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DESIGN OF OPTIMAL SEWERAGE SYSTEMS

By Yerachmiel Argaman,[1] A. M. ASCE, Uri Shamir,[2] and Eldad Spivak[3]

INTRODUCTION

The cost of a sewage collection network constitutes a major fraction of the overall cost of waste disposal. Thus, substantial sums of money can be saved by improving network design. Current design procedures involve the selection of pipe sizes and slopes so as to ensure adequate capacity for peak flows and adequate scouring velocities at minimum or average flows. Several alternative systems (each meeting the physical and hydraulic requirements) are analyzed, and the least-cost system is selected. Obviously the outcome of such a procedure depends to a large extent on the designer's experience, and an optimal solution is not necessarily reached.

The present work constitutes a step toward the development of a calculation technique by which the selection of the least-cost combination of pipes layout, diameters, and slopes can be made in a single computation procedure. It is hoped that the availability of advanced design tools such as the method presented herein, will bring about an examination of the existing practices of sewers design, with the aim of improving the systems economics and performance.

PREVIOUS WORK ON OPTIMIZATION OF SEWER NETWORK

Optimization of a sewer network can be divided into two aspects: (1) Optimization of the layout; and (2) optimization of the design (diameters, elevations, slopes) for a fixed layout. It would be desirable to tackle both aspects

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[1] Sr. Lect. Technion, Israel Inst. of Tech., Haifa, Israel.

[2] Visiting Sci., IBM Thomas J. Watson Research Center, Yorktown Heights, N.Y.; on leave as Assoc. Prof., Technion—Israel Inst. of Tech., Haifa, Israel.

[3] Sr. San. Engr., Environmental Engrg. and Tech. (Israel) Ltd., Ramat Gan, Israel.

simultaneously, since together they constitute the entire design problem; but because of the complexity of the resultant formulation, each of the works reported in the past has addressed itself to only one of the aspects.

Haith (3) developed a program for the optimal design of a single sewer pipe. The length is divided into reaches. Each reach is a section of the line in which the discharge and the cost parameters remain constant. The decision variables are the invert elevation at the ends of each reach, and the (commercial) pipe diameters needed to carry the given flow. The solution is obtained by dynamic programming. The program has several limitations; but these result from programming considerations, rather than from the method used.

Holland (4) developed a method for optimizing the design with a given layout. In this case the discharge in each pipe is fixed, and the decision variables are invert elevations and pipe diameters. Using an elegant formulation of the flow equations, and assuming that the objective function can be expressed in a specific form, Holland succeeds in applying an available general optimization program. This requires, however, that the objective function be cast in a form acceptable by the optimization routine—a form that does not allow for several real world components of the cost function. The diameters obtained are arbitrarily rounded up to commercially available sizes, a process that may move the design away from the optimum. Holland suggests a random search around the optimum to try and locate the best solution having commercial diameters.

Liebman (5) suggested a heuristic method for optimizing the layout, assuming the pipe diameters to be fixed. The "best" layout is found by a search procedure. At each step one "branch" of the network is changed. The change is retained if it results in a decrease in the cost. The method suffers from several shortcomings, the most important being that the network is never designed hydraulically, and, therefore, may not be feasible.

Statement of Problem

The present paper is addressed to the simultaneous optimization of layout and design of a sewerage network. The hydraulic considerations and the assumptions made to facilitate the solution are given subsequently.

Hydraulics.—Sewers are designed as open channels, capable of accommodating the peak flow at a partially filled cross section. Based on Manning's formula, the following relationship can be derived for a pipe flowing full

$$Q_f = \frac{1}{n_m k_m} D^{2.67} S^{0.5} \quad \text{(1)}$$

in which Q_f = flow rate in the pipe flowing full; D = diameter; S = slope; k_m = a numerical factor which depends on the system of units used; and n_m = Manning's coefficient. The flow in a partially filled pipe, Q, would be given by Eq. 1 with n_m replaced by n_d, the apparent Manning's coefficient defined by

$$n_d = \frac{n_m}{C} \quad \text{(2)}$$

in which $C = Q/Q_f$, the ratio of the actual flow rate to the flow rate when the pipe is full; and C = a function of the depth to diameter ratio only. To facilitate computations, this function was approximated by four linear relationships over various ranges of C between 0 and 1.

The flow velocity can be calculated by continuity, once the flow rate and the depth of flow are known. However, for computation purposes in this investigation Pomeroy's (7) formula proved very useful. His formula, which is valid for pipes 24 in. in diameter or less reads:

$$V = k_p S^{0.41} Q^{0.24} \qquad (3)$$

in which V = velocity; and k_p = a constant which depends on the smoothness of the pipe, and the system of units used.

The average sewage flow in a pipe was determined from the population (or population equivalents) served, and the average per capita flow. For design purposes the maximum and minimum flow rates are of major importance. These were estimated by the following formulas:

$$Q_{max} = K_{max} Q \qquad (4)$$

$$Q_{min} = K_{min} Q \qquad (5)$$

$$K_{max} = \frac{5}{\left(\frac{N_p}{1{,}000}\right)^{0.2}}; \quad K_{max} \leq 5.0 \qquad (6)$$

$$K_{min} = 0.05\, N_p^{0.2}; \quad K_{min} \geq 0.3 \qquad (7)$$

in which Q_{max}, Q_{min}, and Q = the maximum, minimum, and average flow rates, respectively; K_{max} and K_{min} = the coefficients of maximum and minimum flow rates; and N_p = the populations served by the pipe. Based on Eqs. 3 through 7, the ratio between the maximum and minimum velocity is

$$\frac{V_{max}}{V_{min}} = \left(\frac{K_{max}}{K_{min}}\right)^{0.24} \qquad (8)$$

Assumptions.—Several assumptions must be made before the problem can be solved. These may relate to engineering practices which are not universal. They can, however, be removed or changed by making the appropriate modifications in the computer program. Some of the simplifications which were introduced are not inherent to the method used and can be removed at some cost in computer space, computation time, or programming effort. The assumptions and simplifications are:

1. The network serves a single drainage area, i.e., it operates entirely by gravity.
2. The network is tree-shaped, i.e., it contain no loops, with its final collection point (the sink) being specified in advance by the designer.
3. For every pipe of the network the direction of flow is fixed in advance. This may seem a rather restrictive assumption, whereas in reality one can in most cases fix the direction of flow from topographic considerations. When there is a pipe for which this assumption seems too limiting, one has to solve the problem twice, each time fixing the flow in a different direction. When

several such cases arise in one network the number of solutions needed to exhaust all combinations may be too large, and one has to use judgment in selecting the cases to be tried.

4. Costs are uniform throughout the drainage area, and all pipes are of the same material. Manholes are spaced uniformly along all pipes at fixed intervals. Their cost, which depends on the depth at which the pipe is laid, is added to the cost of the pipe.

5. The depth of the pipe is considered uniform throughout its length, and equals the average of its values at the upstream and downstream nodes.

6. Hydraulic losses at the nodes have been neglected. The crown of the pipe leaving the node was restricted to be lower than the crowns of all pipes entering the node, and this justifies neglecting the minor losses.

7. The minimum velocity is fixed for all pipes. Velocities are computed by Pomeroy's (7) approximate formula during the optimization. Once the optimal design has been determined, the velocities are computed accurately. This procedure saves a considerable amount of computer time and still does not modify the design that is found to be optimal.

8. Minimum cover for the pipes, i.e., the minimum depth from the ground surface to the crown, is fixed for all pipes.

9. The minimum diameter is 6 in., with sizes available above it at 2-in. increments.

10. The maximum size of an internal drop inside a node for which no extra cost is incurred is 35 cm. Beyond that there is an extra cost which is a function of the drop size.

11. All drop structures are at nodes.

12. Costs of the sewerage system include materials, trenching, laying, covering, manholes, and node structures.

Definitions.—The following definitions and abbreviations were used throughout the paper:

1. Node—A point at which a pipe or several pipes either start or end. All pipes terminating at a node drain into the node. Of the pipes starting at the same node only one carries out the discharge. The ground elevation at each node is known, and it is assumed to vary linearly between nodes.

2. Node Elevation (NE)—Invert elevation at the inlet to the pipe draining the node. Incoming pipes may each have a different outlet elevation, higher than the NE, with a drop occurring at the node.

3. Node Drainage Direction (NDD)—The direction (i.e., the downstream node) of the pipe leading out of the node. This direction is one of several possible drainage directions (PDD's) as demonstrated in Fig. 1. For example, flow enters node 5 from two pipes, and in turn 5 may be drained by a pipe leading to 7 or a pipe leading to 8. These are the two possible drainage directions, one of which is selected during the optimization (5-8 is the drainage direction of node 5 shown in Fig. 1).

4. Local and Main Pipes—A local pipe (LP) may start next to a node but does not connect to it. Its inlet elevation is not affected by the pipes leading into that node, but rather by considerations of the local drainage it has to provide along its route, and of the node it discharges into. A main pipe (MP)

is one which leads out of a node, and it also collects local drainage along its route. Referring to Fig. 1, if 2 is drained to 5 then 2-5 is a main pipe and 2-4 is a local pipe. Local pipes are needed to collect sewage along their route. For design purposes, all the flow a local pipe collects is assumed to enter it at the upstream end, and flow its entire length. The characteristics of each pipe, local or main, are: (1) The population it serves; (2) the discharge contributed by this population; (3) the pipe length; and (4) the nodes between which it lies.

5. Drainage Lines (DL)—These are imaginary lines used in the optimization. Each drainage line passes through all nodes which are at the same *link-distance* from the sink. Referring to Fig. 1, drainage line 6 passes through all the nodes which are one link away from the sink (node 15), i.e., nodes 13 and 14. Similarly, line 5 passes through the nodes which are two links away from 15 and line 1 through all the nodes removed 6 links from the sink. Thus, if there are a total of N drainage lines, then line n connects all nodes removed $(N - n)$ links from the sink. On line 1 are only those nodes into which no other nodes drain. On line N there is only one node: i.e., the sink. It follows from the preceding definition that nodes on drainage line n can drain only into nodes on line $(n + 1)$.

The selection of the drainage lines by the designer, which is a prerequisite to the optimization procedure, is restrictive only in that it fixes the link-distance

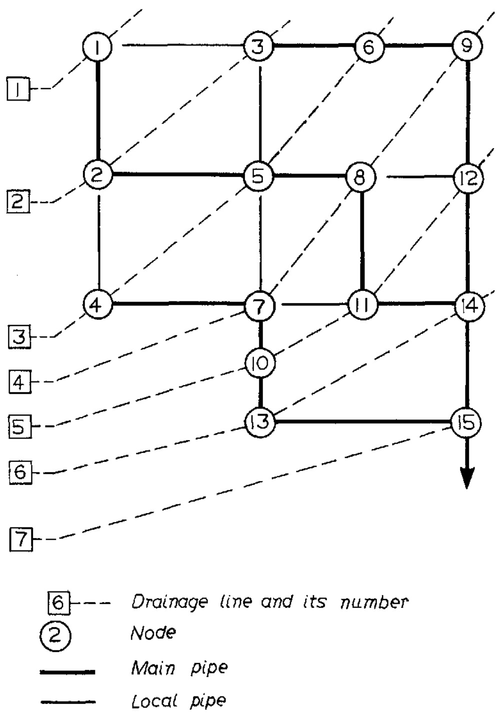

FIG. 1.—Sewerage Network

of each node. The rest of the topology of the network, i.e., the way in which the "tree" is constructed, is still to be determined by the optimization procedure. Fixing the link-distance of each node is actually acheived by fixing the direction of flow in each pipe. The two are thus analogous and should be viewed as a single assumption.

Optimization Problem

The problem to be solved is: find the layout, pipe diameters, elevations and slopes, which will minimize the total cost of the network, subject to given constraints.

The objective function is

$$\underset{T_i, Hu_i, Hd_i}{\text{Min}} \left[\sum_{\text{all pipes}} CP_i(D_i, HU_i, Hd_i, T_i) + \sum_{\text{all drops}} CD_i(D_i) \right] \quad \ldots \ldots \ldots (9)$$

in which $T_i = 1$ when pipe i is a main pipe, $T_i = 0$ if it is a local pipe; Hu_i and Hd_i = the upstream and downstream elevations, respectively, of the ith pipe invert; D_i = ith pipe diameter; CP_i = cost of the ith pipe; and CD_i = cost of the drop structure at the end of the ith pipe.

The minimization is over the connectivity of the network, T, and the elevations. Pipe diameters are dependent on the elevations. The constraints are:

$$D_i \in \bar{D} \quad \ldots \ldots \ldots (10)$$

in which D_i belongs to the set of available diameters, $\bar{D}$, and

$$D_i \geq \frac{k_m (n_d K_{\max i} Q_i)^{0.375}}{S_i^{0.1875}} \quad \ldots \ldots \ldots (11)$$

This constraint, based on Eq. 1, insures that the commercial diameter selected, will be adequate hydraulically.

The velocities are constrained as follows:

$$\max V_i \leq V_{\max}; \quad \text{for} \quad T_i = 1 \quad \ldots \ldots \ldots (12)$$

$$\min V_i \geq V_{\min}; \quad \text{for} \quad T_i = 1 \quad \ldots \ldots \ldots (13)$$

$$\max V_i \geq V_{\min}; \quad \text{for} \quad T_i = 0 \quad \ldots \ldots \ldots (14)$$

The constraints in Eqs. 12 and 13 insure velocities in the admissible range for a main pipe, and in Eq. 14, insures a self-cleaning velocity at least once a day in a local pipe. The maximum and minimum velocities are computed from Eqs. 3 and 8, respectively. An additional constraint is

$$NE_j \leq Hd_i; \quad \text{for} \quad i \in I_j \quad \ldots \ldots \ldots (15)$$

which requires the node elevation to be lower than the downstream elevations of all pipes i, denoted by the set, I_j, which drain into node j. Finally, exactly one pipe may lead out of a node.

Method of Solution

The solution is obtained by dynamic programming (2,6). The independent decision variables are the drainage directions of all nodes and the upstream and downstream elevations of all pipes. Pipe diameters are also decision variables, but they depend directly on the elevations and drainage directions. The solution proceeds in stages; in this case from one drainage line to the next. The recursive equation for the optimization, once the drainage lines have been decided, is

$$F^*_{n+1}(H^{n+1}) = \operatorname*{Min}_{\mathbf{H}^n} \left\{ F^*_n(H^n) + \sum_i \operatorname*{Min}_{T(i,n+1)} f^*_i \, [H^n_i, H^{n+1}, T(i, n+1)] \right\} \quad (16)$$

in which $\mathbf{H}^n$, $\mathbf{H}^{n+1}$ = vector of quantized node elevations of drainage lines n and $(n + 1)$, respectively; H^n_i = elevation of node i on line n; $\mathbf{T}(i, n + 1)$ = vector of connectivity between node i on line n and nodes on line $(n + 1)$. It is a vector whose length is equal to the number of possible drainage directions from node i. It has a 1 in the position of the main line draining node i and zeroes for all other lines which originate near node i but are local lines; $f^*_i \, [H^n_i, H^{n+1}, T(i, n + 1)]$ = the cost of the cheapest feasible pipes lying between node i on line n and nodes on line $(n + 1)$, when all node elevations and the drainage direction of node i have been fixed; $F^*_n(H^n)$ = the cost of the optimal network ranging from the 1st to the nth drainage line, for fixed values of the node elevation, $\mathbf{H}^n$ on drainage line n; and $F^*_{n+1}(H^{n+1})$ = same for line $(n + 1)$.

In order to clarify the details of the solution process, each operation of Eq. 16 is considered separately.

First the computation of f^*_i and its minimization over the possible drainage directions of node i. Term f^*_i is in itself a least-cost (optimal) solution because even when all node elevations on the two drainage lines and the drainage direction from node i are fixed, it is still necessary to determine upstream and downstream elevations of all the pipes which have i as their upstream node—compatible with the node elevations—which yield the least cost. For each pipe in this group separately one seeks the upstream and downstream elevation, pipe size, trench depth, and drop structure. Term f^*_i is the sum of these costs. The operation, $\text{Min}_{\mathbf{T}(i,n+1)} f^*_i \, [H^n_i, H^{n+1}, T(i, n + 1)]$ is performed by simply trying all feasible vectors $\mathbf{T}(i, n + 1)$, i.e., all possible drainage directions from node i, to determine the optimal. This is done separately for each node i, and the results are added to give the summation term in Eq. 16. If there are m nodes on line n, and each has p possible drainage directions, the total number of cases examined in forming the summation is only $m \times p$, even through there are p^m possible combinations from all nodes on line n to line $(n + 1)$. This is because the optimal drainage direction of each node is independent of those of the other nodes on the same drainage line.

Next, consider the operation of minimizing the right-hand side over all feasible values of $\mathbf{H}^n$. This is done for a given value of $\mathbf{H}^{n+1}$, and one thus has to consider as feasible only elevations on line n which are greater than those fixed on $\mathbf{H}^{n+1}$. This is due to the fact that flow from line n to $(n + 1)$ is by gravity. For each $\mathbf{H}^n$ which is feasible with the fixed $\mathbf{H}^{n+1}$, one performs the internal sum, Σ_i Min f^*_i, and adds to it the value of $F^*_n(H^n)$ which has been stored in a table in the previous stage of the process. One ranges over all feasible $\mathbf{H}^n$ and finds the one that gives the minimum. This value is now stored in a table of $F^*_{n+1}(H^{n+1})$; on entry for each vector point $\mathbf{H}^{n+1}$, together with a pointer indicating which value of H^n it was that gave the minimum. In the next stage, $F^*_{n+1}\ (H^{n+1})$ is used on the right-hand side of Eq. 16 in computing $F^*_{n+2}\ (H^{n+2})$. The process is continued until the last drainage line, where $F^*_N(H^N)$ is computed for the outlet node (the sink). Of all the values of F^*_N the best is selected, and the associated outlet elevation is $(H^N)^*$. This completes the "forward" part of the process.

Next, using the tables of pointers one determines the optimal node elevations as follows: going "backward," i.e., from stage N to stage 1, $(H^n)^*$ is that value of $\mathbf{H}^n$ which caused $F^*\ [(H^{n+1})^*]$ to be optimal. Having thus determined $(H^N)^*$, $(H^{N-1})^*$, ..., $(H^3)^*$, $(H^2)^*$ the optimal network has been found.

To summarize, the procedure starts by setting $\mathbf{H}^1$ to the highest possible values, as determined by the minimum cover for the pipe. These values are taken because it is obvious that there is nothing to be gained by putting the upstream ends deeper than absolutely necessary. Associated with this $\mathbf{H}^1$ is also $F^*_1(H^1) \equiv 0$, as there is no cost upstream of this line. One then computes $F^*_2(H^2)$, using Eq. 16, for each feasible $\mathbf{H}^2$. At this stage only the internal minimization is performed, as $\mathbf{H}^1$ is fixed. Next one computes $F^*_3(H^3)$ for each feasible $\mathbf{H}^3$, ranging over feasible $\mathbf{H}^2$; i.e., allowing for gravity flow between lines 2 and 3. A pointer is stored, along with $F^*_3(H^3)$, to indicate which $\mathbf{H}^2$ gave the optimum. The process is continued in the same way for all stages (drainage lines) until $F^*_N(H^N)$ has been found. Its minimum is the least cost of the entire network, and it is associated with the optimal elevation $(H^N)^*$. Now the pointers are used to find the optimal node elevations $(H^n)^*$ for $n = (N - 1), ..., 2$.

During the "forward" phase, one stores at each stage the flow rate in each pipe and the population it serves, as a function of the node elevations. These values are then used to compute the flow rate and population served by pipes of the next stage. The flow rates and population figures are given by the recursive relations:

$$Q^*_{j,n+1}\ (H^{n+1}) = \sum_i Q^*_{i,n}\ (H^n)\ T_i + Q_{ij} \quad \text{. . . . .} \quad (17)$$

$$N^*_{j,n+1}\ (H^{n+1}) = \sum_i N^*_{i,n}\ (H^n)\ T_i + N_{ij} \quad \text{. . . . .} \quad (18)$$

in which i = a node on line n and j on line $(n + 1)$; $Q^*_{i,n}\ (H^n)$ = the flow rate out of node i; T_i = 1 only if the pipe from i to j is a main pipe, T_i = 0 otherwise; Q_{ij} = the flow rate serviced by the pipe between nodes i and j, whether local or main; $Q^*_{j,n+1}\ (H^{n+1})$ = the flow rate out of node j; and similar definitions relating to population, N, in Eq. 18.

Computation Time and Storage Space

Assume that there are m nodes on each of the $(N - 1)$ drainage lines, each having p possible drainage directions. If the continuous elevations at each node are quantized into t values, then the term in braces in Eq. 16 has to be evaluated $[(N - 1)\ p\ m\ t^{2m}]$ times in the "forward" phase. For example, for m = 5, p = 3, t = 4, and N = 10 there are a total of $[(10 - 1)\ 3\ 5\ 4^{10}]$ = 1.5

10^8 evaluations to be performed. Each entails the determination of $\Sigma_i \min_{T(i,n+1)} f_i^*$, in itself comprising quite a few operations. Thus, if it is assumed, e.g., that one evaluation takes 10^{-5} sec on a high speed computer, some 25 min of computer time are necessary to solve a problem of size given in the preceding example (the "backward" phase takes very little time, as it contains mostly logical operations). If in the preceding example, one increases t from 4 to 5, the computation time increases by a factor of 10; whereas, if t is decreased to 3, computation time decreases by a factor of 18. The other important parameter is obviously m. Although t is to be selected by the designer, due consideration being given to the balance between computation time, storage space (reviewed subsequently), and the resultant accuracy of the solutions, m is fixed for the problem. Nevertheless, it is relevant to try to decrease m by judiciously separating the network into subnetworks and solving them separately. This method, which should be based on experience and engineering judgment, is beyond the scope of this presentation.

More crucial than computation time is storage space. When $F_{n+1}^*(H^{n+1})$ is being computed by Eq. 16, it is necessary to access the table of $F_n^*(H^n)$ many times, and it, therefore, has to reside in the computer's high speed core memory. Space has also to be allocated to the table of $F_{n+1}^*(H^{n+1})$ which is being filled. During this stage one also makes frequent use of the tables of $Q^*(H^n)$ and $N^*(\mathrm{H}^n)$ to construct the tables of $Q^*(H^{n+1})$ and $N^*(H^{n+1})$ by Eqs. 17 and 18. After the tables for line $(n + 1)$ have been filled, those of line n are not needed for the remainder of the "forward" phase, and can thus be transferred to lower speed secondary memory (e.g., disk). The tables for line $(n + 1)$ are now shifted to occupy the vacated core space and used in computing the tables for line $(n + 2)$. Thus one needs core space for six tables, their dimension being given by the number of points in **H**. If there are $m = 5$ nodes on each line, and at each node the elevation can take on $t = 4$ discrete values, the total number of entries in the tables is $6\ 4^5 = 6{,}166$ words of storage. To this one must add several tables for pointers and for the decision variables, yielding a very high space requirement for even a modest size problem.

Analysis of Optimization Procedure

As can be seen from Eq. 16, F_n^* depends only on the elevations $\mathbf{H}^n$, whereas it should also be a function of the connectivities, $\mathbf{T}(n, n + 1)$, between all nodes on line n and nodes on line $(n + 1)$. A formulation more complete than that of Eq. 16 would have been

$$F_{n+1}^*[H^{n+1}, T(n, n+1)] = \operatorname*{Min}_{\mathbf{H}^n} \Big\{ F_n^*[H^n, T(n-1, n)] + \sum_i \operatorname*{Min}_{T(i,n+1)} f_i^*[H_i^n, H^{n+1}, T(i, n+1)] \Big\} \quad (19)$$

with similar modifications being made to Eqs. 17 and 18. For $m = 5$ nodes on each line, each having $\hat{p} = 3$ possible drainage directions the formulation according to Eq. 19 would require $3^5 = 250$ times as much computer space as does the formulation according to Eq. 16. This is obviously not feasible, and therefore, the more restricted formulation was used. This may lead to a suboptimal solution, but for practical reasons one cannot do better with the use of a dynamic programming formulation. One of the conclusions of this work is that other methods should be attempted, because although the answers obtained by the method reported herein are useful, as will be demonstrated subsequently, computer space and computation time requirements restrict the applicability of the method.

Computer Program

The program was written in FORTRAN IV, and run on the IBM 360/50. The program made use of compact storage of integers (INTEGER*2) to save space, and maintained all tables except the ones in current use on a disk. The program was limited to problems with $N = 10$ drainage lines, each having up to $m = 5$ nodes, with up to $p = 3$ possible drainage directions from each node. The maximum number of discrete elevations tested at each node is $t = 4$. In this form, the program uses 50,000 words (~400KB) of core and another 500,000 words (~4MB) of disk space. The listing of the program is contained Ref. 1.

Application of Proposed Method

The aforementioned optimization procedure was first applied to a hypothetical network in order to test its usefulness, and to carry out a sensitivity analysis. It was later used to design a portion of a real sewerage system. Cost data were obtained from local consulting firms and expressed by third degree polynomials. Thus, the following cost functions were obtained: (1) Unit cost of pipe as a function of diameter; (2) unit cost of trenching as a function of depth and the pipe diameter; (3) cost of manholes as a function of depth and type of soil; and (4) cost of drops as a function of pipe diameter.

Application to Hypothetical Network.—This network consisted of 10 nodes, arranged on five drainage lines. The problem was solved under various conditions (over 20 runs in all), from which the following was concluded:

1. When the ground surface is steep the optimal network is practically parallel to the surface—provided the velocity constraints do not become binding—and the optimization is primarily of the layout. As the surface slope becomes milder there is increasing dependence of the optimal node elevations on minimum velocity, ratio of pipe cost to trenching cost, and the number of elevations examined at each node.
2. As the cost of trenching increases the optimal network tends to be as close as possible to the surface.
3. The optimal solution tends to allocate larger slopes to pipes in the downstream section of the system than to pipes in the upstream sections. This is due to the fact that discharges are larger in the downstream section, and a large slope there yields savings which more than outweigh the extra cost incurred in decreasing the slope of upstream pipes. The effect is that the optimal system has typically more trenching for the larger pipes.
4. The minimum and maximum velocity constraints have most effect when

the terrain is either very steep or very mild. If the V_{max} constraint becomes binding for some pipe this has only a localized effect, i.e., it changes the design parameters of that pipe (reduces its slope and increases the drop at its end), but does not affect the other pipes. On the other hand, if the V_{min} constraint becomes binding for some pipe it will affect this pipe and possibly all others downstream from it, since the pipe's slope has to be increased, lowering its downstream elevation.

5. The design of short pipes is more sensitive to the size of the elevation increments checked at each node, since a fixed size increment represents a greater change in slope for a shorter pipe.

Application to Real Sewerage System.—The optimization program was used to design the sewerage network of an urban region, serving a population of

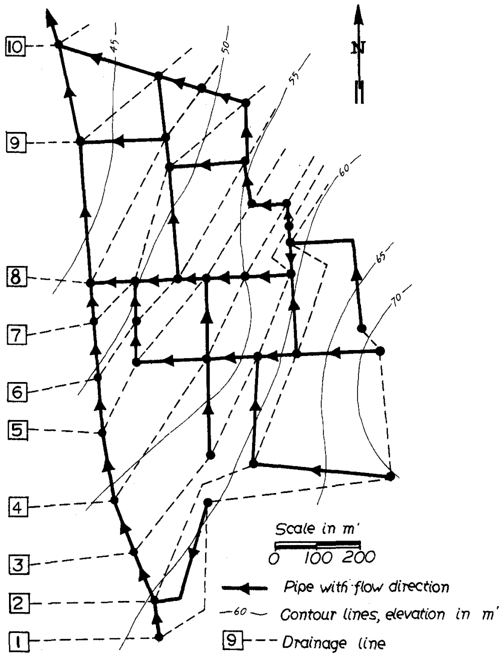

FIG. 2.—Map of Sewerage Area with Preliminary Layout

40,000. A map of the area is shown in Fig. 2. The map includes the topography, flow directions, and the drainage lines. The solution obtained by the optimization program is depicted in Fig. 3. Due to the topography, a main collector was obtained along the western boundary of the region. A similar solution was obtained by a local consultant who designed the system by conventional methods. In a second run, an attempt was made to change this solution artificially by changing the direction of flow in some of the lateral pipes so that an additional main collector will be formed. The solution thus obtained was less economical. The transition from one set of initial conditions to another proved to be quite simple.

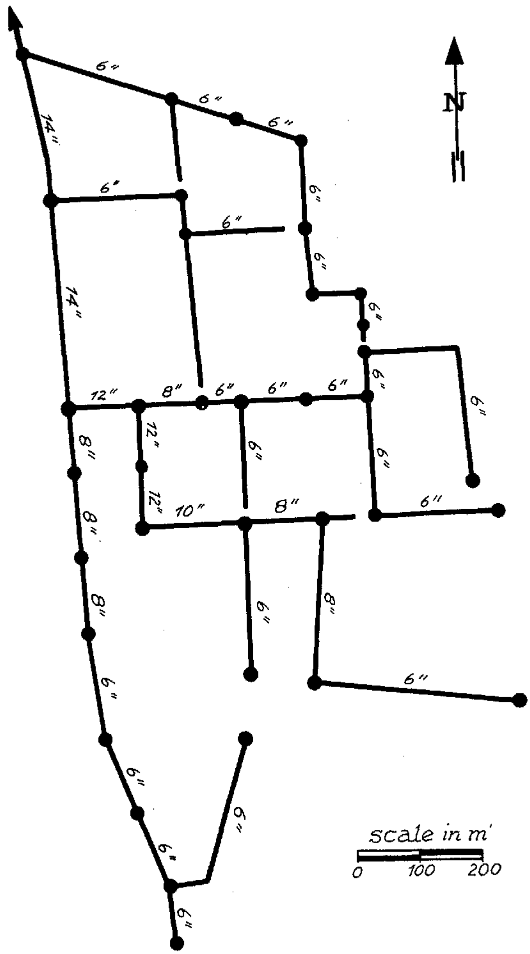

FIG. 3.—Optimal Sewerage Network

This justifies running the program several times in order to obtain the optimal solution.

Summary and Conclusions

The problem of obtaining an optimal design of a sewerage network has been solved, in principle. The solution was obtained by employing the dynamic programming technique as formulated in Eq. 19. Due to limitations in computer space and computation time, the applicability of this solution is restricted to very small networks. A more practical, although not as perfect, solution has been proposed as expressed by Eq. 16. In this solution a suboptimal design is obtained. The usefulness of this optimization procedure was demonstrated by its application to hypothetical and real sewerage systems. The main shortcoming of the method is the need for large computer space and long computation time, as the dimensions of the network increase. These restrictive requirements are inherent in the dynamic programming technique, and cannot be avoided unless a different approach is adopted. On the other hand, dynamic programming enables complete freedom in selecting the objective functions, constraints, cost functions, etc. In preparing for a solution of a particular network, the user has to make several preliminary assumptions. These include the selection of the nodes, drainage lines, and direction of flow in each pipe. It was found that the logical starting network is quite easily selected in most cases. Where difficulties in making these assumptions are encountered, several alternative starting networks must be tried. Once a solution has been obtained for a given network, the transition to another set of starting conditions is relatively simple.

In conclusion, it may be stated that the optimization procedure developed in this work can be applied successfully to the design of small sewerage systems. Large systems may be decomposed to smaller subsystems, each of which is optimized internally, and later combined to a single optimal network. Development of more advanced computers may render the method more applicable to large systems. Until such equipment is available, the optimization of large sewerage systems should be attempted by other methods.

Appendix I.—References

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Appendix II.—Notation

The following symbols are used in this paper:

C = ratio of actual flow rate to flow rate of pipe flowing full;
CP = cost of pipe;
CD = cost of drop structure;
D = diameter of pipe;
$\bar{D}$ = set of available pipe diameters;
DL = drainage line;
F_n^* = cost of optimal network from 1st to nth drainage line;

f_i^* = cost of cheapest feasible pipes between node i to nodes on next drainage line;
H = vector of quantized node elevation on drainage line;
Hd, Hu = downstream and upstream elevations;
I_j = set of all pipes which drain to node j;
K_{max}, K_{min} = *coefficient of maximum and minimum flow rate;*
k_n = numerical constant;
k_p = numerical constant;
LP = local pipe;
MP = main pipe;
m = number of nodes on drainage line;
N = total number of drainage lines;
NDD = node drainage direction;
NE = node elevation;
N_{ij} = population served by pipe between nodes i and j;
$N_{i,n}^*$ = population out of node i;
N_p = population;
n_d = apparent Manning's coefficient;
n_m = Manning's coefficient;
PDD = possible drainage direction;
p = number of possible drainage directions from node;
Q, Q_{max}, Q_{min} = average, maximum, and minimum flow rates;
Q_{ij} = flow rate serviced by pipe between nodes i and j;
$Q_{i,n}^*$ = flow rate out of node i;
S = slope;
T = connectivity vector;
t = number of elevations checked in node; and
V, V_{max}, V_{min} = average, maximum, and minimum velocity.