large scale problem is straightforward.

In this example we consider the problem of optimal allocation in WSS (Figure 1) fed from two aquifers and a desalination plant with cost of 1 M\$/MCM, where the aquifers' recharges are uncertain. The demand is 80 MCM for each year, the initial water level is zero, $\hat{h}_a$ the desired final water level is 30 (m), SA_a the storativity multiplied by area is 0.8 (MCM/m) and E_a the deviation penalty from this level is 0.3 (M\$/m) in both aquifers.

The objective is to minimize multiyear operation costs to provide a given demand over two years. The operation satisfies bounds on the water levels of the aquifers and non-negativity constraints on the flows (decision variables) and the water levels (state variables). The uncertain recharges in both aquifers are given by the ellipsoidal set:

$$U = \left\{ \begin{pmatrix} R_{11} \\ R_{21} \\ R_{12} \\ R_{22} \end{pmatrix} = \begin{pmatrix} 40 \\ 40 \\ 40 \\ 40 \end{pmatrix} + \begin{pmatrix} 12 & 0 & 0 & 0 \\ 4 & 9 & 0 & 0 \\ 0 & 0 & 12 & 0 \\ 0 & 0 & 4 & 9 \end{pmatrix} \begin{pmatrix} \varsigma_1 \\ \varsigma_2 \\ \varsigma_3 \\ \varsigma_4 \end{pmatrix} ; \|\varsigma\| \leq 2 \right\} \qquad (16)$$

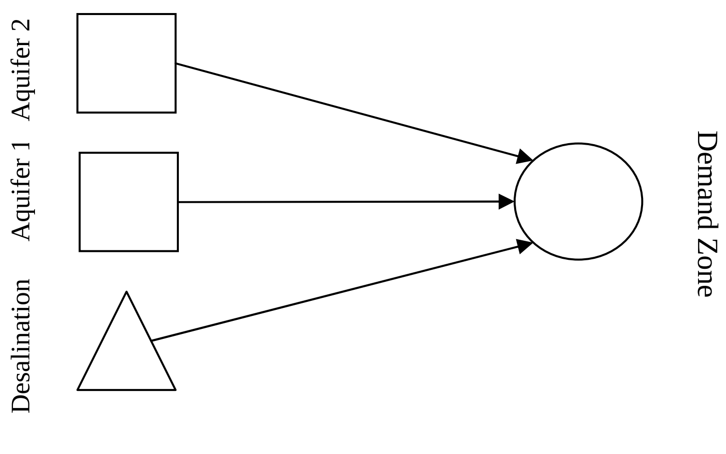

Figure 1: WSS scheme: demonstrated example

In the above AARC model, we account for the fact that decisions on the extracted quantities in each year (beyond the first) can be delayed to the beginning of that year. For example, the extracted quantities in year two can be adjusted according to the realization of the recharge in the first year. Moreover, in AARC the dependence of the adjustable decision variables on past realized data is restricted to be linear. Hence, the adjustment is according to a linear decision rule (LDR).

For instance, the extracted quantities in year two is a linear function of one past realization of the recharge and an additional free coefficient, having a total of three coefficients (realization of two aquifers in the first year and a free coefficient) for each flow variable (flow variables are the decision variables in the deterministic formulation).

In order to explicitly write the AARC model, we substituted the LDRs. Thus, in the AARC formulation, the coefficients of the LDR become the actual decision variables.

To implement the AARC solution we need to find the optimal LDR coefficients and then at the beginning of each year we can calculate the quantity by substituting the realized data in the LDRs.

The optimal LDRs obtained from the AARC formulation for the demonstrated example are given in Table 1. For example, the results show that the extraction from the second aquifer in year 2 is given as the LDR: $Q_{a=2,t=2} = 33.9 - 0.48 \cdot R_{11} + 0.52 \cdot R_{21}$. Thus, one cannot determine the amount to be taken from aquifer 2 in year 2 unless the realization of the first year recharge is given. The AARC found the optimal linear function for the second aquifer withdrawal in year 2. Decision in the first year are real values, not functions, since the is no reveled uncertain data in this stage, the first stage decision is taken in the face of the unknown without any extra information about the uncertainty except that it belongs to the ellipsoidal uncertainty set defined in (16). In Table 1 we can see that the first stage decisions are real values and not functions since all the coefficients except the free coefficients are zero.

The decisions in year 2 can only depend on the revealed uncertain data; hence the decisions in stage 2 are taken in the face of unknown recharge in stage 2, this explains the zero coefficients in the last two columns in Table 1.

The optimal solution of the AARC guarantees cost below 74.1 M$ for all realization within the uncertainty set defined in (16). To show the advantage of learning and adjusting the decisions based on revealed uncertainty we compare the solution of the ARCC to the solution of the “static” RC. Table 2 shows the optimal decision obtained by the RC method. Since the method is “static”, all decisions are real values (not functions of the recharge) as can be seen in Table 2. The significant difference between the free coefficients in the AARC solution (Table 1) and the optimal values of the RC (Table 2) shows the advantage of adjusting the decisions based on gathered information about the uncertainty. A particularly noteworthy feature is the difference in the first stage decisions. While both in the AARC and the RC approaches the first stage is taken without any knowledge of revealed data, still we have significant difference between the decisions. This indicates that the AARC approach reflects the advantage of the adjustment of future decisions for the first stage decisions. As can be seen by comparing Table 1 and Table 2, the solution of the AARC is less conservative by less relying on the desalination plant.

The optimal solution of the RC is given as fixed values for all the horizon, these values are the optimal decisions to be taken in the future if we are forced to take the decisions in the first stage before any uncertainty is revealed. This formulation could be seen as more constrained version of the original problem, namely, by adding the constraint for taking all decision for all years in advance. Adding constraints can only worsen the optimal objective, hence we should expect inferior results compared to the AARC.

The optimal solution of the RC guarantees cost below 77.2 M$ as opposed to the AARC which guarantees cost below 74.1 M$ for all realization within the uncertainty set.

To discuss the difference between the decisions in the first stage let us analyze the range of possible values for the second aquifer withdrawal. Since the decision is given as LDR $Q_{a=2,t=2} = 33.9 - 0.48 \cdot R_{11} + 0.52 \cdot R_{21}$ and the uncertainly is represented by (16):

$$Q_{a=2,t=2} = 33.9 + [-0.48,\ 0.52]\left(\begin{bmatrix} 40 \\ 40 \end{bmatrix} + \begin{bmatrix} 12 & 0 \\ 4 & 9 \end{bmatrix}\begin{bmatrix} \varsigma_1 \\ \varsigma_2 \end{bmatrix}\right) ;\|\varsigma\| \le 2 \qquad (17)$$

which implies:

$$35.5 \le Q_{a=2,t=2} \le 47.6 \qquad (18)$$

The value obtained from the RC approach is 36.42 (MCM) nearly as the lower bound in (18) this demonstrate the over conservativeness of the RC solution which decrease the reliance on the less expensive source because of uncertainty.

		Coefficients of the LDRs				
	Decision	Free	R_{11}	R_{21}	R_{12}	R_{21}
Year 1	Aquifer 1	12.48	0.00	0.00	0.00	0.00
	Aquifer 2	17.51	0.00	0.00	0.00	0.00
	Desalination	50.01	0.00	0.00	0.00	0.00
Year 2	Aquifer 1	41.49	0.41	-0.59	0.00	0.00
	Aquifer 2	33.90	-0.48	0.52	0.00	0.00
	Desalination	4.61	0.06	0.06	0.00	0.00

Table 1: Optimal solution of the AARC, demonstrated example

	Decision	Optimal Solution
Year 1	Aquifer 1	11.79
	Aquifer 2	15.30
	Desalination	52.91
Year 2	Aquifer 1	33.57
	Aquifer 2	36.42
	Desalination	10.01

Table 2: Optimal solution of the RC, demonstrated example

5.1. Simulation results

The guaranteed value of the AARC is lower than the guaranteed value obtained value of the RC approach. This indicates that if the worst case realization within the uncertainty set is realized the performance of the AARC is better than the performance of the RC. However, we it is important to investigate the average performance of both approaches. the average performance could be evaluated by simulation. We generate a recharge sample within the ellipsoidal uncertainty set then we compare the performance of the approached considering the sample as the real life realization of the recharge. The results may differ according to the scattering pattern within the uncertainty set. Here, we consider two cases for demonstration (a) Uniform distribution and (b) Normal distribution. By means of 1000 samples Figure 2 presents histogram of the cost resulted from the AARC and the RC solutions.

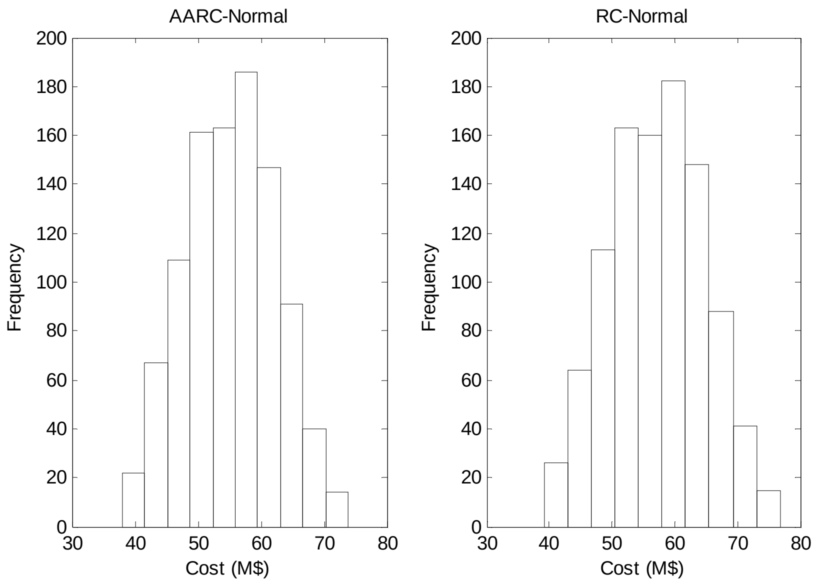

Figure 2-a: Performance comparison of the AARC and the RC under normal distribution.

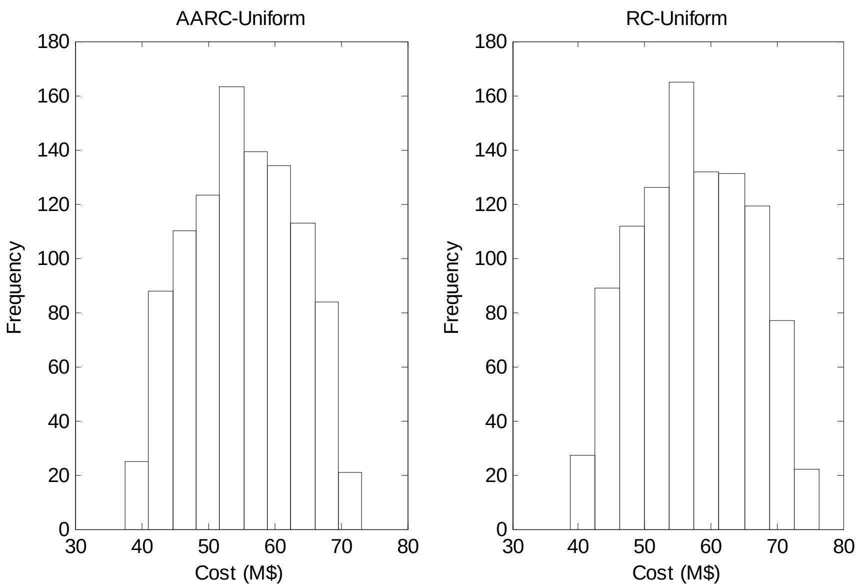

Figure 2-b: Performance comparison of the AARC and the RC under uniform distribution.

Figure 2-a shows that the AARC obtained (a) lower best cost value (lower bound), 37.98 (M$) compared to 39.3 (M$) in the RC solution. (b) lower worst cost value (upper bound) 73.89 (M$) compared to 76.75 (M$) in the RC. (c) lower average cost value 55.32 (M$) compared to 57.31 (M$) in the RC solution.

One can also notice that the peak frequency obtained in the AARC is with smaller cost than in the RC solution.

The simulation results in the uniform distribution shows the same results as the normal distribution as summarized in Table 3.

	Approach	**Min**	**Average**	**Max**
Normal	**AARC**	73.89	37.98	55.32
	RC	76.75	39.28	57.31
Uniform	**AARC**	73.22	37.58	55.30
	RC	76.30	38.79	57.32

Table 3: Performance of the AARC and the RC

6. Conclusions

The concepts of the RC and the AARC methods show considerable promise, regarding the tractability of the models and the results obtained. The results obtained by Housh et al., [2011] demonstrate the advantage of being able to replace the stochastic behavior of the uncertainty by specifying a user-defined set within which the resulting policies are immunized. In this paper we present paper we formulated the AARC as the dynamic variant of the RC method. The method was applied to a small hypothetical system to show the nature of the decision compared to the "static" RC method. The results show the advantage of adjusting the decision variables and utilizing the revealed uncertainty. Here we only presented a comparison between the AARC and the RC for realization within the designed ellipsoidal uncertainty set. However, another evaluation for the performance of the approaches when the realizations are outside the uncertainty set is need. The results obtained here encourage us to compare the performance of the AARC with other dynamic approach which were consider in Housh et al., [2011], such as the dynamic variant of the RC Folding RC and multistage stochastic programming.

7. References

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