Engineering Analysis of Water-Distribution Systems

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In an attempt to bridge the gap between technological advancement and application, a nonmathematical explanation of modeling for analysis of water-distribution systems is given.

The analysis of water-distribution systems has attracted considerable interest, particularly during the past ten years. Many computer programs are now available for this type of work, and a large body of expertise exists within the profession, providing both the experience with the use of computers and the practical know-how required for constructive and economical solutions to problems in water-distribution systems. The approach taken in these studies depends upon the nature of the problem itself as well as the tools available for analysis. In 1966, for an investigation of the Boston, Mass., water-distribution systems, an entirely new technology was introduced. The mathematical details were subsequently published,[1] and a computer program was deposited with the ASCE for public distribution. Unfortunately, this new technology is based on a mathematical formulation that is not grasped easily by many practicing engineers dealing with water-distribution problems. The result has been a gap between the time of the technological development and its acceptance within the profession.

Since the intent of this paper is purely communicative, the basic mathematical structure is not described. Interested readers should refer to the earlier paper.[1] That paper describes, in some detail, the precise mathematical formulations and gives the results of application to the Boston engineering study.

Review of Previous Work

A network of pipes and other hydraulic elements (pumps, valves, reservoirs) is considered solved when the heads and consumptions at all nodes and the flows through all pipes and elements are known. Such a solution relates to a specific loading condition and describes the network's state at a particular time.

A previous work[1] presented a method for obtaining the solution of a hydraulic network. Obtaining the solution consisted of finding the values of a specified set of unknowns, given all other data of the network. The physical laws, whose fulfillment guaranteed the solution, are

1. A known relationship between discharge and the energy gradient for each pipe and element
2. Conservation of mass (continuity) at each node

The method used the Newton-Raphson technique to solve the set of nonlinear equations that express these laws directly for combinations of unknowns, which may include heads, consumptions, and element characteristics. The method can incorporate pumps, valves, and other element characteristics into the mathematical formulation and, by allowing a solution for various combinations of unknowns, achieves simplicity and power that makes it attractive for engineering applications.

A solution is not mathematically possible for every combination of unknowns. The following rules for the assignment of the unknowns, whose total number must be equal to the number of nodes in the network, must be observed:

1. A node having an unknown consumption should be connected to at least one other node with a known consumption.
2. A subsystem consisting of an element with unknown characteristic (i.e., resistance) and its two end nodes should have no more than one additional unknown—one of the heads or consumptions at the two end nodes.
3. Considering any node, at least one of the following should be unknown: the consumption at the node; the head at the node itself or at any adjacent node; or the characteristic of an element that is connected at the node.

If these rules are followed the network is solvable in principle. The method of solution outlined in the previous paper generally converges from any set of initial guesses for the unknowns and is rapid when a reasonable set is given. Subsequent modifications of the method and the computer code have improved the overall stability of the method (its ability to converge to a solution from a poor initial guess) and its convergence rate.[2-4] In some cases, depending mainly on the configuration of the network and the combination of unknowns but also on the quality of the data, the solution converges slowly.

The possibility of divergence or slow convergence is of great concern in programs for automatic optimization of network design or operation[2,3] or for real-time network operation. Although divergence has been shown to be a possibility,[5] it has not caused practical problems in engineering analysis work. In practice, solutions are never allowed to diverge; runs always are terminated if convergence is not achieved after a prescribed number of iterations. Using sensitivity analysis[1] after such a termination, modifications are made in the set of initial guesses for the unknowns, and divergence is avoided. Problems occasionally occur the first time a new network is being solved. Further solutions of the same network, for different data, usually converge rapidly because familiarity with the network ensured reasonable initial guesses and easier detection of faulty data.

Convergence is poorest when resistances are specified as unknowns because of high sensitivity of the continuity equations to resistance values. Poor convergence may also occur in the presence of certain types of hydraulic elements—ones that have discontinuous or irregular characteristics.

The conceptual simplicity of this procedure and ultimately its power falls out of the full set of network equations that are dealt with at one time by the computer. The set of simultaneous equations describing the network can be solved by a direct procedure, such as Newton-Raphson, if the computer is large enough to store the entire set at one time. The procedure was not possible (except for very small networks) until large memory computer hardware became generally available. The procedure was not practical until the machines became very fast because, for a system of equations, computing time varies as the square of the number of equations. Thus, the full potential of the method could not be realized in practice until the mid-1960s.

Having stored the entire system of equations describing the network solution it is a minor mathematical manipulation to store the entire system of partial derivatives of these equations. These derivatives describe the rate of change of the solution with respect to a change in any fixed performance criterion; this provides an automatic sensitivity analysis.

In many cases, the engineer is interested in the sensitivity of certain heads, consumptions, and estimated characteristics to changes in other variables. Using the Newton-Raphson method, this sensitivity analysis is easily obtained. The sensitivity values, which are partial derivatives of certain variables with respect to other variables, are computed directly with the aid of a Jacobian matrix. When the variables whose sensitivity is being investigated are the unknowns of the flow solutions, as is often the case, then the final Jacobian of the Newton-Raphson method is used in the sensitivity analysis. This Jacobian contains the numbers that are used to calculate directly the rate of change of all of the unknowns with respect to a change in any known parameter. When known parameters are design criteria or data from field measurements, multiplication of the Jacobian matrix gives the rate of change of all unknowns (sensitivity) with respect to changes in a design criterion or an error in data. When sensitivity to variables not in the current list of unknowns is required, the list of unknowns may be redefined, the Jacobian recalculated using the current solution, and any desired sensitivity set obtained.

Sensitivity analysis has application to design and operation. For design, it serves as a practical guide for selecting alternatives. It has been made into a formal optimization procedure[3] using the Jacobian to compute a gradient move for making systematic changes in design variables.

For real-time operation, the sensitivity analysis can directly locate changes that will result in the greatest improvement in system performance. Here too, a formalized optimization procedure can be developed using the results of sensitivity analysis as coefficients in an algorithm for locating optimal operational decisions. For automatic control such a procedure, which avoids successive network analysis, appears to be the only available practical approach.

Other computer codes for solving networks are available. Such codes may solve particular problems using less computer time. These codes are based on other methods (e.g., a loop-based formulation of the equations on a minimal-spanning tree, solved by the Newton-Raphson method[6]) or on the same solution technique[4] but without the flexibility in assignment of unknowns. Also, for a specific problem, a special purpose solver—a computer program[2] written to solve the particular network—can be used. These methods have their advantages, which rest mainly in their computational speed. An economically meaningful trade-off among various methods of analysis and computer codes includes the cost of engineering time in addition to computer costs and the knowledge of the network gained from the analysis. Pure computational efficiency, measured in computer time per run, is a poor measure of a method's overall efficiency. There are practical benefits of flexibility in the assignment of unknowns and convenient sensitivity testing.

The techniques described in the previous paper have been implemented in a computer program, designated SDP. The program has been devised so that it can simulate the behavior of a system over time, including the operation of storage reservoirs. If, in a simulation run, a resistance is unknown, then each network solution yields a different value for this resistance. These values can be used to determine a characteristic for the hydraulic element in question such that the network behaves as prescribed.

The computer program, in several variations, has been applied to the analysis of water distribution systems in Boston, Mass.; Calgary, Alta.; Minneapolis, Minn.; Seattle, Wash.; and several smaller cities. Versions of the code have been available since 1969 and probably have been used other studies. Stoner [7,8] has programmed similar formulation for gas-distribution systems.

The Basic Engineering Problems

In the analysis of water-distribution systems, there are several types of analysis: calibration, operation and control, design, and optimization (of the design and operation).

Calibration. Calibration is used here to describe a process that is both a modeling and an engineering problem. Alternatively, it may be called identification. It consists of determining the physical and operational characteristics of an existing system and determining the data that when input to the computer model, will yield realistic results. This type of analysis is the first step in studying an existing network and forms part of the data assembly.

Much of the essential data is gathered, examined, and evaluated while the computer model is being calibrated. In practice, the instantaneous demands in the network, the consumptions, are unknown, except perhaps at a few points in the system. On the other hand, the water inflows at the principal supply points and at the pumpimg stations usually are measured. An important part of the calibration process is estimation of the consumptions at all nodes in the network, using the flows at these principal points and whatever other data are available. The SDP code is convenient for this purpose because of its capability for dealing with mixtures of unknowns.

In the Boston and Minneapolis studies, a minimal program of field measurements was undertaken to help in the calibration. In Boston, the main emphasis was on estimation of pipe resistance by head-loss and discharge measurements because approximately twenty supply points provided basic flow patterns with which the demands at nodes were highly correlated. In Minneapolis, on the other hand, the field prograrm was aimed mainly at simultaneous measurement of total heads throughout the network since many pipe resistances were known, whereas supplies were measured at only four points. The computer analysis was relied upon to indicate the local instantaneous consumption at most nodes and to obtain a fine tuning for the resistances of a few important pipes.

A field program that includes simultaneous discharge measurements at key locations within the distribution system is important where storage in the system plays an important role. Discharge estimates computed on the basis of total head measurements are not generally reliable and can lead to excessive accumulated errors over time.

Fine tuning of the computer model is facilitated through the use of sensitivity analysis. This analysis can rank pipe resistances with respect to their influence on total heads and consumptions at points in the network where there is significant disagreement between observed and computed values, eliminating many trial-and-errors runs.

Operation and control. The second type of analysis—operation of an existing network and the possibility for on-line control—is a continuation and extension of the calibration. Once the model of the network has been calibrated satisfactorily, the engineer can use it to study various options for operating the system. This study could be performed through a sequence of computer runs, each for a different loading condition or operational option, but can be expedited by the use of sensitivity analysis. The sensitivity analysis indicates potentially efficient alternative operations to achieve desired results relative to the current status of the network. Procedures for on-line control, either manual or automatic, should be based on an appropriate model and sensitivity analysis.

Design. The design of an entirely new system or of additions to an existing one is a third type of analysis. In this case, the locations of new network elements are prescribed, and the conveyance characteristics are sought. Designating the new elements' characteristics as unknowns allows the program to solve directly for these values, thereby eliminating much tedious trial-and-error work. Sensitivity analysis is also useful in reaching an acceptable design because not all resistances can be solved for simultaneously when many hydraulic elements are being designed.

If a booster pump or some other special hydraulic element is being designed, its required characteristic can be approximated through a series of solutions covering the range of discharges expected—for example, by simulating the operation of the network over time. For this type of problem, labeling such an element as having an unknown resistance is necessary. The computed numerical value of the resistance is of no interest; the characteristic of the element is determined from the computed pairs of values of discharge and head differential. Since no specific functional form for the characteristic is assumed, any type of device—pumps, heat exchangers, valves, pressure reducers—can be handled in this way, with the results suggesting the type of device required. With this approach the designer can specify pressure and consumption criteria for an acceptable design and then use the program to obtain guidelines for the types of devices needed. He then can select the actual devices to match these indications as closely as possible and run a revised model containing the selected characteristics to examine the network's actual behavior with these devices.

Optimization. Finally, design and operation decisions are sought that are optimal in some sense. The method described above has been used as a component in an optimization scheme.[3] The formulation is general and can accomodate optimal design and operation problems. The optimization method, which uses the generalized reduced gradient technique, has been implemented in a computer program that can be used for all the tasks mentioned above as well as optimization.

Design Criteria

A design problem can be approached in different ways, depending mostly on the capabilities of the analytical tools which are used. If the computer program is capable of solving only for unknown heads, the analysis must be carried out by trial-and-error, changing design variables, such as pipe sizes, between successive runs. In this case, the criteria for acceptable designs can be stated rather loosely and used to guide the process, since the computation is not directly concerned with these criteria.

On the other hand, if the design is approached by solving for a mixture of unknown element characteristics (head-discharge characteristics of pipes, heat exchangers, pressure reducing valves, check valves, pumps), consumptions, and total heads, using the procedures discussed above, then the design criteria must be specified explicitly. These criteria are used directly in the computations. They appear as fixed values, or knowns, in the formulation. For example, areas in which the minimum allowable pressure is to occur must be identified and the head at appropriate nodes specified as known. Unknown consumptions then may be assigned to supply nodes, and the solution will give these directly along with the design of selected elements. If different criteria are used, the operation of sources as well as element design will change; each set of criteria leads to a unique solution that satisfies the network equations. In some distribution systems with certain demand conditions, the design may be sensitive to the assumed fixed heads and allocations at the supply nodes. The sensitivity analysis provides a means for determining if this is the case; intuition alone often is misleading. Design criteria should be selected with care, both with regard to the quality of service and the resulting costs in hardware or operation.

The design criteria should not be arbitrary or based on untested rules of thumb. They should, on the other hand, reflect local conditions, alternatives, and practices and be based on the proper economic considerations and data. For example, investment in improved fire-fighting equipment may be a more cost-effective means for achieving better fire protection than additional pressure in the region near a low pressure node.

Surrogate Models, Skeletonized Networks

The inclusion of every pipe and hardware feature of the real network in the computer model is not necessary. In practice, real distribution systems often are schematized prior to analysis, leaving only the most important features, such as the larger pipes. This step in the analysis is particularly important when analyzing distribution systems in larger cities having a wide range of pipe sizes. Many of the smaller pipes may not be important to the capability of the network for delivery of large quantities of water to the various regions of the system. Furthermore, the detailed local description of demands may not be important, and these can be aggregated at nodes representing demand over neighboring areas. The procedure for deciding what to include in the model of the network and what to leave out is often imprecise, depending on the judgment of the engineer performing the analysis.

A design problem can be approached in different ways, depending mostly on the capabilities of the analytical tools used.

In the interest of minimizing computation time, common practice is to reduce a number of pipes to one having an equivalent carrying capacity. This procedure is well worth the effort even when using modern high speed computers. If a 100-node network can be reduced to a 50-node one. the computer cost will be reduced by approximately a factor of four because computation time varies roughly as the square of the number of nodes. This can be a very important consideration in the cost-effectiveness of computer analysis, especially if simulation of internal storage operations, network-optimization analysis, or real-time on-line control are considerations.

A network containing equivalent pipes still may appear more complex than is required to solve the engineering problem at hand. The capacity of a network to convey water from the sources to a few major points of demand can be evaluated from a surrogate network of fictitious pipes. The conveying capability of the surrogate network over the anticipated discharge range should be the same as that of the real distribution system.

One technique for developing a surrogate network is to obtain solutions over a range of loadings. These solutions should include a number of pipes with unknown resistances in a skeleton network containing only the few most important pipes of the real network: this defines the surrogate network. The results of an analysis of a more realistic, larger network model or actual field data can be used to determine values of known consumptions and total heads for the surrogate model analysis. The method discussed above allows direct solution for the required resistances of the fictitious pipes to match the behaviour of the more complex network. Surrogate demands and heads may be determined by a similiar procedure.

The results of a computer analysis of a 200-node model of the Calgary distribution system were used to develop an entirely fictitious surrogate nineteen-node model suited to studies of reservoir operations and alternative points of new major supplies.[9] Sensitivity analysis was used extensively as an aid in selecting the resistances of the fictitious pipes of this surrogate network.

Developing a surrogate network is equivalent to calibrating a network model to represent adequately the real distribution system. The goal is not to replicate the physical geometry of the real system but to duplicate its performance. If modifications to the physical geometry of the real system appear to be recommended, then geometrical duplication will be necessary to ensure that the modified network, for which no data exist, is modeled properly. Even this problem can be dealt with, however, by combining analysis of an approximate surrogate model, convenient for computation, with analysis of a more realistic geometrical representation for checking the effects of proposed network changes. Both such representations are approximations of the real distribution system, and as such both are surrogate models—each is chosen to reflect the trade-offs between engineering and computer costs and the necessary quality of the results.

Surrogate models are necessary when the computational efficiency of solution is crucial. This is the case for on-line control. Attempts have been made to use other types of surrogate models (although they were never given this name), such as that by DeMoyer et al.[10] who used a regression-type set of equations.

Modeling Special Elements

Some networks contain booster pumps and check valves and, in industrial and defense systems, heat exchangers and other exotic equipment. All elements that have continuous monotonically varying relationships between discharge and head loss can be incorporated into the Newton-Raphson analysis by assigning the appropriate head-loss equation and its derivatives to the corresponding element of the network. Rao et al.[4] successfully have incorporated a variety of hydraulic elements into a Newton-Raphson network solver. The previous paper[1] illustrated this approach for pumps. Since that time the authors have never found the need to program such a procedure for a computer and do not recommend doing so, except as an area of research, for two reasons.

Firstly, with pipes alone the system of equations is well behaved in a numerical sense, and the programs have worked well on a large number of different distribution systems. Introducing exotic functions to this proven reliable scheme of equations could limit the general applicability of the method. Research into these possibilities is justified, however.

Secondly, it is not necessary to use exotic functions to model non-pipe network elements. The method for determining a pump characteristic using an unknown negative resistance was described above. Check valves are modeled in the same way—the, solution indicating very little or no discharge through the pipes and very high resistances. If an element is known to behave differently than indicated by the solution, then the data imposed or other features of the surrogate network were incorrectly stated. Specifying unknown characteristics allows the computer program to determine which devices are required and how existing ones should operate to satisfy the network equations.

Simulation and Reservoirs

It is often desirable to obtain a sequence of network solutions that represent the behavior of the distribution system over a full cycle of time. For the Boston Water Study[11] solutions were obtained for each of the 24 hr of a "study day." Dimensionless demand curves were obtained from analysis of discharge measurements in the major pipes entering and leaving certain areas of Boston. These curves were prorated to other areas on the basis of the fraction of similar land use. The study day was chosen considering the availability and suitability of data for calibration of the surrogate models. (Three separate distribution systems existed in Boston, the high service, the low service, and the high pressure fire service.) The 24 solutions permitted calibration over a range of pressure and flow conditions.

The analysis of higher future demands was also carried over a 24-hr cycle and showed that the minimum pressures did not occur simultaneously at all nodes but depended on the distribution pattern, which changed with time. The hour of maximum total demand did not cause the lowest pressures to occur at all nodes, as would be the case if all consumptions followed the same pattern.

The 24 solutions obtained during the Boston Water Study made efficient use of computer time as the starting values of unknowns for each new run were those given by the previous solution. There is no internal storage in the Boston distribution system, and none was indicated to be required. Had there been, the 24-hr simulation could have been used to analyze the operation of storage, as was necessary for the work in Calgary.

The Calgary study was concerned with the combined effects of alternative points of new supply and the operation of a major storage facility within the distribution system. Dimensionless 24-hr demand curves were developed for each node of the surrogate model. The head and consumption were designated as unknowns at the node representing the reservoir. The 24-hr solutions then showed the required active storage volumes and the total head and direction of flows into and out of the reservoir. From these results the required pumping capability was defined. The minimum pressure criteria for the distribution system's performance were important in this application because they explicitly determined the reservoir and pumping station requirements. The results showed that, in order to achieve effective operation at different times, pumping water into the reservoir as well as out of it would be necessary, depending on the demand pattern in the distribution system.

If reservoirs operate without pumping, responding passively to the ambient pressure gradient, they can be modeled by specifying either heads or consumptions as unknowns at the corresponding nodes and determining the other variable, designated as known, from the geometry of the storage facility. At each hour of the simulation, the solution is used to update reservoir demand or the reservoir level before proceeding to the next hour. The updating procedure is entirely separate from the network analysis; designating a node as a reservoir requires no modification to the basic network solver described above.

The choice of time intervals (e.g., hourly) for simulation or analysis of reservoir operation should not be arbitrary but should reflect considerations for the size of storage facilities and the rate of change of demand.

Behavior of Water-Distribution Systems

From experiences in application of the methodology, a number of observations can be made about the behavior of water-distribution systems.

Experience with a particular network always allows the engineer to develop a capability for good guesses for initial values of unknowns—a reason why numerical or mathematical methods are not essential for this purpose. Intuition, however, is not always a good guide for predicting how a network will react to different demand patterns or changes in elements. In the studies described here, there have been many surprises resulting from attempts at trial-and-error improvements to the calibrations.

The sensitivity analysis seems to be an essential aid to judgment. For example, if under a certain pattern of demand, operation of a valve to redistribute pressure from areas of excess to areas of deficiency without causing unwanted side-effects elsewhere would be desirable. This occasion might arise as a necessity during an emergency condition or, where using remotely operated automatic equipment, as a daily routine. Choosing the best location for such a valve is a task well suited to the sensitivity analysis, which gives the rate of change of head at all nodes with respect to changes in resistance of the elements that are candidates for such a valve.

The sensitivity analysis can be used to extrapolate results from a known solution over a wide range of alternative conditions. In studies of one small distribution system, investigations found that fire-flow problems could be assessed throughout the network by examining only the sensitivity results: the rate of change of head at all nodes with respect to changes in consumption at fire-prone locations was a constant over a wide range of possible fire flows. This conclusion has been checked using complete network solutions and found to be true on every network that the authors have analyzed. While the individual elements have nonlinear head-loss characteristics, the network as a whole behaves very nearly linearly. An area for fruitful research would define the full range of conditions for which this conclusion is valid and develop appropriate techniques of linear systems analysis for use in optimization of network operation.

The above comments suggest that water-distribution systems have certain special mathematical properties that have yet to be fully exploited by research workers. There are probably additional useful analytical techniques to be developed. Real distribution systems also have certain behavioral properties that can be exploited by engineers concerned with improved operations or rehabilitation of existing networks.

Distribution systems of major cities have grown over the years by periodically adding new pipes. The cumulative effect has been to render some pipes ineffective because of local pressure gradients.

Often there are large pipes that convey essentially zero flow under all conditions of demand and during most potential emergency conditions. These pipes are redundant under the existing demand level and pattern, but to locate them is important, nevertheless, because they can be used. They offer a starting point for programs of rehabilitation that will not interfere with the quality of service normally provided by the distribution system. The Boston Water Study showed that the entire trunk system of corroded cast-iron pipe could be cleaned and lined by carefully sequencing the work, starting with presently unused large diameter pipes. A series of computer solutions were made to demonstrate that pipes could be taken out of service successively as the program of improvements progressed.

In old systems, if pipes are not effective in the distribution system, they should be valved off because they are a source of leakage.

Some distribution systems contain reservoirs or other storage facilities that are allowed to fill and empty in passive response to the surrounding pressure gradient. Such storage sometimes is termed as "floating on the system." At the time these facilities were constructed, they may have operated over their full range, but subsequently, as demand patterns changed and as new pipes were constructed, they may have become ineffective. This can be determined by examining the recorded storage levels or nearby pressure charts (if such data is available) or by a computer simulation as discussed above. When improvements to the system are necessary because of increasing peak demand, the efficiency of allowing the storage to float should be examined critically.

Finally, an unusual application for distribution-system analysis should be mentioned. In some communities in the north, burial of utilities below the frost line is impossible or uneconomical because of the presence of exposed bedrock or permafrost to great depth. Insulated, above-ground utilidors are used in such cases. To avoid freezing, continuous circulation throughout the water system and back to the relatively warm lake source can be desirable. An analysis is required to tune the system to avoid local areas of sluggish flow and redundant pipes as described above. In this case, some local pumping capacity is sized not to meet peaks but to raise minimum flow rates. The economic trade-offs in such a distribution system require analysis of low demand periods.

Conclusions

The described method[1] of solving water distribution networks is practical for engineering analysis.

The capability to solve for a mixture of unknowns influences the way in which engineers approach distribution-analysis problems and acquire field data.

For evaluations of network or data modifications, the sensitivity analysis is more effective than the solution itself because water-distribution networks behave linearly over a practical range of operation.

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